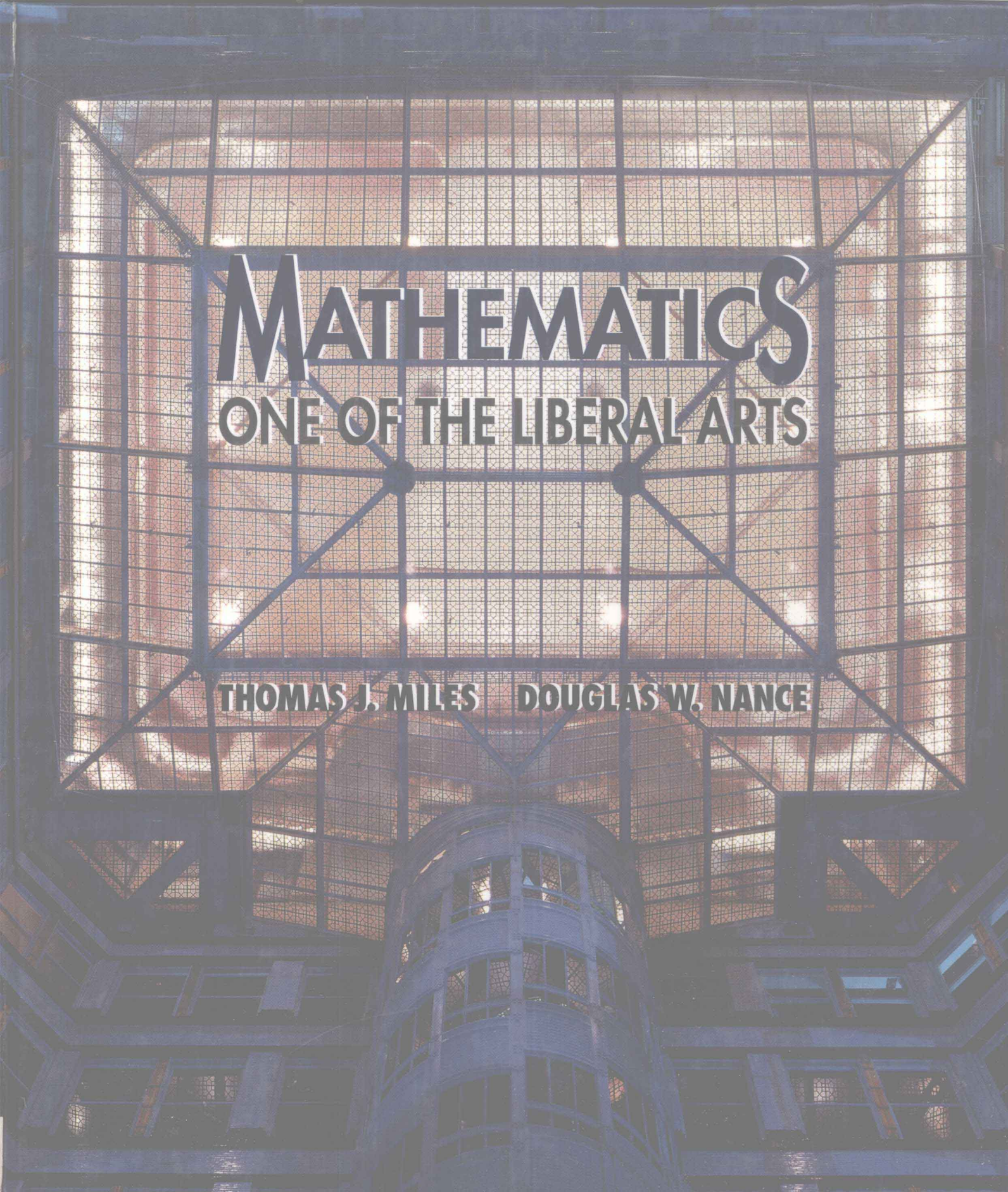
MATHEMATICS
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Copyediting
Elliot Simon, Simon and Associates

Interior Design
Diane Beasley

Page Layout and Cover Design
DeNee Reiton Skipper

Photo Research
Kathy Ringrose

Cover Image
Restored historic skylight of the Rookery Building. Owner: Baldwin Development Company. Restoration architect: McClier Corporation. Photo by: Nick Merrick of Hedrich Blessing.

Etymologies
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Production, Prepress, Printing and Binding by West Publishing Company

Text and Photo Credits Follow Index

FOR MORE INFORMATION, CONTACT

BROOKS/COLE PUBLISHING COMPANY
511 Forest Lodge Road
Pacific Grove, CA 93950
USA

International Thomson Publishing Europe
Berkshire House 168-173
High Holborn
London WC1V 7AA
England

Thomas Nelson Australia
102 Dodds Street
South Melbourne, 3205
Victoria, Australia

Nelson Canada
1120 Birchmount Road
Scarborough, Ontario
Canada M1K 5G4

International Thomson Editores
Campos Eliseas 385, Piso 7
Col. Polanco
11560 México D. F. México

International Thomson Publishing GmbH
Königswinterer Strasse 418
53227 Bonn
Germany

International Thomson Publishing Asia
221 Henderson Road
#05-10 Henderson Building
Singapore 0315

International Thomson Publishing Japan
Hirakawacho Kyowa Building, 3F
2-2-1 Hirakawacho
Chiyoda-ku, Tokyo 102
Japan

∞

British Library Cataloguing-in-Publication Data. A catalogue record for this book is available from the British Library.

Printed in the United States of America

04 03 02 01 00 99 98 97 8 7 6 5 4 3 2

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Miles, Thomas J.
Mathematics : one of the liberal arts / Thomas J. Miles, Douglas W. Nance.
p. cm.
Includes bibliographical references and index.
ISBN 0-314-09576-4 (hard : alk. paper)
1. Mathematics. I. Nance, Douglas W. II. Title.
QA39.2.M548 1997
510—dc21 96-29496
CIP

To my family.
They stuck with me while
Mathematics: One of the Liberal Arts
was one of the laboral arts.
TJM

To Helen.
She understands, accepts, and
participates in the seemingly endless
process of textbook preparation.
DWN

PREFACE

This text is written for students who are taking mathematics for liberal arts or general education purposes, including quantitative literacy and mathematics competency. A variety of courses, some described below, can be taught from its contents. We presume students have taken at least one year of high school algebra. A year of geometry is preferred, but not presumed.

OVERVIEW

We have included topics that many college mathematics teachers believe are important. We develop some algebra, geometry, and set theory topics that students may have seen before. We then extend these topics to material students probably have not seen such as population growth, non-euclidean geometry, fractal geometry, and power in weighted voting. In addition to basic concepts that review high school mathematics, the text presents mathematics as an integral part of the development of intellectual ideas and as a subject which is used to model real world situations in almost every discipline.

STATEMENT OF PHILOSOPHY

The present day liberal arts have their roots in the *quadrivium* of the ancient Greeks. It included arithmetic and geometry. Through 25 centuries a liberal arts background has been regarded as an essential component of a well-rounded education and mathematics has been regarded as a vital part of that liberal arts background. In order to emphasize this, several features have been included. They are:

- *Doing mathematics.* Students are encouraged to become active learners by formulating some concepts themselves, either cooperatively in groups or individually. Nearly every section has one or more "Your Formulation" activities to encourage this. In addition, numerous exercises are included for doing mathematics.
- *Historical development.* Topics are presented in the context of their historical development.
- *Cultural development.* How have societal changes prompted or hindered mathematical development? How have mathematical developments affected society?
- *Human ideas.* Mathematics is something developed by people. We describe those people and we describe mathematics as a creative human activity.

- *Written assignments.* The formation and subsequent expression of ideas is fundamental to educated individuals. Learning is assisted by expressing what we know. Accordingly, we have included suggestions for written assignments throughout the text.

MAJOR THEMES

Problem solving is a major objective of students' mathematical experiences. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* and related publications put problem solving as a main objective of K–12 mathematics instruction. The report *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus,* published by the American Mathematical Association of Two-Year Colleges (AMATYC) in 1995, has as Standard I-1, "Students will engage in substantial mathematical problem solving." For students who have already developed problem solving skills, this book offers material to add to those skills. For those who have not worked on developing problem solving skills, they can begin here. The book begins with two sections on problem solving. Throughout the book students are presented with a number of new and unfamiliar situations in which they are asked to apply problem solving skills in order to arrive at a solution.

Mathematical modeling is synonymous with applications of mathematics. The authors have found that students respond much better to modeling-oriented mathematics than to just extending their algebra, geometry, trigonometry, or mathematical functions development. In the report *Quantitative Reasoning for College Graduates: A Complement to the Standards,* approved by the Committee on the Undergraduate Program (CUPM) of the Mathematical Association of America (MAA), conclusion 2 is that "Colleges and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real-world problems." AMATYC's Standard I-2 says, "Students will learn mathematics through modeling real-world situations." Accordingly, we have developed basic mathematical concepts and then used them to model various real world situations. Section 1.3 is devoted to mathematical modeling, but ideas of modeling are used throughout the book.

POSSIBLE COURSES

The material in this book and the above emphases can be organized in several different ways to fit several types of courses.

- *Mathematical modeling.* Selections from Sections 1.3 (Mathematical Modeling), 2.5 (Check Digits), part of 3.2 on weighted voting, 3.6 (Applications of Sets), much of Chapter 5 (Algebra), Chapter 8 (Probability), Chapter 9 (Statistics), Chapter 10 (Consumer Mathematics), Chapter 11 (Discrete Mathematics), and Chapter 13 (Mathematics and the Other Liberal Arts).
- *Quantitative literacy.* The MAA's *Quantitative Reasoning for College Graduates* makes clear that a required mathematics course or two is not sufficient for quantitative literacy, but the report lists five capabilities desired in a quantitatively literate college graduate (p. 10). These capabilities can be developed by material in Chapter 1 (Problem Solving and Mathematical Modeling), Section 3.6 (Applications of Sets), Chapter 5 (Algebra), Chapter 6 (Geometry), and Chapter 9 (Sta-

tistics). In addition, such a course should "immerse [students] in doing quantitative reasoning of a nonroutine nature." (p. 14) Such topics can be selected from Section 2.5 (Check Digits), Chapter 11 (Discrete Mathematics), and parts of Chapter 13 (Mathematics and the Other Liberal Arts).

- *Mathematical competency.* Chapter 1 (Problem Solving and Mathematical Modeling), Chapter 3 (Sets), Chapter 4 (Logic), Chapter 5 (Algebra), and Chapter 6 (Geometry).
- *Survey of mathematics.* Topics selected from any of the chapters.
- *Historical or cultural approach to mathematics.* Chapter 2 (Numbers and Numerals), Section 4.1 (Introduction [to Logic]), part of Section 5.1 ([Algebra as] Generalized Arithmetic), Section 5.4 (Further Developments in Algebra), Section 6.1 (Overview and History [of Geometry]), part of Section 7.1 (Geometry as an Axiomatic System), Section 7.2 (Non-Euclidean Geometry), part of Section 7.3 (Fractals), Section 8.1 (Historical Background [of Probability]), much of Chapter 12 (Mathematics and Computers), parts of Chapter 13 (Mathematics and the Other Liberal Arts).

FEATURES

The text has the following features designed to motivate and aid learning.

- *Prologue.* Each chapter begins with a prologue designed to pique the student's interest by relating some of the material in the chapter to a concept that the student might not have associated with the chapter material.
- *Etymology.* Each chapter contains the history of the key word or words in the chapter title. The authors are grateful to Steven Schwartzman and the MAA for permission to use lightly edited etymological references from Schwartzman's delightful book *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English.*
- *Goals and Objectives.* Each section begins with a statement of up to four goals for the section. The instructor's manual contains a more detailed list of objectives. It can be modified to meet the objectives of your course. This modified list can serve as a study guide for students.
- *Your Formulation.* Most sections have one or more "Your Formulation" activities in which students are asked to take an active part in formulating concepts. These are ideal for cooperative learning activities.
- *Margin Notes.* These are brief comments relating to adjacent text material. Some are notes on people, some are light verse, and some take other forms. All are intended to help keep mathematics interesting.
- *On a Tangent.* These are the comments that take off from the text's comment about a person or concept in the same way that a tangent to a curve takes off from a curve. They are longer than the Margin Notes.
- *Exercises.* The exercises are designed to have a number of routine computational-type exercises as well as some applications that go beyond the examples and some exercises that challenge the student's problem solving ability.
- *Written Assignments.* The written assignments include exercises for students to summarize material that has been covered, to write about their personal reactions to material they have studied, to debate some issues, and to do research on historical and cultural developments and applications in mathematics. Numerous teachers have reported on the advantages of having students write to learn mathematics.
- *Chapter Review Exercises.* At the end of each chapter are review exercises. These are directed at measuring the student's mastery of the objectives for the chapter.

- *Bibliography.* A bibliography is included at the end of each chapter. It includes references to books and articles that (1) have been cited in the chapter, (2) have served as background material, of (3) can serve as references for further development.
- *Color.* Extensive use of color helps accent concepts and make the book more attractive.
- *Liberal arts look.* Headings and borders are designed to emphasize visually the location of mathematics within the liberal arts.

It is our viewpoint that the best liberal arts mathematics courses are those in which the instructor is really excited about the material being presented. Students catch that excitement either from the teacher or from a sense of the value of the course to them. We hope that this textbook can be the reference for conveying excitement about knowledge and ways of knowing that should be part of every liberally educated person.

ANCILLARIES

The following supplementary materials are available to assist instruction and learning.

- *Instructor's manual.* This contains a detailed list of objectives for each section. These often expand upon the goals printed at the start of each section in the textbook. It also contains considerations in teaching material in a section, including suggestions for cooperative learning activities.
- *Test bank.* This is available in print and computerized form.
- *Color acetates.* This set contains 150 key figures from the text.
- *Student solutions manual.* This contains worked out solutions to the even-numbered exercises in the text.

ACKNOWLEDGEMENTS

This book is shaped, in part, by the thoughtful suggestions of the following reviewers, to whom we offer our thanks.

H. Rux Allonce
Miami Dade Community College
Jeffrey S. Allbritten
Middle Tennessee State University
George Bradley
Duquesne University
Janis M. Cimperman
St. Cloud State University
Lisa DeMeyer
University of North Carolina—Chapel Hill
Joe S. Evans
Middle Tennessee State University
Marc Franco
South Seattle Community College
Gary Grabner
Slippery Rock University
John S. Haverhals
Bradley University
Evan Innerst
Canada College
Howard Jones
Lansing Community College
Maureen Kelley
North Essex Community College
Thomas Lada
North Carolina State University

Anne F. Landry
Dutchess Community College
Tom Linton
Willamette University
Lee McCormick
Pasadena City College
Janice McFatter
Gulfcoast Community College
Donna Massey
Talahassee Community College
Gael Mericle
Mankato State University
Harald M. Ness
University of Wisconsin–Center
Sunny Norfleet
St. Petersburg Junior College
Stephen K. Prothero
Willamette University
Joe Rappoport
Miami-Dade Community College
Dennis Reissig
Suffolk Community College
Judith Rice
Viollanova University
Stewart M. Robinson
Cleveland State University
Howard L. Rolf
Baylor University
Marsha Self
El Paso Community College–Rio Grande Campus
Henry Mark Smith
University of New Orleans–Loyola University
Raymond F. Smith
Whittier College
Litsa St. Amand
Mesa Community College
Joseph F. Stokes
Western Kentucky University
Hugo S.H. Sun
California State University–Fresno
Richard G. Vinson
University of Southern Alabama
Bruce Williamson
University of Wisconsin–River Falls
Jim Wooland
Florida State University

Jerry Westby, Executive Editor, prompted the writing of this book, saw it through the review process, encouraged us, aided us with his keen sense of what is useful to students and instructors, and served as the monitor of good taste. His personal perspective is of mathematics as one of the liberal arts.

Deanna Quinn, Senior Production Editor, has handled the production of the book from copyediting, through photo and art review, to its finished form with affability, helpfulness, patience, and a higher regard for pedagogical concerns than ease of production. One of us has worked with her before and was delighted to work with her again; the other of us is writing his first book and can't imagine a better production editor.

Elliot Simon, Copyeditor, was thorough and creative in removing grammatical flaws and overreliance on certain words and phrases. Mathematics is sometimes difficult to read. Elliot's work has made it easier by polishing our writing.

Halee Dinsey, Developmental Editor, has provided numerous suggestions and background material for consideration as well as summarizing suggestions for reviewers. In the early stages of writing, Betsy Friedman, Developmental Editor, compiled and analyzed reviewer responses and offered suggestions.

Helen Nance has provided efficient and insightful assistance in all phases of the textbook-preparation process from word-processing early versions of the manuscript to proofreading galleys. Her keen eye for detail and her sense of what looks right have greatly assisted the production process.

To all the people named above we offer our thanks. All of you have played a role in shaping this book.

Thomas J. Miles

Douglas W. Nance

BRIEF CONTENTS

CHAPTER 1 PROBLEM SOLVING AND MATHEMATICAL MODELING 1
CHAPTER 2 NUMBERS AND NUMERALS 19
CHAPTER 3 SETS 65
CHAPTER 4 LOGIC 105
CHAPTER 5 ALGEBRA 153
CHAPTER 6 FUNDAMENTALS OF GEOMETRY 215
CHAPTER 7 TOPICS IN GEOMETRY 261
CHAPTER 8 PROBABILITY 295
CHAPTER 9 STATISTICS 337
CHAPTER 10 CONSUMER MATHEMATICS 389
CHAPTER 11 DISCRETE MATHEMATICS 423
CHAPTER 12 MATHEMATICS AND COMPUTERS 473
CHAPTER 13 MATHEMATICS AND THE OTHER LIBERAL ARTS 531
ANSWERS TO SELECTED ODD-NUMBERED EXERCISES 573
INDEX 609

CONTENTS

PREFACE xvii

CHAPTER 1

PROBLEM SOLVING AND MATHEMATICAL MODELING 1

Prologue 1

1.1 INTRODUCTION TO PROBLEM SOLVING 2
Goals 2
What Is Problem Solving? 2
Historical Development 2
Psychological Aspects 3
What Constitutes a Problem? 4
What Is Not a Problem? 4
Written Assignments 1.1 5

1.2 MODERN HEURISTICS AND SOME EXAMPLES 6
Goal 6
Exercises 1.2 10

1.3 MATHEMATICAL MODELING 12
Goals 12
Exercises 1.3 15
Written Assignments 1.3 16

Chapter 1 Review Exercises 16
Bibliography 17

CHAPTER 2

NUMBERS AND NUMERALS 19

Prologue 19

2.1 ANCIENT NUMERATION SYSTEMS 20
Goals 20
Egyptian Hieroglyphics 22
Roman Numerals 23
African Numeration Systems 25
Hindu-Arabic Numerals 26
Exercises 2.1 28
Written Assignments 2.1 29

2.2 DIFFERENT BASES 29
Goals 29
Computer Numeration Systems 33
Application: The Game of Nim 35
Exercises 2.2 36
Written Assignments 2.2 37

2.3 FRACTIONS AND DECIMALS 38
Goals 38
Unit Fractions 38
Comparing Fractions 40
Development of Decimals 40
Converting from a Decimal to a Fraction 41
Exercises 2.3 42
Written Assignments 2.3 43

2.4 NUMBER SYSTEMS 43
Goals 43
Natural Numbers 44
Integers 45
Rational Numbers 46
Real Numbers 47

The Pythagoreans 48
Complex Numbers 50
Exercises 2.4 52
Written Assignments 2.4 53

2.5 CHECK DIGITS 54
Goals 54
U.S. Postal Service Scheme 54
Universal Product Code (UPC) 56
International Standard Book Number (ISBN) 57
Comparison of Schemes 59
Summary 59
Exercises 2.5 61
Written Assignments 2.5 62

Chapter 2 Review Exercises 62
Bibliography 64

CHAPTER 3

SETS 65

Prologue 65

3.1 BASIC IDEA AND NOTATION 66
Goals 66
History 66
Notation and Terminology 67
Exercises 3.1 70
Written Assignments 3.1 70

3.2 SUBSETS 71
Goals 71
Definition and Notation 71
Universal Set 72
Set Equality 72
Proper Subsets 73
An Application to Classification 73
Number of Subsets 73
Conjectures and Inductive and Deductive Reasoning 75
An Application Involving Weighted Voting 76
Exercises 3.2 79
Written Assignments 3.2 79

3.3 SET OPERATIONS 79
Goals 79
Union 80
Intersection 80
Difference 81
Complement 82
Combining Set Operations 83
Exercises 3.3 84
Written Assignments 3.3 85

3.4 VENN DIAGRAMS 85
Goals 85
Diagrams for Two Sets 86
Venn Diagrams and Set Operations 88
Diagrams for Three Sets 89
Exercises 3.4 92
Written Assignments 3.4 93

3.5 SET PROPERTIES 94
Goals 94
DeMorgan's Laws 94
Distributive Properties 96
Exercises 3.5 97
Written Assignments 3.5 98

3.6 APPLICATIONS OF SETS 98
Goal 98
Exercises 3.6 101
Written Assignments 3.6 102

Chapter 3 Review Exercises 102
Bibliography 104

CHAPTER 4

LOGIC 105

Prologue 105

4.1 INTRODUCTION 106
Goals 106
Written Assignments 4.1 112

4.2 TERMINOLOGY AND NOTATION 113
Goals 113
Statements 113
Connectives 114
Negation 116

Multiple Connectives 117
Exercises 4.2 118
Written Assignments 4.2 119

4.3 TRUTH TABLES 119
Goals 119
Truth Tables for Two Statements 120
Exclusive or 122
Truth Tables for Three Statements 122
Exercises 4.3 124
Written Assignments 4.3 124

4.4 CONDITIONAL SENTENCES 125
Goals 125
Terminology and Notation 125
Truth Tables 125
Biconditional Sentences 128
Contrapositive 129
Converse and Inverse 130
Exercises 4.4 131
Written Assignments 4.4 133

4.5 VALID ARGUMENTS 133
Goals 133
Basic Form of a Valid Argument 133
Using Truth Tables 136
Invalid Arguments 137
Arguments and Truth 138
Exercises 4.5 140
Written Assignments 4.5 141

4.6 EQUIVALENT STATEMENTS 142
Goals 142
DeMorgan's Laws 142
Implication 143
Distributive Properties 146
Expressions That Are not Equivalent 147
Exercises 4.6 148
Written Assignments 4.6 148

Chapter 4 Review Exercises 149
Bibliography 151

CHAPTER 5

ALGEBRA 153

Prologue 153

5.1 GENERALIZED ARITHMETIC AND APPLICATIONS OF LINEAR EQUATIONS 154
Goals 154
Concept of a Variable 154
General Arithmetic 156
Applications of Linear Equations 159
More Examples 161
Exercises 5.1 163
Written Assignments 5.1 164

5.2 LINEAR AND QUADRATIC FUNCTIONS AND QUADRATIC EQUATIONS 164
Goals 164
Definitions 165
Functional Notation 166
Single-Variable Functions 168
Multiple-Variable Functions 168
Linear Functions 168
Quadratic Functions 169
Parabolas 169
Zeros of Quadratic Functions 171
The Quadratic Formula 172
Applications of Quadratic Functions 173
Exercises 5.2 176
Written Assignments 5.2 176

5.3 EXPONENTIAL FUNCTIONS 177
Goals 177
Exponential Functions 177
Calculator Use 180
A Special Exponential Function 181
Exercises 5.3 195
Written Assignments 5.3 196

5.4 FURTHER DEVELOPMENTS IN ALGEBRA 197
Goals 197
Historical Interlude 197
Inequalities 203
Solving Inequalities in One Variable 203
Application 207
Linear Inequalities in Two Variables 208
Abstract Algebra 210

Exercises 5.4 210
Written Assignments 5.4 211

Chapter 5 Review Exercises 211
Bibliography 213

CHAPTER

FUNDAMENTALS OF GEOMETRY 215

Prologue 215

6.1 OVERVIEW AND HISTORY 216
Goals 216
Written Assignments 6.1 218

6.2 TERMINOLOGY AND NOTATION 219
Goals 219
Points 219
Lines 220
Angles 222
Planes 222
Exercises 6.2 224
Written Assignments 6.2 224

6.3 ANGLES 225
Goals 225
Terminology and Notation 225
Measure of an Angle 226
Special Angles 228
Angles and Parallel Lines 228
Exercises 6.3 230
Written Assignments 6.3 231

6.4 POLYGONS 231
Goals 231
Terminology 231
Triangles 233
Quadrilaterals 238
General Polygons 240
Exercises 6.4 241
Written Assignments 6.4 243

6.5 PERIMETER AND AREA 243
Goals 243
Perimeters of Polygons 244
Areas of Polygons 245
Circles 249
Exercises 6.5 255
Written Assignments 6.5 257

Chapter 6 Review Exercises 257
Bibliography 259

CHAPTER

TOPICS IN GEOMETRY 261

Prologue 261

7.1 GEOMETRY AS AN AXIOMATIC SYSTEM 262
Goals 262
Axiomatic Systems 262
Euclid's *Elements* 264
Euclidean Geometry 265
A Finite Geometry Axiomatic System 265
Variations in Geometry 267
Exercises 7.1 270
Written Assignments 7.1 270

7.2 NON-EUCLIDEAN GEOMETRY 271
Goals 271
Dominance of Euclidean Geometry 272
The Parallel Postulate 272
Two Millennia of Work 272
Results of the 1800s 274
Ramifications of Non-Euclidean Geometry 275
Exercises 7.2 276
Written Assignments 7.2 277

7.3 FRACTALS 278
Goals 278
Historical Development 280
Examples of Fractals 281
Fractals in Nature 289
Exercises 7.3 290
Written Assignments 7.3 291

Chapter 7 Review Exercises 291
Bibliography 292

CHAPTER

PROBABILITY 295

Prologue 295

8.1 HISTORICAL BACKGROUND 296
- Goals 296
- The Birth of Probability Theory 297
- Why Probability Theory Emerged in the Seventeenth Century 297
- Written Assignments 8.1 300

8.2 BASIC CONCEPTS OF PROBABILITY 300
- Goals 300
- Experiments, Outcomes, and Sample Spaces 301
- Mathematical Modeling and Assigning Probabilities 302
- Probability Distributions 303
- Exercises 8.2 306
- Written Assignments 8.2 308

8.3 SOME COUNTING RULES 308
- Goals 308
- The Multiplication Rule 309
- Permutations 310
- Combinations 313
- Which Counting Method Do I Use? 314
- Exercises 8.3 315
- Written Assignments 8.3 316

8.4 DETERMINING PROBABILITIES USING COUNTING RULES 317
- Goals 317
- Exercises 8.4 319
- Written Assignments 8.4 320

8.5 PROBABILITIES OF COMPOUND EVENTS 320
- Goals 320
- Conditional Probability 322
- Dependent and Independent Events 323
- Relation Between Mutually Exclusive and Independent Events 324
- The Probability of A and B 325
- The Birthday Problem 327
- Exercises 8.5 328
- Written Assignments 8.5 329

8.6 ODDS AND EXPECTED VALUE 330
- Goals 330
- Odds 330
- Expected Value 331
- Exercises 8.6 333
- Written Assignments 8.6 334

Chapter 8 Review Exercises 335
Bibliography 336

CHAPTER

STATISTICS 337

Prologue 337

9.1 GRAPHICAL PRESENTATION OF DATA AND SAMPLING 338
- Goals 338
- Histograms 339
- Stem-and-Leaf Display 343
- Sampling 344
- Exercises 9.1 346
- Written Assignments 9.1 347

9.2 MEASURES OF CENTRAL TENDENCY AND LOCATION 347
- Goals 347
- Measures of Central Tendency 347
- Measures of Location 351
- Exercises 9.2 353
- Written Assignments 9.2 353

9.3 MEASURES OF SPREAD 354
- Goals 354
- Range 354
- Box Plots 354
- Standard Deviation 356
- Empirical Rule 359
- Summary 360
- Exercises 9.3 361
- Written Assignments 9.3 362

9.4 NORMAL DISTRIBUTIONS 362
- Goals 362
- Continuous Random Variables 362
- Continuous Probability Distributions 362
- The Normal Distribution 363

Applications of the Normal Distribution 370
Exercises 9.4 372
Written Assignments 9.4 373

9.5 STATISTICAL INFERENCE 373
Goals 373
Sampling Distribution for $\bar{x}$ 374
Sampling Distribution for $\hat{p}$ 379
Summary 384
Exercises 9.5 384
Written Assignments 9.5 385

Chapter 9 Review Exercises 385
Bibliography 388

CHAPTER 10

CONSUMER MATHEMATICS 389

Prologue 389

10.1 INTRODUCTION 390
Goals 390
Written Assignments 10.1 392

10.2 UNIT COSTS, SALES, AND COUPONS 392
Goals 392
Unit Costs 392
Sales 394
Coupons 395
Exercises 10.2 396
Written Assignments 10.2 397

10.3 SIMPLE INTEREST 397
Goals 397
Interest 397
Amount Owed or Earned 398
Credit Cards 399
Exercises 10.3 400
Written Assignments 10.3 401

10.4 COMPOUND INTEREST 401
Goals 401
Compound Amount 401
Exponential Growth 403
Present Value 405
Effective Rate 406
Inflation 407
Rule of 72 407
Summary 408
Exercises 10.4 408
Written Assignments 10.4 409

10.5 ANNUITIES 409
Goals 409
Ordinary Annuity 410
Annuity Due 412
Present Value of an Ordinary Annuity 413
Summary 414
Exercises 10.5 414
Written Assignments 10.5 415

10.6 LOAN PAYMENTS 415
Goals 415
Amortization Schedule 416
Long-Term Loans 418
Taking Out a Loan 419
Exercises 10.6 419
Written Assignments 10.6 420

Chapter 10 Review Exercises 420
Bibliography 422

CHAPTER 11

DISCRETE MATHEMATICS 423

Prologue 423

11.1 DISCRETE MATHEMATICS 424
Goals 424
Written Assignments 11.1 425

11.2 EULER CIRCUITS 426
Goals 426
Königsberg Bridge Problem 428
When Does a Graph Have an Euler Circuit? 428
Exercises 11.2 430
Written Assignments 11.2 432

11.3 HAMILTONIAN CYCLES 432
Goals 432
Traveling Salesperson Problem (TSP) 436
Exercises 11.3 445
Written Assignments 11.3 446

11.4 THE MARRIAGE PROBLEM 447
Goals 447
Stable Marriages 447
Deferred Acceptance Algorithm 450
Unequal Numbers of Men and Women 454
Best Set of Stable Marriages 455
The Marriage Problem in Other Contexts 455
Strategy 455
Exercises 11.4 456
Written Assignments 11.4 456

11.5 VOTING METHODS 457
Goals 457
Binary Voting 459
Runoff Elections 462
Borda Count 463
Arrow's Theorem 465
Exercises 11.5 468
Written Assignments 11.5 469

Chapter 11 Review Exercises 470
Bibliography 472

CHAPTER 12

MATHEMATICS AND COMPUTERS 473

Prologue 473

12.1 MECHANICAL AND MATHEMATICAL AIDS TO COMPUTATION 474
Goals 474
The Abacus 475
Logarithms 478
The Slide Rule 483
Summary 484
Exercises 12.1 485
Written Assignments 12.1 485

12.2 AUTOMATIC CALCULATING MACHINES AND COMPUTERS 486
Goals 486
Mechanical Calculating Machines 487
Early Development of Computers 488
Logic Machines 491
Tabulating Machines 493
Electromechanical Calculators 493
Electronic Computers 495
Stored-Program Electronic Computers 497
Technological Changes 497
Exercises 12.2 498
Written Assignments 12.2 498

12.3 PROGRAMMING CONCEPTS 499
Goals 499
Languages 499
Algorithms 500
Variables and Expressions 500
Assignment Statements 500
Conditional Statements 501
While Loops 504
Exercises 12.3 506
Written Assignments 12.3 507

12.4 SPEED AND ALGORITHMS 507
Goals 507
Speed 507
Some Algorithms 509
Summary 515
Exercises 12.4 516
Written Assignments 12.4 516

12.5 SOME USES OF COMPUTERS IN MATHEMATICS 517
Goals 517
Exploration 517
Visualization 520
Simulation 524
Proof 525
Summary 526
Exercises 12.5 526
Written Assignments 12.5 527

Chapter 12 Review Exercises 528
Bibliography 530