LYNN H. LOOMIS

Student Supplement to Accompany

CALCULUS

THIRD EDITION

STUDENT SUPPLEMENT TO ACCOMPANY

calculus third edition

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preface

This supplement to CALCULUS, Third Edition, includes a separate group of applied problems which is for students in the management, life and social sciences. The second part of the supplement contains complete solutions to most of the even-numbered problems in the text.

The applied problems are taken, with the author's permission, from Marvin Bittinger's book, CALCULUS: A MODELING APPROACH (1976, Addison-Wesley Publishing Company, Inc.). I am extremely grateful to Dr. Bittinger for his kindness in letting me borrow his problems.

L.H.L.
January 1982

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part I applied problems

CHAPTER 3

Section 4

Problems

1. The initial population in a bacteria colony is 10,000. After t hours the colony grows to a number $P(t)$ given by

   $$P(t) = 10{,}000(1 + 0.86 + t^2) .$$

   a. Find the rate of change of the population P with respect to time t. This is also known as the growth rate.

   b. Find the number of bacteria present after 5 hours. Also find the growth rate when $t = 5$.

2. The initial population of a bacteria colony is 10,000. After t hours the colony grows to a number $P(t)$ given by

   $$P(t) = 10{,}000(1 + 0.97t + t^2) .$$

   a. Find the growth rate of the population.

   b. Find the number of bacteria present (the population) when $t = 5$ hr. Find the growth rate at $t = 5$ hr.

   c. Find the number of bacteria present when $t = 6$ hr. Find the growth rate when $t = 6$.

3. The spherical volume V of a cancer tumor is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the tumor, in centimeters. Find

   a. The rate of change of the volume with respect to the radius.

   b. Find the rate of change of volume at $r = 1.2$ cm.

4. Stopping Distance on Glare Ice. The stopping distance (at some fixed speed) of regular tires is given by a linear function of the air temperature F: $D(F) = 2F + 115$, where $D(F)$ = stopping distance, in feet, when the air temperature is F, in degrees Fahrenheit. Find the rate of change of the stopping distance D with respect to the air temperature F.

5. Percentage of the Population in College. The percentage of the population in college is given by a linear function $P(t) = 1.25t + 15$, where $P(t)$ = percentage in college the tth year after 1940. Find the rate of change of the percentage P with respect to time t.

6. The circular area A, in square centimeters, of a healing wound is given by $A = \pi r^2$, where r is the radius, in centimeters. Find the rate of change of the area with respect to the radius.

7. The circumference C, in centimeters, of a healing wound is given by $C = 2\pi r$, where r is the radius, in centimeters. Find the rate of change of circumference with respect to the radius.

8. The population of a city grows from an initial size of 100,000 to an amount P given by $P = 100,000 + 2000t^2$, where t is measured in years.

   a. Find the growth rate.

   b. Find the number of people in the city after 10 years (at $t = 10$ years).

   c. Find the growth rate at $t = 10$ years.

9. The temperature T of person during an illness is given by $T(t) = -0.1t^2 + 1.2t + 98.6$, where T is the temperature (^{o}F) at time t, measured in days.

   a. Find the rate of change of the temperature with respect to time.

   b. Find the temperature at $t = 1.5$ days.

   c. Find the rate of change at $t = 1.5$ days.

10. A firm estimates that it will sell N units of a product after spending a dollars on advertising, where $N(a) = -a^2 + 300a + 6$, and a is measured in thousands of dollars.

    a. What is the rate of change of the number of units sold with respect to the amount spent on advertising?

    b. How many units will be sold after spending $10 thousand dollars on advertising?

c. What is the rate of change at $a = 10$?

11. For a certain dosage of x cc (cubic centimeters) of a drug, there is a resultant blood pressure B given by $B(x) = 0.05x^2 - 0.3x^3$. Find the rate of change of the blood pressure with respect to the dosage.

12. The <u>home range</u> H of an animal is defined to be the region to which it confines its movements. The area of that region is related to its body weight by $H = W^{1.41}$. Find $\frac{dH}{dW}$.

13. The <u>territory area</u> T of an animal is defined to be its defended, or exclusive, region. The area T of that region is related to its body weight by $T = W^{1.31}$. Find $\frac{dT}{dW}$.

14. Given revenue and cost functions $R(x) = 50x - 0.5x^2$, $C(x) = 4x + 10$, find

a. $P(x)$
b. $R(20)$, $C(20)$, $P(20)$.
c. $R'(x)$, $C'(x)$, $P'(x)$.
d. $R'(20)$, $C'(20)$, $P'(20)$.

15. Given revenue and cost functions $R(x) = 5x$, $C(x) = 0.001x^2 + 1.2x + 60$, find

a. $P(x)$.
b. $R(100)$, $C(100)$, $P(100)$.
c. $R'(x)$, $C'(x)$, $P'(x)$.
d. $R'(100)$, $C'(100)$, $P'(100)$.

CHAPTER 5

Section 5

Problems

1. A stereo manufacturer determines that in order to sell x units of a new stereo its price per unit must be $p = D(x) = 1000 - x$. It also determines that the total cost of producing x units is given by $C(x) = 3000 + 20x$.

a. Find the total revenue $R(x)$.
b. Find the total profit $P(x)$.
c. How many units must the company produce and sell to maximize profit?
d. What is the maximum profit?
e. What price per unit must be charged to make this maximum profit?

2. Fight promoters ride a thin line between profit and loss, especially in determining the price to charge for admission to closed circuit television showings in local theaters. By keeping records, a theater determines that if the admission price is \$20 it averages 1000 people in attendance. But, for every increase of \$1 it loses 100 customers from the average. Every customer spends an average of \$.80 on concessions. What admission price should it charge to maximize total revenue?

In Problems 3-8, find the maximum profit and the number of units which must be produced and sold to yield the maximum profit.

3. $R(x) = 50x - 0.5x^2$, $C(x) = 4x + 10$.

4. $R(x) = 50x - 0.5x^2$, $C(x) = 10x + 3$.

5. $R(x) = 2x$, $C(x) = 0.01x^2 + 0.6x + 30$.

6. $R(x) = 5x$, $C(x) = 0.001x^2 + 1.2x + 60$.

7. $R(x) = 9x - 2x^2$, $C(x) = x^3 - 3x^2 + 4x + 1$.

8. $R(x) = 100x - x^2$, $C(x) = \frac{1}{3}x^3 - 7x^2 + 111x + 100$. $R(x)$ and $C(x)$ are in thousands of dollars, x is in thousands of unit.

9. Raggs, Ltd., a clothing firm, determines that to sell x suits its price per suit must be $p = D(x) = 150 - 0.5x$. It also determines that its total cost of producing suits is given by $C(x) = 4000 + 0.25x^2$.

   a. Find the total revenue $R(x)$.
   b. Find the total profit $P(x)$.
   c. How many suits must the company produce and sell to maximize profit?
   d. What is the maximum profit?
   e. What price per suit must be charged to make this maximum profit?

10. An appliance firm is marketing a new refrigerator. It determines that to sell x refrigerators its price per refrigerator must be $p = D(x) = 280 - 0.4x$. It also determines that its total cost of producing x refrigerators is given by $C(x) = 5000 + 0.6x^2$.

    a. Find the total revenue $R(x)$.
    b. Find the total profit $P(x)$.
    c. How many refrigerators must the company produce and sell to maximize profit?
    d. What is the maximum profit?
    e. What price per refrigerator must be charged to make this maximum profit?

11. A soup company is constructing an open top metal rectangular tank with a square base which will have a volume of 32 cubic feet. What dimensions yield the minimum surface area? What is the minimum area?

12. A university is trying to determine what price to charge for football tickets. At a price of \$6 per ticket it averages 70,000 per game. For every increase of \$1 it loses 10,000 people from the average. Every person at the game spends an average of \$1.50 on concessions. What price per ticket should be charged to maximize revenue? How man people will attend at that price?

13. Suppose you are the owner of a 30-unit motel. All units are occupied when you charge \$20 a day per unit. For every increase of x dollars in the daily rate, there are x units vacant. Each occupied room costs \$2 per day to service and maintain. What should you charge per unit to maximize profit?

14. An apple farm yields an average of 30 bu of apples per tree when 20 trees are planted on an acre of ground. Each time 1 more tree is planted per acre, the yield decreases 1 bu per tree due to the extra congestion. How many trees should be planted to get the highest yield?

15. When a theater owner charges \$3 for admission there is an average attendance of 100 people. For every \$.10 increase in admission, there is a loss of 1 customer from the average. What admission should be charged to maximize revenue?

16. The postal service places a limit of 84 in. on the combined length and girth (distance around) of a package to be sent parcel post. What dimensions of a rectangular box with square cross section, will contain the largest volume that can be mailed?

17. A power line is to be constructed from a power station at point A to an island at point C which is directly 1 mile out in the water from a point B on shore. Point B is 4 miles downshore from the power station at A. It costs \$500 per mile to lay the power line under water and \$300 per mile to lay the line under ground. At what point S downshore from A should the line come to the shore to minimize cost? Note that S could very well be B or A.

18. The total cost function for producing x units of a certain product is given by $C(x) = 2x^2 + 10x + 50$.

a. Find the marginal cost $C'(x)$.

b. Find the average cost $A(x) = \frac{C(x)}{x}$.

c. Find the marginal average cost $A'(x)$.

d. Find the minimum of $A(x)$ and the value x_0 at which it occurs. Find the marginal cost at x_0.

e. Compare $A(x_0)$ and $C'(x_0)$.

19. Consider $A(x) - \frac{C(x)}{x}$.

a. Find $A'(x)$ in terms of $C'(x)$ and $C(x)$.

b. Show that $A(x)$ has a minimum at that value of x_0 such that

$$C'(x_0) = A(x_0) = \frac{C(x_0)}{x_0}.$$

This shows that when marginal cost and average cost are the same, a product is being produced at the least average cost.

20. A retail appliance store sells 2500 TV's per year. It costs \$10 to store one TV for a year. To reorder TV's there is a fixed cost of \$20 plus \$9 for each TV. How many times per year should the store reorder TV's, and in what lot size, so the inventory costs are minimized?

Discussion: Let x = lot size. Now inventory costs are given by

C(x) = YEARLY CARRYING COSTS + YEARLY REORDER COSTS.

We consider each separately.

a. Yearly Carrying Costs. The average amount held in stock is $\frac{x}{2}$, and it costs \$10 per TV for storage. Thus

$$\text{AVERAGE CARRYING COSTS} = \text{Yearly Cost Per Item} \cdot \text{Average Number of Items} = 10 \cdot \frac{x}{2}$$

b. Yearly Reorder Costs. Now x = lot size, and suppose there are N reorders each year. Then $Nx = 2500$, and $N = \frac{2500}{x}$. Thus

$$\text{YEARLY RECORDER COSTS} = \text{Cost of Each Order} \cdot \text{Number of Reorders} = (20 + 9x)\frac{2500}{x}.$$

21. An appliance store sells 600 refrigerators per year. It costs \$30 to store one refrigerator for one year. To reorder refrigerators there is a fixed cost of \$40 plus \$11 for each refrigerator. How many times per year should the store order refrigerators, and in what lot size, to minimize inventory costs?

22. A sporting goods store sells 100 pool tables per year. It costs \$20 to store one pool table for one year. To reorder pool tables there is a fixed cost of \$40 plus \$16 for each pool table. How many times per year should the store reorder pool tables, and in what lot size, to minimize inventory costs?

23. A pro shop in a bowling alley sells 200 bolowing balls per year. It costs \$4 to store one bowling ball for one year. To reorder bowling balls there is a fixed cost of \$1 plus \$.50 for each bowling ball. How many times per year should the shop order bowling balls, and in what lot size, to minimize inventory costs?

24. A retail outlet for Boxowitz Calculators sells 360 calculators per year. It costs \$8 to store one calculator for one year. To reorder calculators there is a fixed cost of \$10 plus \$8 for each calculator. How many times per year should the store order calculators, and in what lot size, to minimize inventory costs?

25. A sporting goods store in southern California sells 720 surf boards per year. It costs \$2 to store one surf board for one year. To reorder surf boards there is a fixed cost of \$5 plus \$2.50 for each surf board. How many times per year should the store order surf boards, and in what lot size, to minimize inventory costs?

MAXIMUM SUSTAINABLE HARVEST. Suppose that a population P will grow to $f(P)$ in one year. If this were a population of fur-bearing animals, then one could "harvest" the amount $f(P) - P$ each year without depleting the initial population P; the harvest $f(P) - P$ is "sustained". For each reproduction curve in problems 26-29 below,

a. Find the population at which the maximum sustainable harvest occurs.

b. Find the maximum sustainable harvest.

26. $f(P) = P(20 - P)$, where P is measured in thousands.

27. $f(P) = P(6 - P)$, where P is measured in thousands.

28. $f(P) = -0.025P^2 + 4P$, where P is measured in thousands. This is the reproduction curve in the Hudson Bay area for the snowshoe hare -- a fur-bearing animal.

29. $f(P) = -0.01P^2 + 2P$, where P is measured in thousands. This is the reproduction curve in the Hudson Bay area for the lynx -- a fur-bearing animal.

30. Find the maximum profit and the number of units which must be produced and sold to yield the maximum profit, if $R(x) = x^2 + 110x + 60$, $C(x) = 1.1x^2 + 10x + 80$.

31. A sporting goods store sells 625 tennis rackets per year. It costs \$1 to store one tennis racket for one year. To reorder tennis rackets there is a fixed cost of \$2 plus \$.50 for each tennis racket. How many times per year should the sporting goods store order tennis rackets, and in what lot size, to minimize inventory costs?

32. Consider the reproduction curve $f(P) = P(100 - P)$, where P is measured in thousands. Find the population at which the maximum sustainable harvest occurs. Find the maximum sustainable harvest.

33. A company begins a radio advertising campaign in New York City to market a new product. The percentage of the "target market" which buys a product is normally a function of the length of the advertising campaign. The radio station estimates this percentage as $(1 - e^{-.04t})$ for this type of product, where t = number of days of the campaign. The target market is estimated to be 1,000,000 people and the price per unit is \$.50. The costs of advertising are \$1000 per day. Find the length of the advertising campaign which will result in maximum profit.

34. Solve the advertising problem where the costs of advertising are \$2000 per day.

35. Solve the advertising problem where the costs of advertising are \$4000 per day.

36. A company's costs, in millions of dollars, is given by

$C(t) = 100 - 50e^{-t}$, where t = time. Find

a. The marginal cost $C'(t)$.
b. $C'(0)$. c. $C'(4)$.

37. A company's costs, in millions of dollars, is given by

$C(t) = 200 - 40e^{-t}$, where t = time. Find

a. The marginal cost $C'(t)$.
b. $C'(0)$. c. $C'(5)$.

Chapter 5

Section 8

Problems

1. The spherical volume of a cancer tumor is given by $V = \frac{4}{3}\pi r^3$, where r is the radius in centimeters. By approximately how much does the volume increase when the radius is increased from 1 cm to 1.2 cm? Use 3.14 for π.

2. The circular area of a healing wound is given by $A = \pi r^2$, where r is the radius in centimeters. By approximately how much does the area decrease when the radius is decreased from 2 cm to 1.9 cm? Use 3.14 for π.

Suppose we have cost function $C(x)$. When $\Delta x = 1$, we have $\Delta C \approx C'(x)$. Whether this is a good approximation depends on the function and on the values of x. If it is, then:

$C'(x) \approx C(x+1) - C(x)$

Marginal cost is approximately the cost of the $(x+1)$, or next, unit. This is the historical interpretation which economists have given to marginal cost.

Marginal revenue and marginal cost have similar interpretations.

3. Consider the total cost function $C(x) = 0.01x^2 + 4x + 500$.
   a. Find ΔC and $C'(x)$ when $x = 5$ and $\Delta x = 1$.
   b. Find ΔC and $C'(x)$ when $x = 100$ and $\Delta x = 1$.

4. For the total cost function $C(x) = 0.01x^2 + 0.6x + 30$, find ΔC and $C'(x)$ when $x = 70$ and $\Delta x = 1$.

5. For the total cost function $C(x) = 0.01x^2 + 1.6x + 100$, find ΔC and $C'(x)$ when $x = 80$ and $\Delta x = 1$.

6. For the total revenue function $R(x) = 2x$, find ΔR and $R'(x)$ when $x = 70$ and $\Delta x = 1$.

7. For the total revenue function $R(x) = 3x$, find ΔR and $R'(x)$ when $x = 80$ and $\Delta x = 1$.

8. a. Using $C(x)$ of Exercise 4 and $R(x)$ of Exercise 6, find the total profit $P(x)$.
   b. Find ΔP and $P'(x)$ when $x = 70$ and $\Delta x = 1$.

9. a. Using $C(x)$ of Exercise 5 and $R(x)$ of Exercise 7, find the total profit $P(x)$.
   b. Find ΔP and $P'(x)$ when $x = 80$ and $\Delta x = 1$.

CHAPTER 6

Section 1

Problems

1. A company determines that the marginal cost, C', of producing the xth unit of a certain product is given by $C'(x) = x^3 - 2x$. Find the total cost function C, assuming fixed costs are $100.

2. A company determines that the marginal cost, C', of producing the xth unit of a certain product is given by $C'(x) = x^3 - x$. Find the total cost function C, assuming fixed costs are $200.

3. A company determines that the marginal revenue R', from selling the xth unit of a certain product is given by $R'(x) = x^2 - 3$.

   a. Find the total revenue function R, assuming $R(0) = 0$.

   b. Why is $R(0) = 0$ a reasonable assumption?

4. A company determines that the marginal revenue R', from selling the xth unit of a certain product is given by $R'(x) = x^2 - 1$.

   a. Find the total revenue function R, assuming $R(0) = 0$.

   b. Why is $R(0) = 0$ a reasonable assumption?

5. The rate at which a machine operator's efficiency E (expressed as a percentage) changes with respect to time is given by $\frac{dE}{dt} = 40 - 10t$, where t = the number of hours the operator has been at work.

   a. Find $E(t)$ if it is known that her efficiency after working 2 hours is 72%. That is, $E(2) = 72$.

   b. Use the answer to a) to find the operator's efficiency after 4 hours, 8 hours.

6. The rate at which a machine operator's efficiency E (expressed as a percentage) changes with respect to time is given by $\frac{dE}{dt} = 30 - 10t$, where t = the number of hours the operator has been at work.

   a. Find $E(t)$ if it is known that his efficiency after working 2 hours is 72%. That is, $E(2) = 72$.

   b. Use the answer to a) to find the operator's efficiency after 3 hours, 5 hours.

7. Raggs, Ltd., goes even further to reduce production costs. In addition to installing new sewing machines, it installs air conditioning, and has the president take a calculus course. This allows the marginal cost per suit to decrease rapidly in such a way that

   $C'(x) = 0.0003x^2 - 0.2x + 50$. Find the total cost of producing 400 suits. Ignore fixed costs.

8. A sound company determines that the marginal cost of producing the xth stereo is given by $C'(x) = 100 - 0.2x$, $C(0) = 0$. It also determines that its marginal revenue from the sale of the xth stereo revenue is given by $R'(x) = 100 + 0.2x$, $R(0) = 0$.

   a. Find the total cost of producing x stereos.
   b. Find the total revenue of selling x stereos.
   c. Find the total profit from the production and sale of x stereos.
   d. Find the total profit from the production and sale of 1000 stereos.

9. A refrigeration company determines that marginal cost of producing the xth refrigerator is given by $C'(x) = 50 - 0.4x$, $C(0) = 0$. It also determines that its marginal revenue from the sale of the xth stereo is given by $R'(x) = 50 + 0.4x$, $R(0) = 0$.

   a. Find the total cost of producing x refrigerators.
   b. Find the total revenue of selling x refrigerators.
   c. Find the total profit from the production and sale of x refrigerators.
   d. Find the total profit from the production and sale of 1000 refrigerators.

CHAPTER 7

Section 1

Problems

1. In a psychological experiment students were shown a set of nonsense syllables, such as PDQ, and asked to recall them every second thereafter. The percentage $R(t)$ who retained the syllables after t seconds was found to be given by $R(t) = 80 - 27 \ln t$, for $t \geq 1$. (Strictly speaking the function is not continuous, but in order to use calculus, we "fill in" the graph with a smooth curve, considering $R(t)$ to be defined for any number $t \geq 1$.

   a. What percentage retained the syllable after 1 second?
   b. Find $R'(t)$, the rate of change of R with respect to t.
   c. Find maximum and minimum values, if they exist.

2. A model for advertising response is given by $N(a) = 500 + 200 \ln a$, $a \geq 1$, where $N(a)$ = number of units sold, a = amount spent on advertising, in thousands of dollars.

   a. How many units were sold after spending 1 thousand dollars? (Substitute 1 for a, not 1000.)
   b. Find $N'(a)$.
   c. Find maximum and minimum values, if they exist.

3. Students in college botany took a final exam. They took equivalent forms of the exam in monthly intervals thereafter. The average score, $S(t)$ in percent, after t months was found to be given by $S(t) = 68 - 20 \ln (t+1)$, $t > 0$.

   a. What was the average score when they initially took the test, $t = 0$?
   b. What was the average score after 4 months?
   c. What was the average score after 24 months?
   d. What percentage of the initial score did they retain after 2 years (24 months)?
   e. Find $S'(t)$.
   f. Find maximum and minimum values, if they exist.

4. A model for advertising response is given by $N(a) = 1000 + 200 \ln a$, $a \geq 1$, where $N(a)$ = number of units sold, a = amount spent on advertising in thousands of dollars.

   a. How many units were sold after spending 1 thousand dollars ($a = 1$) on advertising?
   b. Find $N'(a)$, $N'(10)$.
   c. Find maximum and minimum values, if they exist.