Undergraduate Texts in Mathematics

Robin Hartshorne

GEOMETRY: EUCLID AND BEYOND

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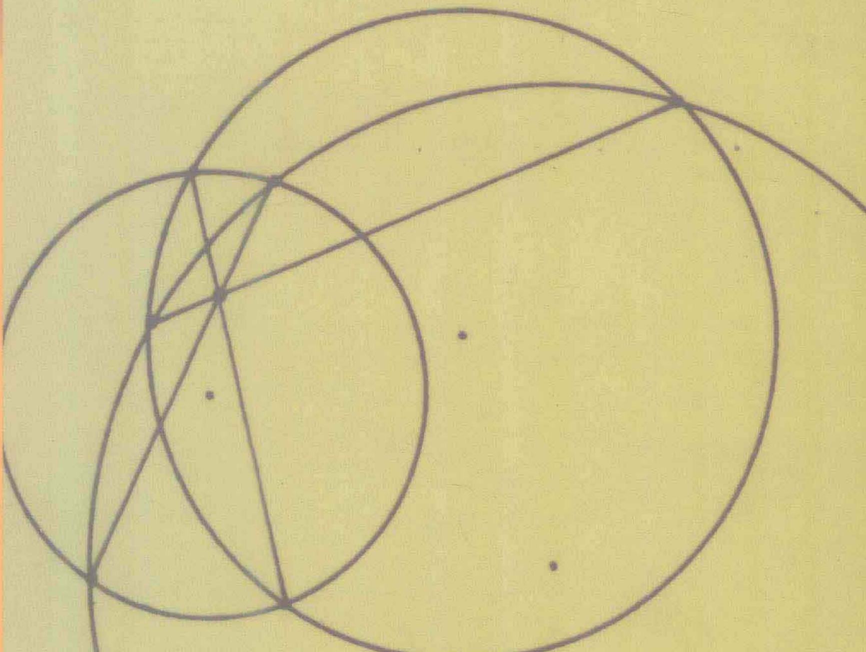

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Robin Hartshorne

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Robin Hartshorne
Department of Mathematics
University of California
Berkeley, CA 94720
USA
robin@math.berkeley.edu

Editorial Board
S. Axler
Mathematics Department
San Francisco State University
San Francisco, CA 94132
USA
axler@sfsu.edu

F.W. Gehring
Mathematics Department
East Hall
University of Michigan
Ann Arbor, MI 48109
USA
fgehring@math.lsa.umich.edu

K.A. Ribet
Mathematics Department
University of California at Berkeley
Berkeley, CA 94720-3840
USA
ribet@math.berkeley.edu

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For
Edie, Ben, and Joemy

and

In Loving Memory of
Jonathan Churchill Hartshorne
1972–1992

I have not found anything in Lobatchevski's work that is new to me, but the development is made in a different way from the way I had started and to be sure masterfully done by Lobatchevski in the pure spirit of geometry.

– letter from Gauss to Schumacher (1846)

Preface

In recent years, I have been teaching a junior–senior-level course on the classical geometries. This book has grown out of that teaching experience. I assume only high-school geometry and some abstract algebra. The course begins in Chapter 1 with a critical examination of Euclid's *Elements*. Students are expected to read concurrently Books I–IV of Euclid's text, which must be obtained separately. The remainder of the book is an exploration of questions that arise naturally from this reading, together with their modern answers. To shore up the foundations we use Hilbert's axioms. The Cartesian plane over a field provides an analytic model of the theory, and conversely, we see that one can introduce coordinates into an abstract geometry. The theory of area is analyzed by cutting figures into triangles. The algebra of field extensions provides a method for deciding which geometrical constructions are possible. The investigation of the parallel postulate leads to the various non-Euclidean geometries. And in the last chapter we provide what is missing from Euclid's treatment of the five Platonic solids in Book XIII of the *Elements*.

For a one-semester course such as I teach, Chapters 1 and 2 form the core material, which takes six to eight weeks. Then, depending on the taste of the instructor, one can follow a more geometric path by going directly to non-Euclidean geometry in Chapter 7, or a more algebraic one, exploring the relation between geometric constructions and field extensions, by doing Chapters 3, 4, and 6. For me, one of the most interesting topics is the introduction of coordinates into an abstractly given geometry, which is done for a Euclidean plane in Section 21, and for a hyperbolic plane in Section 41.

Throughout this book, I have attempted to choose topics that are accessible to undergraduates and that are interesting in their own right. The exercises are meant to be challenging, to stimulate a sense of curiosity and discovery in the student. I purposely do not indicate their difficulty, which varies widely.

I hope this material will become familiar to every student of mathematics, and in particular to those who will be future teachers.

I owe thanks to Marvin Greenberg for reading and commenting on large portions of the text, to Hendrik Lenstra for always having an answer to my questions, and to Victor Pambuccian for valuable references to the literature. Thanks to Faye Yeager for her patient typing and retyping of the manuscript. And special thanks to my wife, Edie, for her continual loving support.

> Of all the works of antiquity which have been transmitted to the present times, none are more universally and deservedly esteemed than the *Elements of Geometry* which go under the name of Euclid. In many other branches of science the moderns have far surpassed their masters; but, after a lapse of more than two thousand years, this performance still maintains its original preeminence, and has even acquired additonal celebrity from the fruitless attempts which have been made to establish a different system.
>
> – from the preface to
> Bonnycastle's Euclid
> London (1798)

Contents

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Introduction

A little after the time of Plato, but before Archimedes, in ancient Greece, a man named Euclid wrote the *Elements*, gathering and improving the work of his predecessors Pythagoras, Theaetetus, and Eudoxus into one magnificent edifice. This book soon became the standard for geometry in the classical world. With the decline of the great civilizations of Athens and Rome, it moved eastward to the center of Arabic learning in the court of the caliphs at Baghdad.

In the late Middle Ages it was translated from Arabic into Latin, and since the Renaissance it not only has been the most widely used textbook in the world, but has had an influence as a model of scientific thought that extends way beyond the confines of geometry. As Billingsley said in his preface to the first English translation (1570), "Without the diligent studie of Euclides *Elementes*, it is impossible to attaine unto the perfecte knowledge of Geometrie, and consequently of any of the other Mathematical Sciences." Even today, though few schools use the original text of Euclid, the content of a typical high-school geometry course is the same as what Euclid taught more than two thousand three hundred years ago.

In this book we will take Euclid's *Elements* as the starting point for a study of geometry from a modern mathematical perspective.

To begin, we will become familiar with the content of Euclid's work, at least those parts that deal with geometry (Books I–IV, VI, and XI–XIII). Here we find theorems that should be familiar to anyone who has had a course of high-school geometry, such as the fact (I.4) that two triangles are congruent if they have two sides and the included angle equal, or the fact (III.21) that a given arc of a circle subtends the same angle at any point of the circle from which it is seen. (Throughout this book, references such as (I.4) or (III.21) refer to the corresponding Book and Proposition number in Euclid's *Elements*.)

Many of Euclid's propositions pose construction problems, such as (I.1), to construct an equilateral triangle, or (IV.11), to construct a regular pentagon inscribed in a circle. Euclid means to construct the required figure using only the ruler, which can draw a straight line through two points, and the compass, which can draw a circle with given center and given radius. These **ruler and compass constructions** are often taught in high-school geometry. Note that Euclid casts these problems in the form of constructions, whereas a modern mathematician would be more likely to speak of proving the existence of the required figure.

At a second level, we will study the logical structure of Euclid's presentation. Euclid's *Elements* has been regarded for more than two thousand years as the prime example of the **axiomatic method**. Starting from a small number of self-evident truths, called postulates, or common notions, he deduces all the succeeding results by purely logical reasoning. Euclid thus begins with the simplest assumptions, such as Postulate 1, to draw a line through any two given points, or Postulate 3, to draw a circle with given center and radius. He then proceeds step by step to the culmination of the work in Book XIII, where he gives the construction of the five regular solids: the tetrahedron, the cube, the octahedron, the icosahedron, and the dodecahedron.

Upon closer reading, we find that Euclid does not adhere to the strict axiomatic method as closely as one might hope. Certain steps in certain proofs depend on assumptions that, however reasonable or intuitively clear they may seem, cannot be justified on the basis of the stated postulates and common notions. So, for example, the fact that the two circles in the proof of (I.1) will actually meet at some point seems obvious, but is not proved. The **method of superposition** used in the proof of (I.4), which allows one to move the triangle ABC so that it lies on top of the triangle DEF, cannot be justified from the axioms. Also, various assumptions about the relative position of figures in the plane, such as which point lies between the others, or which ray lies in the interior of a given angle, are used without any previous clarification of what such notions should mean.

These lapses in Euclid's logic lead us to the task of disengaging those implicit assumptions that are used in his arguments and providing a new set of axioms from which we can develop geometry according to modern standards of rigor. The logical foundations of geometry were widely studied in the late nineteenth century, which led to a set of axioms proposed by Hilbert in his lectures on the foundations of geometry in 1899. We will examine Hilbert's axioms, and we will see how these axioms can be used to build a solid base from which to develop Euclid's geometry pretty much according to the logical plan that he first laid out.

We will also cultivate an awareness of what additional axioms may be required for certain portions of the theory.

Our third level of reading Euclid's *Elements* involves rather broader investigations than the first two levels mentioned above: We will consider various mathematical questions and subsequent developments that arise naturally from Euclid's presentation.

For example, the modern reader quickly becomes aware that Euclid does not use numbers in his geometry. He speaks of **equality** of line segments, and a notion of one segment being added to another to form a third segment, but he does not mention the **length** of a line segment. When it comes to **area** (I.35 ff.), though Euclid does not say explicitly what he means by equality of area, we can infer from his proofs that he means a notion generated by cutting figures in pieces and adding or subtracting congruent figures. He does not use any number to measure the area of a triangle. So we may note with surprise that the famous Pythagorean theorem (I.47) does not state that the square of the length of the hypotenuse (a number) is equal to the sum of the squares of the lengths of the two sides of a right triangle; rather, it says that the area of a square built on the hypotenuse is equal to the area formed by the union of the two squares built on the sides.

The absence of numbers may seem curious to a student educated in an era in which the real numbers are all-powerful, when an interval is measured by its length (which is a real number), and an area by a certain integral (another real number). In fact some modern educators have gone so far as to build the real numbers into the axioms for geometry with the "ruler postulate," which says that to each interval is assigned a real number, its **length**, and that two intervals are congruent if they have the same length. However, this use of the real numbers at the foundational level of geometry is far from the spirit of Euclid.

So we may ask, what role do numbers play in the development of geometry? As one approach to this question we can take the modern algebraic structure of a field (which could be the real numbers, for example), and show that the **Cartesian plane** formed of ordered pairs of elements of the field forms a geometry satisfying our axioms. But a deeper investigation shows that the notion of number appears intrinsically in our geometry, since we can define purely geometrically an **arithmetic of line segments**. We will show that (up to congruence) one can add two segments to get another segment, and one can multiply two segments (once a unit segment has been chosen) to get another segment. These operations satisfy the usual associative, commutative, and distributive laws, so that we obtain an **ordered field**, whose positive elements are the congruence equivalence classes of line segments.

Thus we establish a connection between the abstract geometry based on axioms and the methods of modern algebra.

I would like to emphasize throughout this course how methods of modern algebra help to understand classical geometry and its associated problems.

For example, in the theory of area, one can formalize Euclid's notion of equality based on adding and subtracting congruent figures. However, we do not know any purely geometric proof that this theory of area is nontrivial, so that, for example, one figure properly contained in another will have a smaller area. Euclid just cites Common Notion 5, "the whole is greater than the part." But unless we are willing to accept this as an axiom, we should give a proof. Such a proof can be provided using algebraic arguments in the field of segment arithmetic.

Concerning ruler and compass constructions, algebraic methods have led to notable results. For example, Gauss made an extraordinary discovery in 1796, when he used roots of unity to show that it is possible to construct a regular 17-sided polygon—the first new polygon construction since Euclid's constructions of the pentagon, hexagon, decagon, and quindecagon. On the other hand, field theory, in particular the Galois theory of finite field extensions of $\mathbb{Q}$, has provided proofs of the impossibility of certain ruler and compass constructions such as the regular 7-sided polygon, the trisection of the angle, or the doubling of the cube. For in the algebraic interpretation, one can construct with ruler and compass only those points whose coordinates lie in successive quadratic extensions of $\mathbb{Q}$, while the three problems just mentioned require the solution of cubic equations. We will see, however, that these three problems can be solved if one allows the use of a **marked ruler**. In fact, constructions using the marked ruler, in addition to ordinary straightedge and compass, correspond exactly to the solution of equations of degrees three and four.

Euclid bases his treatment of similar triangles (Book VI) on a complicated theory of proportion (developed in Book V) where ratios of given quantities are compared by seeing whether arbitrary rational multiples of the one exceed or fall short of the other. This method foreshadows Dedekind's nineteenth-century definition of a real number as a division ("Dedekind cut") of the rational numbers into two subsets, namely those greater than and those less than the given real number. The theory of proportion depends on **Archimedes' axiom**, which states that given any two segments there is an integer multiple of the first that will exceed the second. Using the field of segment arithmetic mentioned above we can give (following Hilbert) an alternative development of the theory of similar triangles that is simpler and does not depend on Archimedes' axiom.

In developing the theory of volume of three-dimensional figures in Books XI and XII, Euclid abandons, remarkably, the finite dissection methods used for the area of plane figures. Instead, he applies the "method of exhaustion" attributed to Eudoxus, which suggests the limiting process used to define the Riemann integral. Gauss (1844) expressed his regret that such an infinite method should be used for something so apparently elementary as the volume of a triangular pyramid (XII.5). Hilbert, in his famous list of problems stated in 1900, asked whether this infinite limiting process was really necessary, and Dehn in the

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