Finite Element Methods for Engineers

2nd Edition

Roger T Fenner

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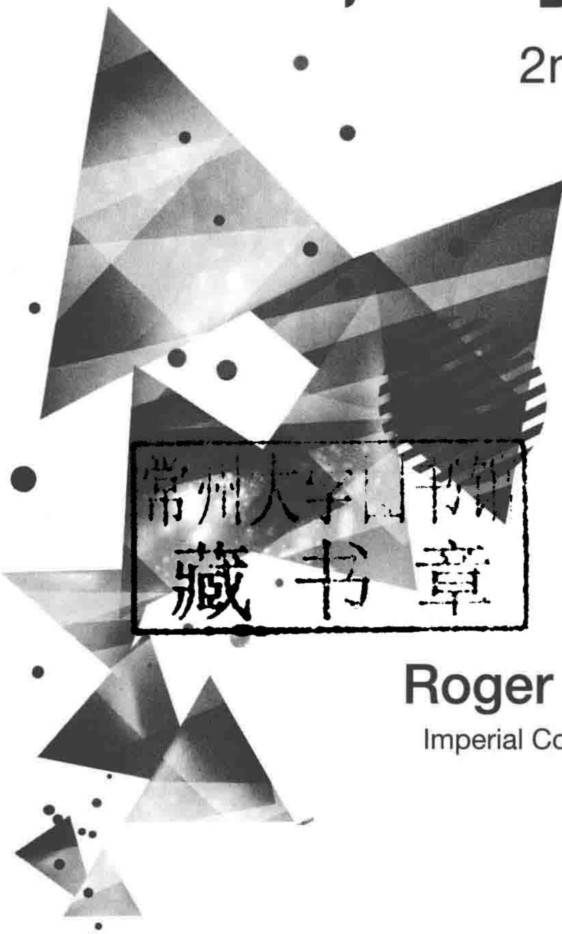

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PREFACE TO THE FIRST EDITION

The advent of high-speed electronic digital computers has given tremendous impetus to all numerical methods for solving engineering problems. Finite element methods form one of the most versatile classes of such methods, and were originally developed in the field of structural analysis. They are, however, equally applicable to continuum mechanics problems in general, including those of fluid mechanics and heat transfer. While some very sophisticated finite element methods have been devised, there is a great deal of very useful analysis that can be performed with the most straightforward types, which are simple to understand and formulate.

The teaching of finite element methods has hitherto been largely confined to university postgraduate courses, particularly those concerned with structural analysis in civil or aeronautical engineering. The purpose of this book is to serve as an introduction to finite element methods applicable to a wider range of problems, particularly those encountered in mechanical engineering. The main emphasis is on the simplest methods suitable for solving two-dimensional problems. Since computer programs form an integral part of the finite element approach they are treated as such in the text. Several programs are presented and described in detail and their uses are illustrated with the aid of a number of practical case studies.

This book is based on courses given by the author to both undergraduate and postgraduate students of mechanical engineering at Imperial College. A prior knowledge of the FORTRAN computer programming language is assumed. The level of continuum mechanics, numerical analysis, matrix algebra and other mathematics employed is that normally taught in undergraduate engineering courses. The book is therefore suitable for engineering undergraduates and other students at an equivalent level.

Postgraduates and practising engineers may also find it useful if they are comparatively new to finite element methods.

The author wishes to thank Miss E.A. Quin for her very skilful typing of a difficult manuscript.

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PREFACE TO THE SECOND EDITION

The first edition of Finite Element Methods for Engineers was published initially in 1975 by Macmillan Press, and from 1996 by Imperial College Press. Despite having been around for so long in a rapidly changing field it has continued to sell steadily. It is this sustained interest in the book that has encouraged me to prepare a substantially revised second edition. Nevertheless, the philosophy of the original is retained — it is a deliberately simple introduction to finite elements in a way that should be readily understandable to engineers, both students and practising professionals. Only the very simplest elements are considered, mainly two-dimensional three-noded 'constant strain triangles', with simple linear variations of the relevant variables.

There have been major and continuing developments in finite element technology and capability in the period since the book first appeared, and the method remains the dominant numerical solution technique in both structural and stress analysis. Nowadays, students and engineers tend to be introduced to finite elements via very sophisticated and powerful software packages: they are trained as users, rather than educated in the details of how and why finite elements work. More importantly, perhaps, the remoteness from the actual workings of the computer programs and any lack of deeper understanding of what can go wrong can lead to serious errors being made. We are definitely in an era of potential 'engineering disaster by computer'. Just because numerical results can be printed out to high precision and plots of deformations and stresses drawn in pretty colours does not mean that the answers are right, or even nearly right.

Not only have finite element methods evolved, but computers have become almost unimaginably more powerful in terms of both computing speed and readily accessible memory for storing data. The programs in the first edition were intended for use on mainframe computers, with data input on punched cards. Such machines, although physically large and requiring a team of operators to keep them running, were actually very limited, particularly in terms of memory, which was very expensive. For general use, anything more than at most a few tens of thousands of numbers required in a calculation had to be stored not in the fast core of the machine but on slow magnetic tapes, requiring rather sophisticated programming techniques to manipulate them. On the other hand, the modern desktop, or indeed laptop, computer is not only much faster but can store up to billions of numbers in its very cheap central memory, and is under the direct personal control of the user.

The revolution in computer capacity has also had an influence on the numerical methods of solutions employed. For example, the large sets of linear algebraic equations generated by the finite element method were in the first edition solved mainly by the iterative Gauss–Seidel method. This was chosen both for simplicity in line with the use of simple triangular elements, and the minimal amount of memory it requires. With much larger available memory it is possible to use a direct method such as Gaussian elimination, even in its simplest form with no allowance for the very high proportion of zero coefficients contained in finite element equations. For finite elements more sophisticated than constant strain triangles, a direct method is usually a better choice, although to make memory usage reasonably efficient for large scale problems some rather complex coefficient manipulation is required. When preparing this second edition I had to decide whether to stay with Gauss–Seidel or change to Gaussian elimination. On reflection I decided on the former, on the grounds of simplicity and the benefit of exposing the reader to both iterative and direct methods of solving equations arising in practical engineering problems. Also, with an iterative solution technique, constant strain triangles provide remarkably powerful and straightforward methods for tackling nonlinear problems.

I also decided to stay with Fortran as the programming language. Although it is somewhat unfashionable these days for general programming purposes, Fortran is still very widely used in engineering computation. I have nevertheless tried to rewrite the programs in such a way as to take advantage of language features offered by contemporary versions of Fortran, and have made them much more convenient for use on desktops and laptops. They are also able to solve much larger problems. On the other hand, I have kept things simple and not tried to introduce any graphical plotting of results. A major new feature is the treatment of problems involving nonlinear materials.

Finally, I have added a selection of problems, with solutions, at the ends of the chapters.

Electronic copies of all the computer programs displayed in the book can be downloaded from the Publisher's website:

http://www.worldscientific.com/doi/suppl/10.1142/p847/suppl_file/p847_program.zip

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NOTATION

The mathematical symbols commonly used in the main text are defined in the following list. In some cases particular symbols have more than one meaning in different parts of the book, although this should not cause any serious ambiguity.

A	constant in Lamé equations for a thick-walled cylinder
$\boldsymbol{A}$	a square matrix
a	radius of a small hole in a flat plate
a_i, a_j, a_k	dimensions of a triangular element
a_x, a_y	semi-axes of an ellipse
a_1, a_2, a_3	constants in general boundary condition Eq. 2.84
B	width of a beam
B	constant in Lamé equations for a thick-walled cylinder
$\boldsymbol{B}$	element dimension matrix
B_{rs}	coefficient of $\boldsymbol{B}$
b	bandwidth of overall stiffness matrix
b	semi-width of a square flat plate
b_i, b_j, b_k	dimensions of a triangular element
C	torsional couple
C_p	specific heat
C_1 to C_{10}	constants in polynomial shape functions
D	flexural rigidity of a flat plate
$\boldsymbol{D}$	element elastic property matrix
E	Young's modulus
E_0	effective modulus for a nonlinear material
e	strain
$\boldsymbol{e}$	element strain vector

e_{T} truncation error
$\boldsymbol{e}_{\mathrm{T}}$ element thermal strain vector
$\boldsymbol{F}$ vector of overall externally applied forces
F_i externally applied force at node i
$\boldsymbol{F}_i$ subvector of externally applied force components at node i
$\boldsymbol{F}_m$ element vector of externally applied forces
$F_{\mathrm{D}}, F_{\mathrm{P}}$ drag and pressure flow shape factors for downstream flow
f coefficient of $\boldsymbol{f}$
$\boldsymbol{f}_i$ self-flexibility submatrix for node i
G shear modulus
$\boldsymbol{G}$ vector of overall body forces applied to the nodes
$\boldsymbol{G}_i$ subvector of overall body force components at node i
$\boldsymbol{G}_m$ element body force vector
g acceleration due to gravity
g heat generated per unit volume
H depth of a beam, lubricating film, channel or solution domain in general
h heat transfer coefficient
h distance between nodal points
h_r distance between nodal points in radial direction
h_x, h_y distance between nodal points in Cartesian co-ordinate directions
I second moment of area for bending
I integral defined in Eqs. 3.46
i nodal point number
i_r counter for circular rings of nodes and elements
i_x, i_y counters for nodes and elements along rows in Cartesian co-ordinate directions
i_θ counter for nodes and elements around a circular ring
j nodal point counter
K ratio of concentric cylinder radii
$\boldsymbol{K}$ overall stiffness matrix
K_{pq} coefficient of $\boldsymbol{K}$
$\boldsymbol{K}_{pq}$ submatrix of $\boldsymbol{K}$
$\widetilde{\boldsymbol{K}}$ rectangular form of $\boldsymbol{K}$
k thermal conductivity
k nodal point counter
$\boldsymbol{k}_m$ element stiffness matrix

k_{rs}	coefficient of $\boldsymbol{k}$
$\boldsymbol{k}_{rs}$	submatrix of $\boldsymbol{k}$
L	length of a beam
L	length of side of an equilateral triangle
$\boldsymbol{L}$	vector storing the number of nonzero coefficients in the rows of matrix $\boldsymbol{M}$
L_i	coefficient of $\boldsymbol{L}$
L_m	length of an element
L_1, L_2	lengths of sides of elements on a solution domain boundary
l_i, l_j, l_k	lengths of the sides of a triangular element
$\boldsymbol{M}$	matrix storing original column numbers of coefficients of $\widetilde{\boldsymbol{K}}$
M_i, M_j	moments applied internally to a beam element at its nodes
M_{ij}	coefficient of $\boldsymbol{M}$
m	element counter
N	bending moment
n	number of nodal points in a mesh
n	outward normal to the boundary of a solution domain
n	power-law index for a nonlinear material
n_c	number of elements at the centre of a circular mesh
n_q	number of a square in a mesh of right-angled triangles
n_r	number of nodes along a horizontal radius of a circular mesh
n_s	number of nodes per side of a triangular mesh
n_x, n_y	numbers of nodes per row of a rectangular mesh in the Cartesian co-ordinate directions
P	an externally applied force
P_x, P_y, P_z	pressure gradients in Cartesian co-ordinate directions
p	pressure
p	number of nonzero coefficients per row of overall stiffness matrix
Q	shear force
Q	volumetric flow rate
Q	externally applied force required to maintain a boundary restraint
q	number of iterations for convergence of the Gauss–Seidel solution process
$\boldsymbol{R}_i$	subvector of internally applied forces (and moments) at node i of an element

$\boldsymbol{R}_m$	vector of internal forces (and moments) applied to an element at its nodes
r	radial co-ordinate
r_1, r_2	relative efficiency parameters for methods of solving linear algebraic equations
$\overline{r}_m$	radius of the centroid of element m
S	mesh scale factor
S_x, S_y	summations involved in the Gauss–Seidel method, defined in Eq. 6.43
s	distance along a solution domain boundary
T	temperature
t	time
U_i, U_j, U_k	force components in x-direction applied internally to an element at its nodes
u	displacement or velocity in x-direction
$\overline{u}$	a mean value of u
V_i, V_j, V_k	force components in y-direction applied internally to an element at its nodes
V_x, V_z	velocity components of a boundary in Cartesian co-ordinate directions
v	displacement or velocity in y-direction
$\overline{v}$	a mean value of v
W	width of a channel or solution domain in general
W	load applied to the end of a cantilevered beam
W_i, W_j, W_k	force components in z-direction applied internally to an element at its nodes
w	displacement or velocity in z-direction
X, Y	global Cartesian co-ordinates
$\overline{X}, \overline{Y}, \overline{Z}$	components of body forces per unit volume in Cartesian co-ordinate directions
x, y, z	Cartesian co-ordinates
α	coefficient of thermal expansion
α	prescribed value of dependent variable at a boundary
β	prescribed value of derivative normal to a boundary
γ	an angle
Δ	difference operator defining change in the subsequent quantity
Δ_m	element area

$\boldsymbol{\delta}$	overall vector of unknowns such as displacements or velocities
δ_i	unknown such as displacement or velocity at node i
$\boldsymbol{\delta}_i$	subvector of unknowns such as displacements or velocities at node i
$\boldsymbol{\delta}_m$	vector of element unknowns such as displacements or velocities
ϵ	constant of proportionality in truncation error term
η	an unknown in a finite element analysis
θ	angle of rotation per unit length of a bar in torsion
θ	angular co-ordinate
$\boldsymbol{\theta}$	overall thermal force vector
$\boldsymbol{\theta}_i$	subvector of thermal force components at node i of an element
$\boldsymbol{\theta}_m$	element thermal force vector
θ_i, θ_j	rotations of the ends of a beam element
$\theta_i, \theta_j, \theta_k$	angles at the corners of a triangular element
κ	permeability of a porous medium
λ	parameter defined in Eq. 2.86 or Eq. 3.41
μ	viscosity
ν	Poisson's ratio
π_{P}	dimensionless pressure gradient
π_{Q}	dimensionless flow rate
ρ	density
σ	stress
$\boldsymbol{\sigma}$	element stress vector
σ_e	equivalent stress in simple tension
σ_0	reference stress for a nonlinear material
ϕ	angular co-ordinate
ϕ_1, ϕ_2	functions of position used in general harmonic and biharmonic Eqs. 2.87 and 2.88
χ	stress function
χ	functional used in variational formulation of finite element analyses
ψ	stream function, or dependent variable in general
ω	vorticity
ω	over-relaxation factor
∇^2	harmonic operator
∇^4	biharmonic operator