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Stochastic Analysis for Gaussian Random Processes and Fields

With Applications

Vidyadhar S. Mandrekar
Leszek Gawarecki

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We dedicate this book to the memory of
Professor R.R. Bahadur
and to the mentor of V. Mandrekar
Professor S.D. Chatterji

Preface

The book studies the Gaussian random fields using the structure of the Hilbert space of functions on the parameter space associated with the covariance, the so-called Reproducing Kernel Hilbert Space (RKHS). The RKHS associated with the covariance kernel was first used by Aronszajn [3]. It was used in the context of equivalence and singularity of Gaussian Random Fields by Parzen [103] and Kallianpur and Oodaira [55]. Their idea was exploited to provide a simplified presentation of the problem in [15]. The beauty of the technique is in using methods of Hilbert space to study deeper analytic properties connecting probabilistic notions. A good example of this approach is the work of Skorokhod [118], which studies differentiation on the space of random variables. Further application in the analysis of differentiation was achieved by Malliavin, who introduced this concept independently in his fundamental work [76] using what is now called the Malliavin derivative. Using the ideas of Nualart [94], [95], we show that the two concepts coincide. For deeper analysis, one can see the publications of Bismut [6]. The work of Kallianpur and Mandrekar [81] on Lévy Markov property connects it to the analytic structure of RKHS of the Gaussian random field. This explains the free field Markov process of Nelson [90] and local operators of Kusuoka [69]. Our purpose in this book is to start with the study of RKHS of Aronszajn and associating a Gaussian subspace with the RKHS, as in Kakutani [51], to explain the concepts introduced above. In addition, we present the applications of these ideas to current research in the areas of finance, spatial statistics, and the filtering and analytic problem related to fractional Brownian motion. The earlier development of some chapters originated in [79].

The book starts with the presentation of preliminary results on covariance and associated RKHS needed in Chapter 1. We then introduce the Gaussian process as a map π between RKHS $K(C)$ and a subspace of square integrable functions on a probability space, $L^2(\Omega,\mathcal{F},P)$. This gives a natural definition of the Wiener integral as in [80]. It is then easy to obtain the integral representation of certain Gaussian processes. We end Chapter 2 by presenting the definition of multiple Wiener integrals for a general Gaussian process as an extension of the map π to tensor products $K(C)^{\otimes n}$ of RKHSs. As a consequence we give a representation of elements of $L^2(\Omega,\mathcal{F},P)$ in terms of multiple Wiener integrals (chaos expansion). This work is based on the approach in [84].

The chaos expansion is used in Chapter 3 to define the Skorokhod integral, which generalizes the Itô integral. We then define the Ogawa integral following [31], which generalizes the Stratonovich integral. In addition, to relate the ideas of Malliavin and Skorokhod we present the concept of Skorokhod differentiation, and in Chapter 4 we show that, in the case of Brownian motion, it coincides with the Malliavin derivative. Therefore, the Skorokhod integral is a dual operator of Skorokhod differentiation and thus it is the divergence operator of Malliavin. Our presentation follows the ideas in [94] and [95]. We end Chapter 4 by showing the role of stochastic differentiation in finance by discussing the concept of stochastic duration [57]. As an application of our approach, we derive the recent results of [64] on the Itô formula for Gaussian processes with a special structure of covariance with application to fractional Brownian motion.

In Chapter 5, we study Gaussian processes indexed by real numbers and obtain a Kallianpur–Striebel Bayes formula for the filtering problem with the measurement error being a general Gaussian process following [77]. In a particular case, which includes fractional Brownian motion, we derive the analogue of the Zakai equation. One can then solve the filtering problem, including Kalman filtering, using fractional Brownian motion noise. The techniques used are based on the work in [33] and [35].

We consider the problem of equivalence and singularity of Gaussian random fields in Chapter 6. As in [14], we obtain general conditions for equivalence and singularity in terms of the relation between RKHSs associated with the covariances of two fields. In a special case of stationary random fields, we use these results to obtain spectral conditions used by Stein [120] in the problem of interpolation of stationary Gaussian random processes and analogue results in [46] for stationary random fields. The approach is as in [19].

At the end of the chapter, we give a generalization of the Girsanov theorem for Gaussian random fields and derive other results in this direction as consequences following [32].

In Chapters 7 and 8, we study the Markov property of Gaussian random fields indexed by measures and generalized Gaussian random fields indexed by Schwartz space. This part is motivated by the work of Nelson [90], Dynkin [21], and Röckner [108]. However, our approach is through general conditions on RKHS of a Gaussian random field using the ideas of Molchan [88] and Kallianpur and Mandrekar [81]. To derive the results in [21] and [108], we use the techniques from [84] and this is accomplished as in [124]. In order to present the concepts involved, we needed to present some results on Dirichlet forms and associated Markov processes from Fukushima [30]. For the convenience of the reader, these are presented in Appendix 7.4 to Chapter 7. The Gaussian random field is associated with Green's function of the Markov process and is indexed by measures. The terminology is also explained.

In the final Chapter 8, we first derive from our general results the work of

Pitt [104] and Künch [66] on Markov property of Gaussian fields indexed by $\mathbb{R}^d$. In addition, we relate the general work on measure–indexed random fields with general conditions associated to the Dirichlet form generating the Markov process. This is a recent work of Albeverio and Mandrekar [2].

We regret that we cannot present here the interesting results of Okabe [101], Kotani [62], and Pitt [105] on Markov property of stationary Gaussian random fields with analytic conditions on spectral density. This involves additional technical structures from complex analysis, like, for example, ultra distributions. However, we refer the interested reader to the unpublished technical report by Mandrekar and Soltani [83].

Vidyadhar S. Mandrekar
Leszek Gawarecki

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