Study Guide to

Finite Mathematics

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STUDY GUIDE TO FINITE MATHEMATICS

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PART ONE LINEAR PROGRAMMING

CHAPTER 1 LINEAR PROGRAMMING: AN INTRODUCTION

1. Since the graph of a two variable linear equation is a line, it suffices, for each of the cited equations, to determine two points. Consider $2x + 4y = 120$, or equivalently, $x + 2y = 60$. If $x = 0$, $y = 30$; if $y = 0$, $x = 60$. Thus (0,30) and (60,0) are on $x + 2y = 60$. Its graph is shown in Figure 1a.

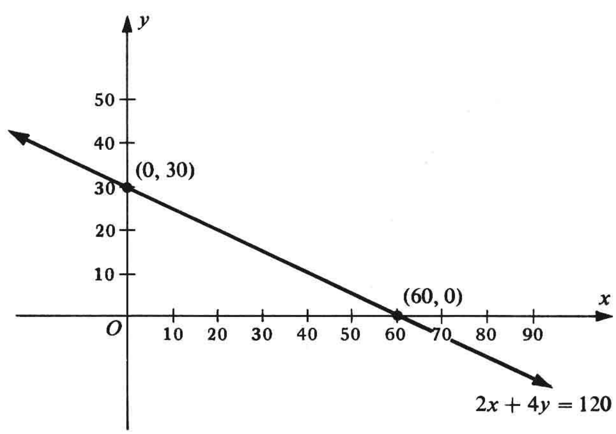

Figure 1a

The graph of $4x = 100$, or equivalently, $x = 25$, is a line parallel to the y-axis and 25 units to the right (see Figure 1b).

(0,55) and $(\frac{55}{2},0)$ are two points on the graph of $4x + 2y = 110$ (shown in Figure 1c). (0,600) and (600,0) are two points on the graph of $x + y = 600$ (shown in Figure 1d).

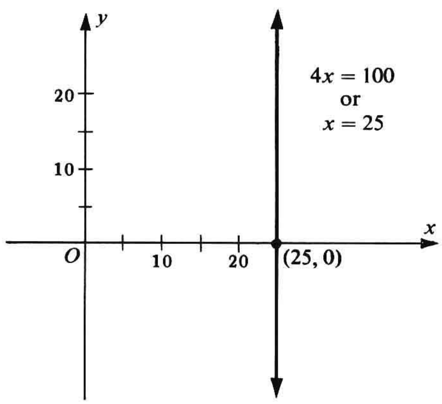

Figure 1b

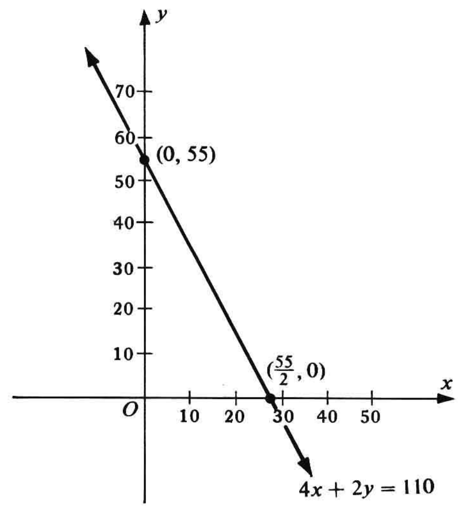

Figure 1c

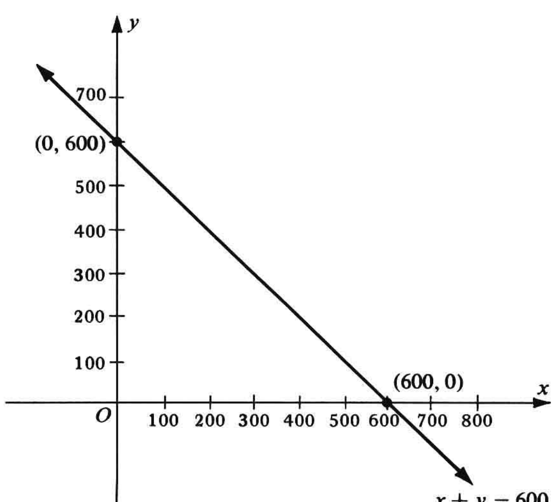

Figure 1d

2. (3,3) is a solution of $3x - 2y \leq 10$ since substitution of 3 for x and 3 for y in $3x - 2y \leq 10$ yields $3 \leq 10$, which is true.
(4,-2) is not a solution of $3x - 2y \leq 10$ since substitution of 4 for x and -2 for y yields $16 \leq 10$, which is not true.
(3,-1) yields $11 \leq 10$, and thus is not a solution.
(-1,6) yields $-15 \leq 10$, and thus is a solution.

3. (1,3,2) is a solution of $2x - 3y + z \leq 12$ since substitution of 1 for x, 3 for y, and 2 for z in $2x - 3y + z \leq 12$ yields $-5 \leq 12$, which is true.
(4,0,4) yields $12 \leq 12$, and thus is a solution.
(3,1,4) yields $7 \leq 12$, and thus is a solution.
(7,1,3) yields $14 \leq 12$, and thus is not a solution.

4. (25,0) is a solution of the system $(i_1) \rightarrow (i_5)$ since it satisfies all of the inequalities of the system. Substitution of 25 for x and 0 for y yields

$$25 \leq 0, \quad 0 \leq 0, \quad 100 \leq 100, \quad 50 \leq 120, \quad 100 \leq 110,$$

all of which are true.
(25,5), (0,30), and $(\frac{50}{3},\frac{65}{3})$ are solutions since they satisfy $(i_1) \rightarrow (i_5)$.
(26,2) is not a solution of $(i_1) \rightarrow (i_5)$ since it does not satisfy (i_3).

5. $(0,0,\frac{15}{4})$, (0,5,0), (6,0,0), $(\frac{9}{2},0,\frac{3}{2})$, and (3,3,0) are solutions since they satisfy all of the inequalities of the system. (0,0,6) and (0,6,0) are not solutions of the system since they do not satisfy (i_4).

6. (0,0,10), (0,20,0), (8,0,0), $(\frac{28}{5},0,\frac{8}{5})$, and (4,8,0) are solutions of the system, while (0,16,0), $(0,0,\frac{16}{3})$, and $(\frac{20}{3},0,0)$ are not. (0,16,0) and $(0,0,\frac{16}{3})$ do not satisfy (i_5). $(\frac{20}{3},0,0)$ does not satisfy (i_4).

7. (2500,2500,0), (3000,2000,0), (3000,2500,0), and (2750,2250,250) are solutions of the system. (0,0,2500) and (0,2500,0) are not solutions of the system since they do not satisfy (i_4).

8. The graph of $x + y \geq 600$ consists of the graph of the boundary line $x + y = 600$ together with the graph of $x + y > 600$. We first sketch the graph of $x + y = 600$ (see Exercise 1d, pp. 1-2). To determine the graph of the inequality $x + y > 600$ we choose a test point, (0,0) for example, and determine if it satisfies $x + y > 600$. Since (0,0) does not satisfy $x + y > 600$ and (0,0) is below the boundary line $x + y = 600$, the graph of $x + y > 600$ consists of all points above $x + y = 600$. The graph of $x + y \geq 600$ is shown in Figure 2.

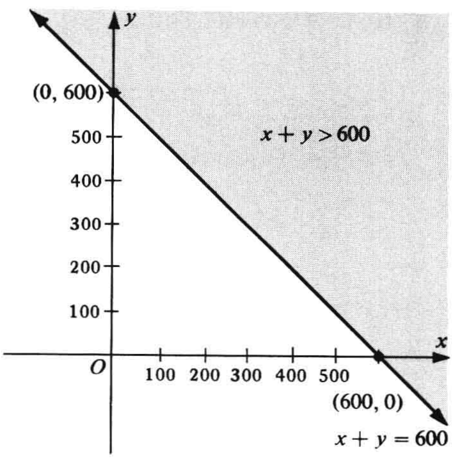

Figure 2

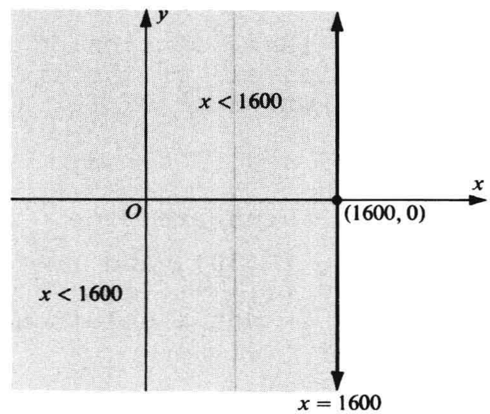

Figure 3

9. The graph of $x \leq 1600$ consists of all points on the boundary line $x = 1600$ and to the left of $x = 1600$ (see Figure 3).

10. The graph of $y \leq 2000$ consists of all points on the boundary line $y = 2000$ and below $y = 2000$ (see Figure 4).

11. The graph of $x + y \leq 2400$ consists of all points on the boundary line $x + y = 2400$ and below $x + y = 2400$ (see Figure 5). That we require points below $x + y = 2400$ can be seen from the fact that the test point (0,0), which lies below $x + y = 2400$, satisfies $x + y < 2400$. Thus all

points which satisfy $x + y < 2400$ lie below $x + y = 2400$.

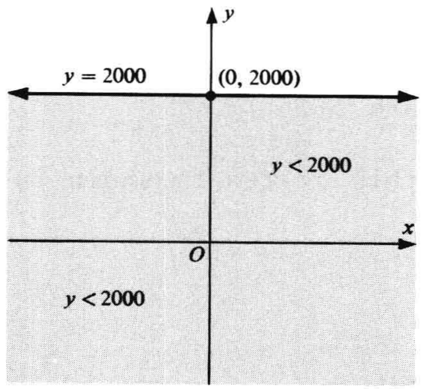

Figure 4

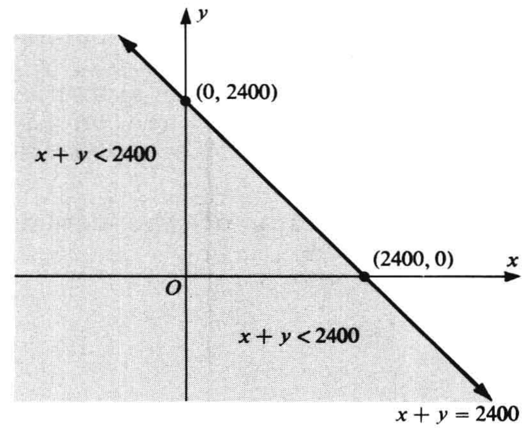

Figure 5

12. The graph of this system is the overlap of the graphs of the members of the system, $x \geq 0$, $y \geq 0$, $x + y \geq 600$. Since $x \geq 0$, $y \geq 0$ specifies the first quadrant and the graph of $x + y \geq 600$ consists of all points on and above the line $x + y = 600$ (see Exercise 8, p. 4), the graph of our system consists of all points in the first quadrant which lie above and on the boundary line $x + y = 600$ (see Figure 6).

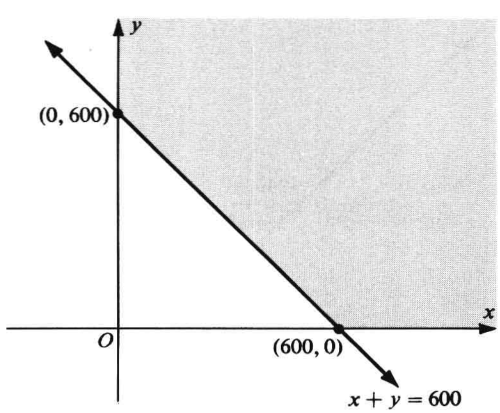

Figure 6

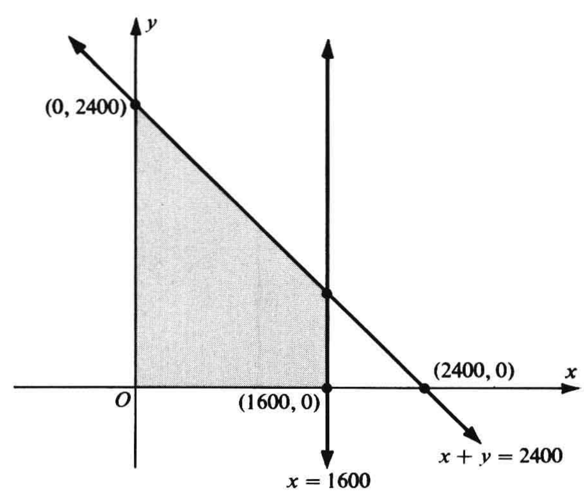

Figure 7

13. The graph of $x + y \leq 2400$ consists of all points on and below the boundary line $x + y = 2400$ (see Exercise 11, pp. 4-5). The graph of $x \leq 1600$ consists of all points on and to the left of the boundary line $x = 1600$ (see Exercise 9, p. 4). Since $x \geq 0$, $y \geq 0$ specifies the first quadrant, the graph of the system consists of all points in the first quadrant which are on or below $x + y = 2400$ and on or to the left of $x = 1600$ (see Figure 7).

14. The overlap of the members of this system is shown in Figure 8.

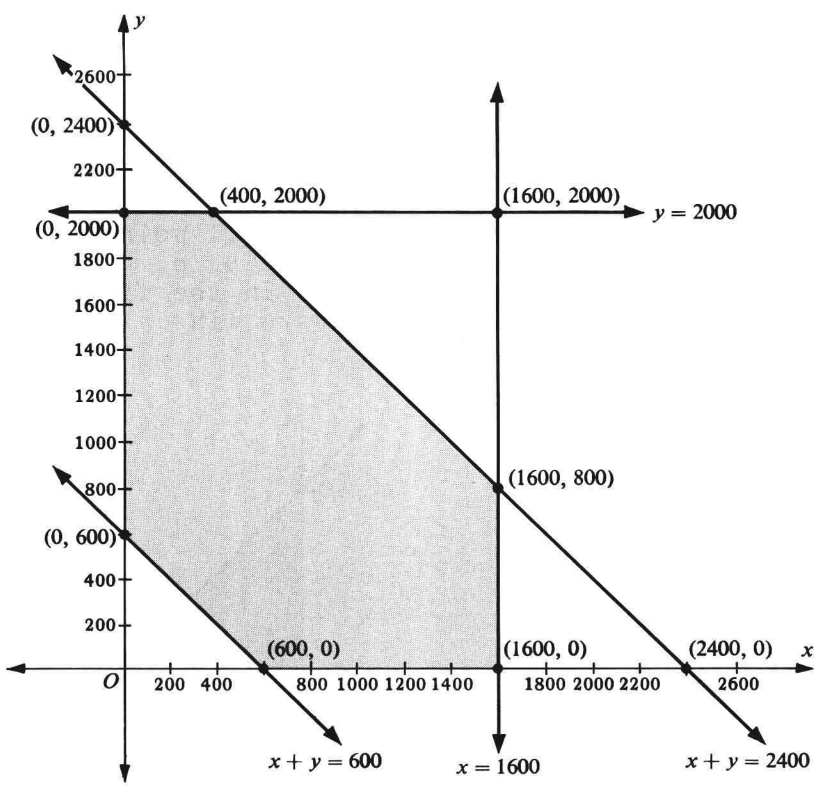

Figure 8

15. The overlap of the members of this system is shown in Figure 9.

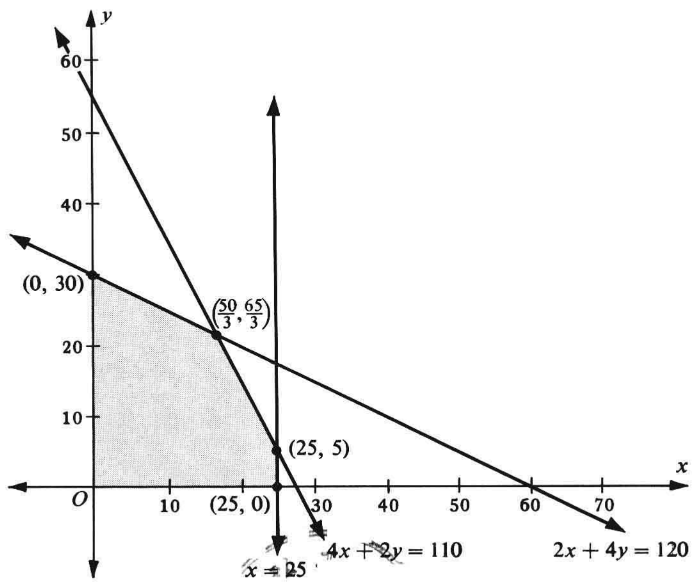

Figure 9

16. The overlap of the members of this system is shown in Figure 10.

17. The overlap of the members of this system is shown in Figure 11.

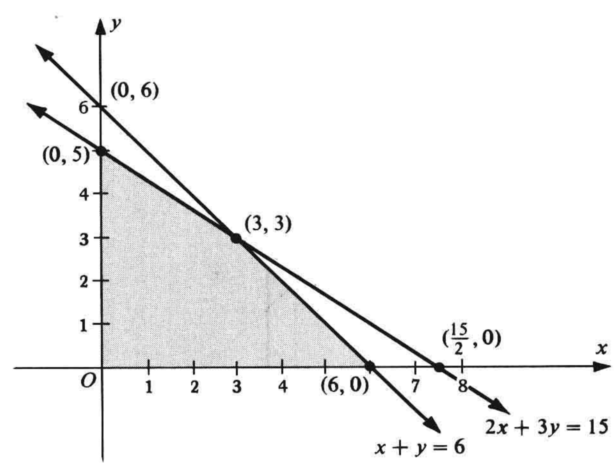

Figure 10

18. The overlap of the members of this system is shown in Figure 12.

19. The overlap of the members of this system is shown in Figure 13.

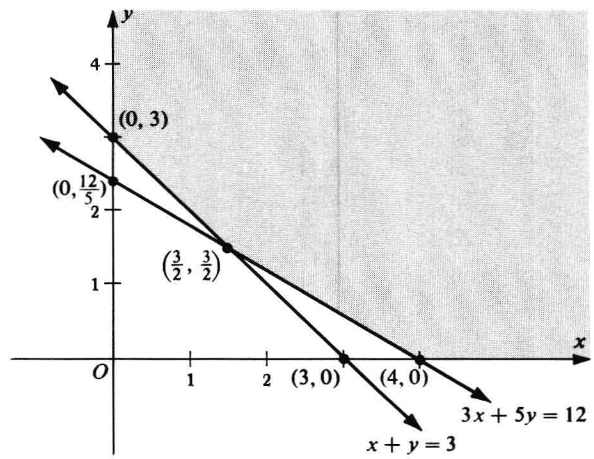

Figure 11

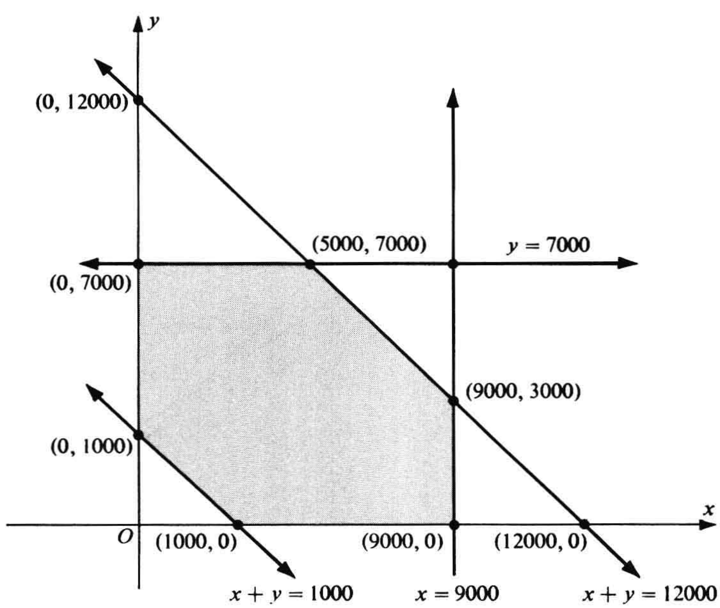

Figure 12

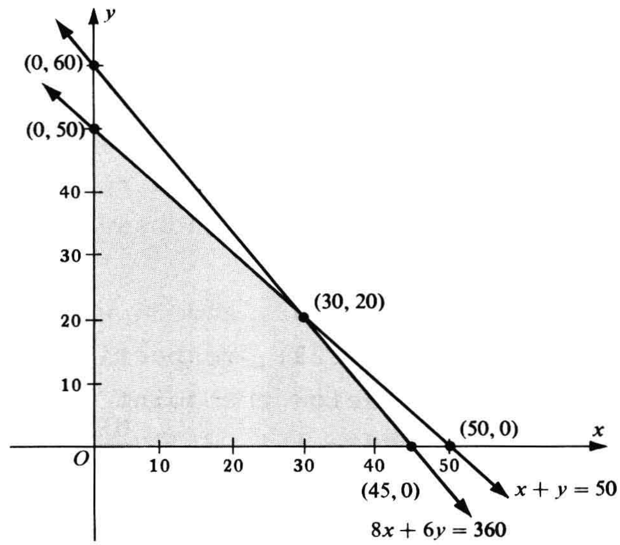

Figure 13

20. The overlap of the members of this system is shown in Figure 14.

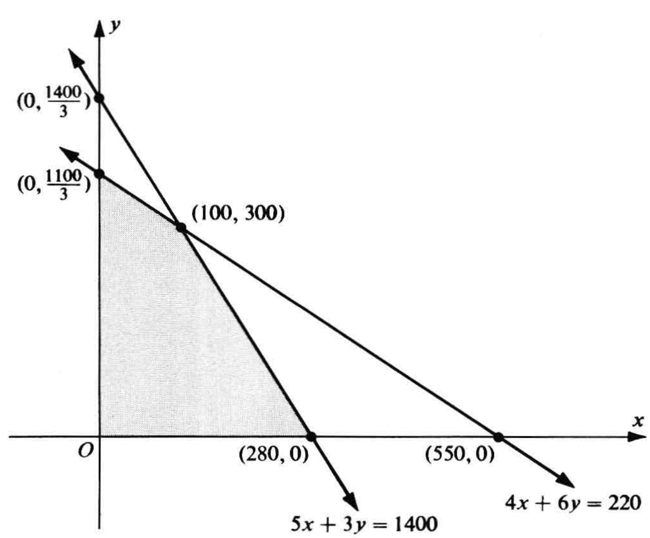

Figure 14

21. Solving $5x + 2y = 220$ for y yields $y = \frac{220 - 5x}{2}$. Let x equal 29, 28, and 26. Then y equals $\frac{75}{2}$, 40, and 45, respectively, so that $(29,\frac{75}{2})$, (28,40), and (26,45) are on $5x + 2y = 220$, above the point (30,35), and in the same direction (see Figure 15). $P(29,\frac{75}{2}) = 2575$, $P(28,40) = 2600$, and $P(26,45) = 2650$. Thus $P(x,y) = 50x + 30y$ takes on values greater than 2550 and increases as we go from $(29,\frac{75}{2})$ to (28,40) to (26,45).
By giving x the values 31, 32, and 34 we obtain the points $(31,\frac{65}{2})$, (32,30), and (34,25), respectively. These points are on $5x + 2y = 220$, below the point (30,35), and in the same direction (see Figure 15). $P(31,\frac{65}{2}) = 2525$, $P(32,30) = 2500$, and $P(34,25) = 2450$. Thus P(x,y) takes on values less than 2550 and decreases as we go from $(31,\frac{65}{2})$, to (32,30) to (34,25).

22. $Q_3(x_1,y_3)$ is on the line $5x + 2y = 220$ means that $5x_1 + 2y_3 = 220$. Solving for y_3 yields

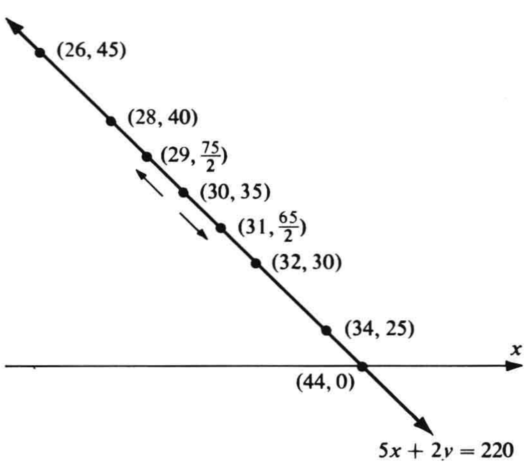

Figure 15

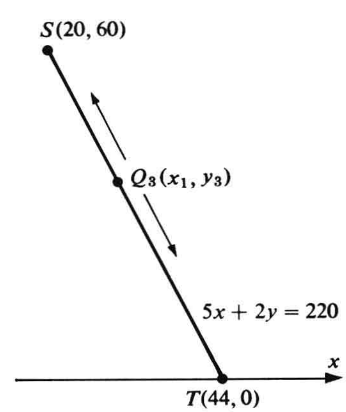

Figure 16

$$y_3 = \frac{220 - 5x_1}{2}.$$

Also $P(x_1,y_3) = 50x_1 + 30y_3$. Substituting $\frac{220 - 5x_1}{2}$ for y_3 in $P(x_1,y_3)$ yields

$$P(x_1,y_3) = 50x_1 + 30\left(\frac{220 - 5x_1}{2}\right) = 3300 - 25x_1.$$

As we move from $Q_3(x_1,y_3)$ toward $S(20,60)$ (see Figure 16) we decrease x_1 and thus increase $P(x_1,y_3)$, since less and less is being subtracted from 3300. As we move from $Q_3(x_1,y_3)$ toward $T(44,16)$ (see Figure 16) we increase x_1 and thus decrease $P(x_1,y_3)$, since more and more is being subtracted from 3300.

23. If $x + y = 600$, then $y = 600 - x$. Substituting $600 - x$ for y in the function $C(x,y) = 2x + 2y + 12{,}400$ yields

$$C(x,y) = 2x + 2(600 - x) + 12{,}400 = 13{,}600.$$

Every point (x,y) on $x + y = 600$ between $(600,0)$ and $(0,600)$ is a feasible point, and is thus a solution, since (x,y) yields the minimum value 13,600.

24. The graph of the feasible points of this linear program is shown in Figure 17. The corner points are $(0,0)$, $(6,0)$,

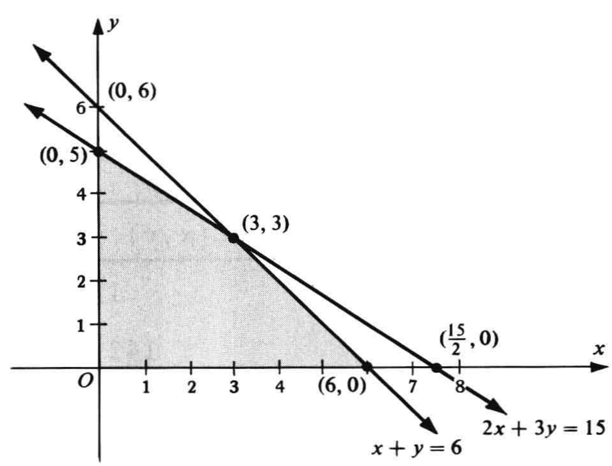

Figure 17

(3,3), and (0,5). From Table 1 we see that (3,3) is the solution and 129 is the maximum value.

Table 1

Corner Point	$P(x,y) = 20x + 23y$
(0,0)	0
(6,0)	120
(3,3)	129
(0,5)	115

25. The graph of the feasible points of this linear program is shown in Figure 18. The corner points are (4,0), $(\frac{3}{2},\frac{3}{2})$, and (0,3). From Table 2 we see that $(\frac{3}{2},\frac{3}{2})$ is the solution and 142.5 is the minimum value.

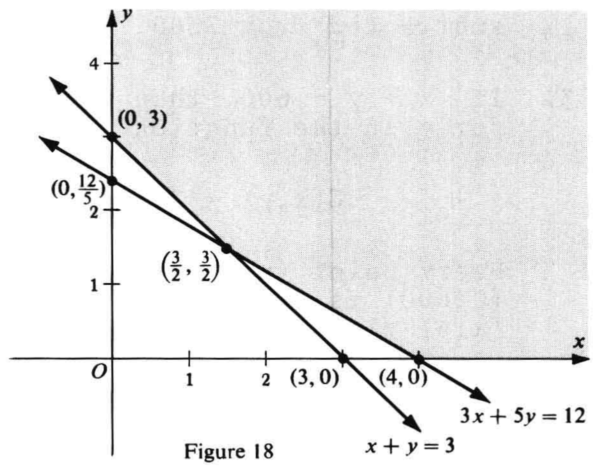

Figure 18

Table 2

Corner Point	$C(x,y) = 45x + 50y$
(4,0)	180
$(\frac{3}{2},\frac{3}{2})$	142.5
(0,3)	150

26. The graph of the feasible points of this linear program is shown in Figure 19. The corner points are (1000,0), (9000,0), (9000,3000), (5000,7000), (0,7000), and (0,1000). From Table 3 we see that (1000,0) is the solution and 4100 is the minimum value.

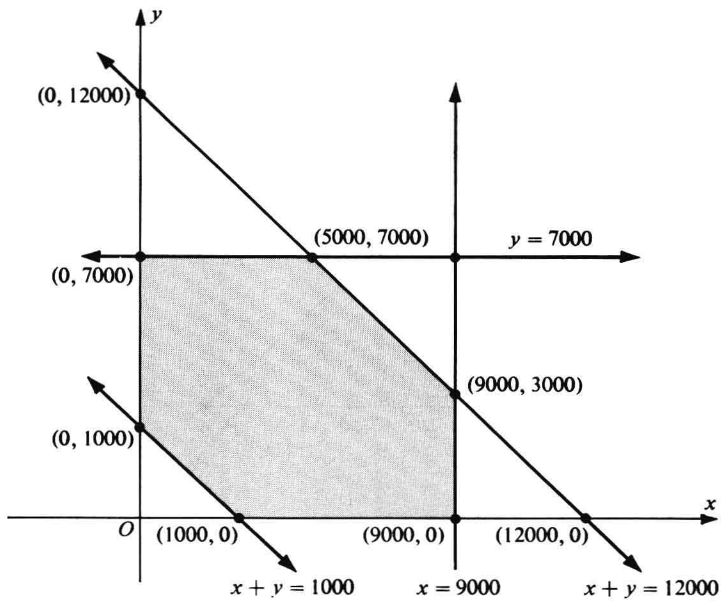

Figure 19

Table 3

Corner Point	$T(x,y) = .15x + .5y + 3950$
(1000,0)	4100
(9000,0)	5300
(9000,3000)	6800
(5000,7000)	8200
(0,7000)	7450
(0,1000)	4450