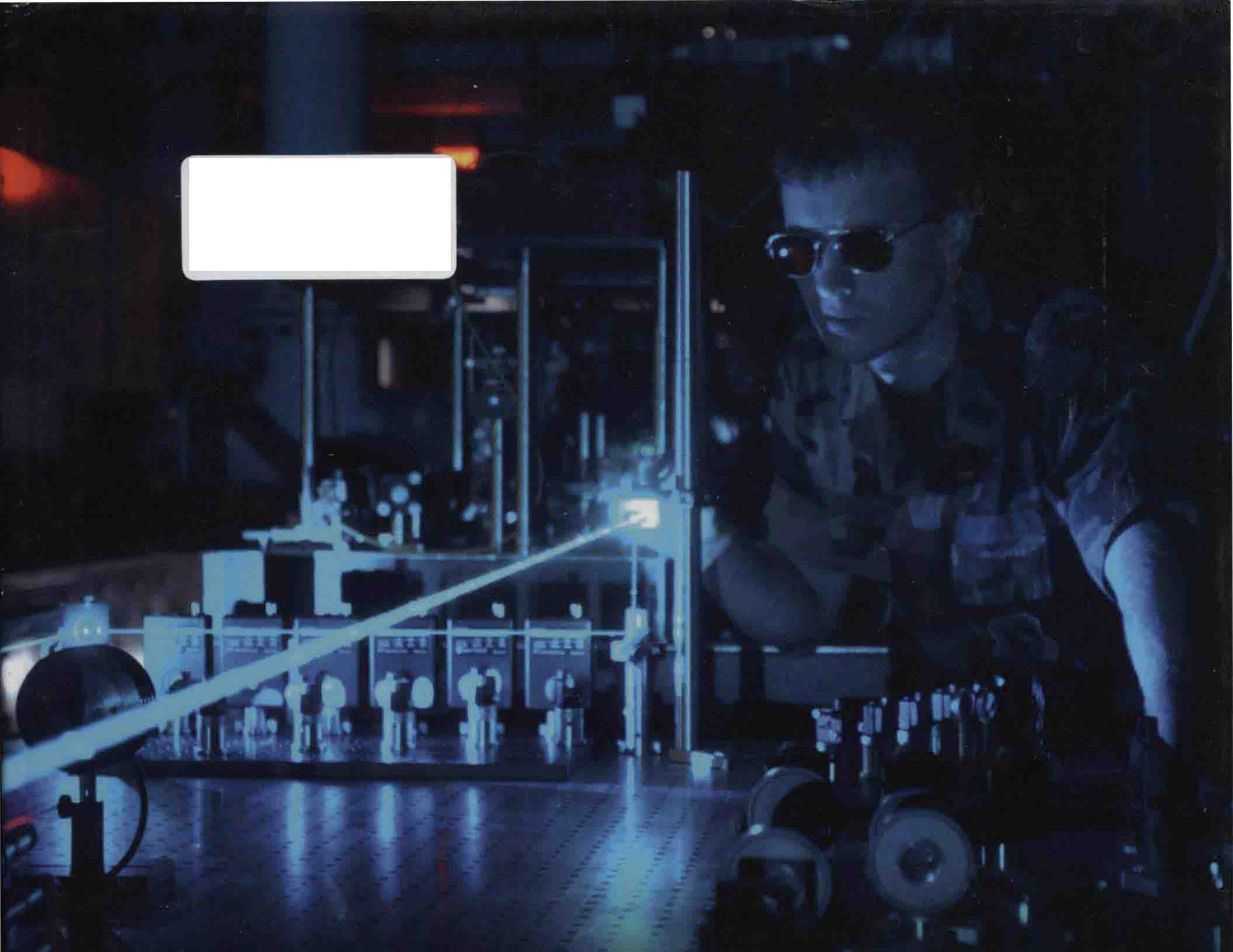

Handbook of

Systematic Approaches in Engineering Physics

Samuel Woodworth
Editor

Handbook of Systematic Approaches in Engineering Physics

Samuel Woodworth
Editor

AURIS Reference
AURIS REFERENCE LTD.
London, UK

Handbook of Systematic Approaches in Engineering Physics

Published by
Auris Reference Ltd., UK
www.aurisreference.com

ISBN: 978-1-78154-406-8

Editor: Samuel Woodworth

Printed in UK

10 9 8 7 6 5 4 3 2 1

Cover Design: Cover Lab

For information about Auris Reference Ltd and its publications, visit our website at www.aurisreference.com

Handbook of
Systematic Approaches in Engineering Physics

Preface

Engineering physics is the study of the combined disciplines of physics, engineering and mathematics in order to develop an understanding of the interrelationships of these three disciplines. Fundamental physics is combined with problem solving and engineering skills, which then has broad applications. This interdisciplinary knowledge is designed for the continuous innovation occurring with technology.

Unlike traditional engineering disciplines, engineering science/physics is not necessarily confined to a particular branch of science or physics. Instead, engineering science/physics is meant to provide a more thorough grounding in applied physics for a selected speciality such as optics, quantum physics, materials science, applied mechanics, nanotechnology, microfabrication, mechanical engineering, electrical engineering, biophysics, control theory, aerodynamics, energy, solid-state physics, etc. It is the discipline devoted to creating and optimizing engineering solutions through enhanced understanding and integrated application of mathematical, scientific, statistical, and engineering principles. The discipline is also meant for cross-functionality and bridges the gap between theoretical science and practical engineering with emphasis in research and development, design, and analysis. Physics is the mother of all sciences. Mathematics and Chemistry are finally the derivates of Physics. Engineering is also a form of applied science with Physics as an important part. Initially, Engineering started with Mechanical and Civil engineering as the main branches. Both the streams are derivatives of Mechanics which in turn is a form of Physics. Hence, the two branches do not have any meaning without Physics. Civil engineering involves a major part of geology, which is also a derivative of Physics. Recently, some other branches like Biomedical engineering and Bio-Technology have also been derived. These two streams are derivatives of Biology. Biology also has got thick links with principles Physics. Both these branches are a blend of applied biology with principles of molecular physics. Thus, it is true that Physics has a significant role in Engineering. Another major part of engineering is Electrical engineering. This is a derivative of Electrical Physics. Since Electrical engineering leads to Electronics engineering and finally to Computer engineering and Information Technology, it can be concluded that the mother of all engineering branches is Physics. New applications of physics can push the boundary of what is possible in electrical engineering, particularly in the areas of materials and devices, both solid-state electronic and optical. Research in applied physics seeks to enable new directions using a combination of theoretical and experimental investigations of novel quantum phenomena, both at a fundamental level, as well as in applications. These include physics and technology of materials at the nanometer scale, engineered crystal growth, low dimensional materials (natural and engineered) with quantum degrees of freedom exhibiting novel many-body phenomena, new lasers, sensors and optical technologies. Together, these fields provide new paradigms for the discovery of exotic new materials, systems, and physical principles, and lead to advancements in computation and communication on the one hand, as well as energy conservation and improved health and the environment on the other.

This book is designed to be used at the graduate level students in physics, applied physics and engineering physics.

—*Editor*

Contents

Chapter 1

Elastic Properties of materials and Waves and Vibrations

Elasticity (Physics)

In physics, elasticity is a physical property of materials which return to their original shape after they are deformed.

Overview

When an elastic material is deformed due to an external force, it experiences internal forces that oppose the deformation and restore it to its original state if the external force is no longer applied. There are various elastic moduli, such as Young's modulus, the shear modulus, and the bulk modulus, all of which are measures of the inherent stiffness of a material as a resistance to deformation under an applied load. The various moduli apply to different kinds of deformation. For instance, Young's modulus applies to uniform extension, whereas the shear modulus applies to shearing.

The elasticity of materials is described by a stress-strain curve, which shows the relation between stress (the average restorative internal force per unit area) and strain (the relative deformation). For most metals or crystaline materials, the curve is linear for small deformations, and so the stress-strain relationship can adequately be described by Hooke's law and higher-order terms can be ignored. However, for larger stresses beyond the elastic limit, the relation is no longer linear. For even higher stresses, materials exhibit plastic behaviour, that is, they deform irreversibly and do not return to their original shape after stress is no longer applied. For rubber-like materials such as elastomers, the gradient of the stress-strain curve increases with stress, meaning that rubbers progressively become more difficult to stretch, while for most metals, the gradient decreases at very high stresses, meaning that they progressively become easier to stretch. Elasticity is not exhibited only by only solids; non-Newtonian fluids, such as viscoelastic fluids, will also exhibit elasticity in certain conditions. In response to a small, rapidly applied and removed strain, these fluids may deform and then return to their original shape. Under larger strains, or strains applied for longer periods of time, these fluids may start to flow like a viscous liquid.

Hooke's Law

As noted above, for small deformations, most elastic materials such as springs exhibit linear elasticity. This idea was first formulated by Robert Hooke in 1675 as a Latin anagram, "ceiiinosssttuv". He published the answer in 1678: "*Ut tensio, sic vis*" meaning "*As the extension, so the force*", a linear relationship commonly referred to as Hooke's law. This law can be stated as a relationship between force F and displacement x,

$$F = -kx,$$

where k is a constant known as the *rate* or *spring constant*. It can also be stated as a relationship between stress σ and strain ε:

$$\sigma = E\varepsilon,$$

where E Is known as the elastic modulus or Young's modulus.

Although the general proportionality constant between stress and strain in three dimensions is a 4th order tensor, systems that exhibit symmetry, such as a one-dimensional rod, can often be reduced to applications of Hooke's law.

Factors Affecting Elasticity

For isotropic materials, the presence of fractures affects the Young and the shear modulus perpendicular to the planes of the cracks, which decrease (Young's modulus faster than the shear modulus) as the fracture density increases, indicating that the presence of cracks makes bodies brittler. Microscopically, the stress-strain relationship of materials is in general governed by the Helmholtz free energy, a thermodynamic quantity. Molecules settle in the configuration which minimizes the free energy, subject to constraints derived from their structure, and, depending on whether the energy or the entropy term dominates the free energy, materials can broadly be classified as *energy-elastic* and *entropy-elastic*. As such, microscopic factors affecting the free energy, such as the equilibriumdistance between molecules, can affect the elasticity of materials: for instance, in inorganic materials, as the equilibrium distance between molecules at 0 K increases, the bulk modulus decreases. The effect of temperature on elasticity is difficult to isolate, because there are numerous factors affecting it. For instance, the bulk modulus of a material is dependent on the form of its lattice, its behaviour under expansion, as well as the vibrations of the molecules, all of which are dependent on temperature.

Bending Moment

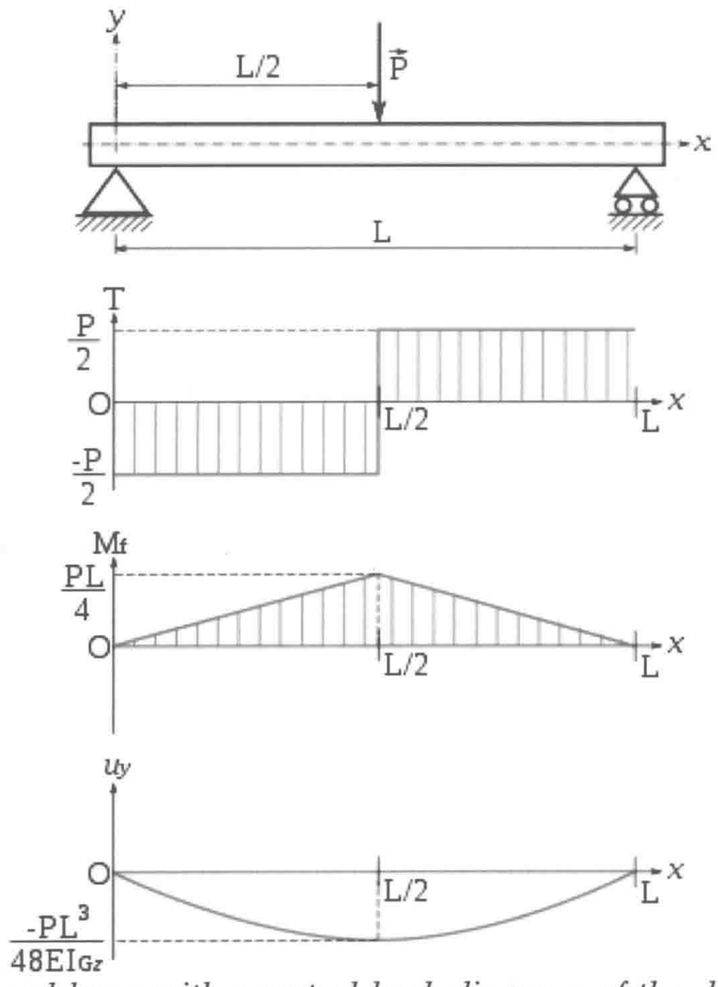

Figure: *Hinged beam with a central load: diagrams of the shearing force T, the bending moment Mf and the deflection uy; the stresses are calculated at the right of the cut.*

A bending moment exists in a structural element when a moment is applied to the element so that the element bends. Moments and torques are measured as a force multiplied by a distance so they have as unit newton-metres (N·m), or pound-foot or foot-pound (ft·lb). The concept of bending moment is very important in engineering (particularly in civil and mechanical engineering) and physics.

Discussion

Tensile and compressive stresses increase proportionally with bending moment, but are also dependent on the second moment of area of the cross-section of the structural element. Failure in bending will occur when the bending moment is sufficient to induce tensile stresses greater than the yield stress of the material throughout the entire cross-section. It is possible that failure of a structural element in shear may occur before failure in bending, however the mechanics of failure in shear and in bending are different.

The bending moment at a section through a structural element may be defined as "the sum of the moments about that section of all external forces acting to one side of that section". The forces and moments on either side of the section must be equal in order to counteract each other and maintain a state of equilibrium so the same bending moment will result from summing the moments, regardless of which side of the section is selected.

Moments are calculated by multiplying the external vector forces (loads or reactions) by the vector distance at which they are applied. When analysing an entire element, it is sensible to calculate moments at both ends of the element, at the beginning, centre and end of any uniformly distributed loads, and directly underneath any point loads. Of course any "pin-joints" within a structure allow free rotation, and so zero moment occurs at these points as there is no way of transmitting turning forces from one side to the other.

If clockwise bending moments are taken as negative, then a negative bending moment within an element will cause "sagging", and a positive moment will cause "hogging". It is therefore clear that a point of zero bending moment within a beam is a point of contraflexure—that is the point of transition from hogging to sagging or vice versa. It is more common to use the convention that a clockwise bending moment to the left of the point under consideration is taken as positive. This then corresponds to the second derivative of a function which, when positive, indicates a curvature that is 'lower at the centre' i.e. sagging. When defining moments and curvatures in this way calculus can be more readily used to find slopes and deflections.

Critical values within the beam are most commonly annotated using a bending moment diagram, where negative moments are plotted to scale above a horizontal line and positive below. Bending moment varies linearly over unloaded sections, and parabolically over uniformly loaded sections.

Dynamic Bending of Beams

The dynamic bending of beams, also known as flexural vibrations of beams, was first investigated by Daniel Bernoulli in the late 18th century. Bernoulli's equation of motion of a vibrating beam tended to overestimate the natural frequencies of beams and was improved marginally by Rayleigh in 1877 by the addition of a mid-plane rotation. In 1921 Stephen Timoshenko improved the theory further by incorporating the effect of shear on the dynamic response of bending beams. This allowed the theory to be used for problems involving high frequencies of vibration where the dynamic Euler-Bernoulli theory is inadequate. The Euler-Bernoulli and Timoshenko theories for the dynamic bending of beams continue to be used widely by engineers.

Euler-Bernoulli Theory

The Euler-Bernoulli equation for the dynamic bending of slender, isotropic, homogeneous beams of constant cross-section under an applied transverse load $q(x,t)$ is

$$EI\frac{\partial^4 w}{\partial x^4}+m\frac{\partial^2 w}{\partial t^2}=q(x,t)$$

where $E$is the Young's modulus, $I$is the area moment of inertia of the cross-section, $w(x,t)$ is the deflection of the neutral axis of the beam, and m is mass per unit length of the beam.

Free Vibrations

For the situation where there is no transverse load on the beam, the bending equation takes the form

$$EI\frac{\partial^4 w}{\partial x^4}+m\frac{\partial^2 w}{\partial t^2}=0$$

Free, harmonic vibrations of the beam can then be expressed as

$$w(x,t)=\mathrm{Re}[\hat{w}(x)e^{-i\omega t}] \quad \Rightarrow \quad \frac{\partial^2 w}{\partial t^2}=-\omega^2\, w(x,t)$$

and the bending equation can be written as

$$EI\frac{\mathrm{d}^4\hat{w}}{\mathrm{d}x^4}-m\omega^2\hat{w}=0$$

The general solution of the above equation is

$$\hat{w}=A_1\cosh(\beta x)+A_2\sinh(\beta x)+A_3\cos(\beta x)+A_4\sin(\beta x)$$

where A_1, A_2, A_3, A_4 are constants and $\beta := \left(\frac{m}{EI}\omega^2\right)^{1/4}$

Timoshenko-Rayleigh Theory

In 1877, Rayleigh proposed an improvement to the dynamic Euler-Bernoulli beam theory by including the effect of rotational inertia of the cross-section of the beam. Timoshenko improved upon that theory in 1922 by adding the effect of shear into the beam equation. Shear deformations of the normal to the mid-surface of the beam are allowed in the Timoshenko-Rayleigh theory.

The equation for the bending of a linear elastic, isotropic, homogeneous beam of constant cross-section beam under these assumptions is

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} - \left(J + \frac{EIm}{kAG}\right)\frac{\partial^4 w}{\partial x^2\,\partial t^2} + \frac{Jm}{kAG}\frac{\partial^4 w}{\partial t^4} = q(x,t) + \frac{J}{kAG}\frac{\partial^2 q}{\partial t^2} - \frac{EI}{kAG}\frac{\partial^2 q}{\partial x^2}$$

where $J = \frac{mI}{A}$ is the polar moment of inertia of the cross-section, $m = \rho A$ is the mass per unit length of the beam, ρ is the density of the beam, A is the cross-sectional area, G is the shear modulus, and k is a shear correction factor. For materials with Poisson's ratios (v) close to 0.3, the shear correction factor are approximately

$$k = \frac{5+5\nu}{6+5\nu} \quad \text{rectangular cross-section}$$

$$= \frac{6+12\nu+6\nu^2}{7+12\nu+4\nu^2} \quad \text{circular cross-section}$$

Free Vibrations

For free, harmonic vibrations the Timoshenko-Rayleigh equations take the form

$$EI\frac{\mathrm{d}^4\hat{w}}{\mathrm{d}x^4} + m\omega^2\left(\frac{J}{m} + \frac{EI}{kAG}\right)\frac{\mathrm{d}^2\hat{w}}{\mathrm{d}x^2} + m\omega^2\left(\frac{\omega^2 J}{kAG} - 1\right)\hat{w} = 0$$

This equation can be solved by noting that all the derivatives of w must have the same form to cancel out and hence as solution of the form e^{kx} may be expected. This observation leads to the characteristic equation

$$\alpha k^4 + \beta k^2 + \gamma = 0\ ;\ \alpha := EI\ ,\ \beta := m\omega^2\left(\frac{J}{m} + \frac{EI}{kAG}\right),\ \gamma := m\omega^2\left(\frac{\omega^2 J}{kAG} - 1\right)$$

The solutions of this quartic equation are

$$k_1 = +\sqrt{z_+}\ ,\ k_2 = -\sqrt{z_+}\ ,\ k_3 = +\sqrt{z_-}\ ,\ k_4 = -\sqrt{z_-}$$

where

$$z_+ := \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}, \; z_- := \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$

The general solution of the Timoshenko-Rayleigh beam equation for free vibrations can then be written as

$$\hat{w} = A_1 e^{k_1 x} + A_2 e^{-k_1 x} + A_3 e^{k_3 x} + A_4 e^{-k_3 x}$$

Cantilever

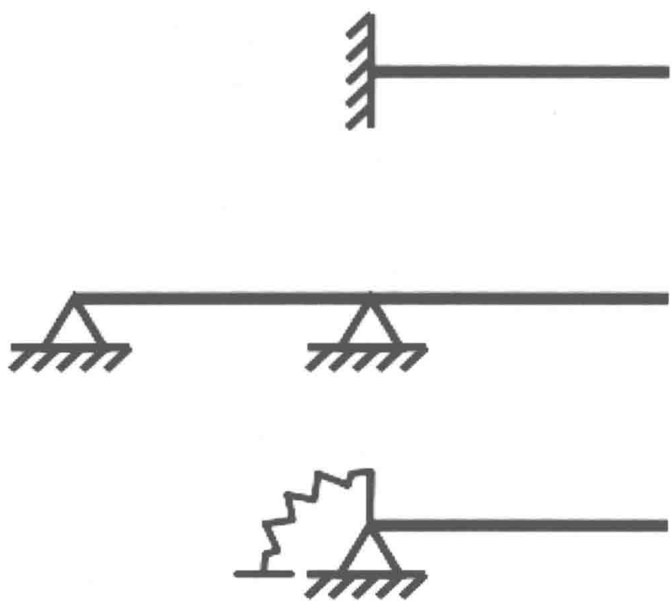

Figure: *A schematic image of three types of cantilevers. The top example has a full moment connection (like a horizontal flag pole bolted to the side of a building). The middle example is created by an extension of a simple supported beam (such as the way a diving-board is anchored and extends over the edge of a swimming pool). The bottom example is created by adding a Robin boundary condition to the beam element, which essentially adds an elastic spring to the end board. The middle and bottom example may be considered structurally equivalent, depending on the effective stiffness of the spring and beam element*

A cantilever is a beam anchored at only one end. The beam carries the load to the support where it is forced against by moment and shear stress. Cantilever construction allows for overhanging structures without external bracing. Cantilevers can also be constructed with trusses or slabs.

This is in contrast to a simply supported beam such as those found in a post and lintel system. A simply supported beam is supported at both ends with loads applied between the supports.

In Bridges, Towers, and Buildings

Cantilevers are widely found in construction, notably in cantilever bridges and balconies. In cantilever bridges the cantilevers are usually built as pairs, with each cantilever used to support one end of a central section. The Forth Bridge in Scotland is an example of a cantilever truss bridge. A cantilever in a traditionally timber framed building is called a jetty or forebay. In the southern United States a historic barn type is the cantilever barn of log construction.

Temporary cantilevers are often used in construction. The partially constructed structure creates a cantilever, but the completed structure does not act as a cantilever. This is very helpful when temporary supports, or falsework, cannot be used to support the structure while it is being built (e.g., over a busy roadway or river, or in a deep valley). So some truss arch bridges are built from each side as cantilevers until the spans reach each other and are then jacked apart to stress them in compression before final joining. Nearly all cable-stayed bridges are built using cantilevers as this is one of their chief advantages. Many box girder bridges are built segmentally, or in short pieces. This type of construction lends itself well to balanced cantilever construction where the bridge is built in both directions from a single support.

These structures are highly based on torque and rotational equilibrium.

In an architectural application, Frank Lloyd Wright's Fallingwater used cantilevers to project large balconies. The East Stand at Elland Road Stadium in Leeds was, when completed, the largest cantilever stand in the world holding 17,000 spectators. The roof built over the stands at Old Trafford Football Ground uses a cantilever so that no supports will block views of the field. The old, now demolished Miami Stadium had a similar roof over the spectator area. The largest cantilever in Europe is located at St James' Park in Newcastle-Upon-Tyne, the home stadium of Newcastle United F.C.

Less obvious examples of cantilevers are free-standing (vertical) radio towers without guy-wires, and chimneys, which resist being blown over by the wind through cantilever action at their base.

Aircraft

Another use of the cantilever is in fixed-wing aircraft design, pioneered by Hugo Junkers in 1915. Early aircraft wings typically bore their loads by using two (or more) wings in a biplane configuration braced with wires and struts. They were similar to truss bridges,