

Ane's Student Edition

Business Mathematics

Theory & Applications

J.K. Sharma



BUSINESS MATHEMATICS

Theory and Applications

Text Book Useful for

- BBA, NCA, BIT, BIT
- PGDBM, MBA, M Com
- B Com (H), BA (Eco. Hons)
- CA, ICWA

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BUSINESS MATHEMATICS

Theory and Applications

Preface

The primary objective of this text is to provide the basic concepts and applications of mathematics suited to the needs of students appearing in the examinations of BCom (Hons.), BA (Eco. Hons.), MCom, MA (Eco.), BBA, BCA, MBA, BIT, CA, ICWA and other professional examinations. The development of this book reflects a special concern to present mathematical concepts in terms that the reader can understand and practice with a variety of illustrations and examples. Thus the approach is to communicate the practical aspects of mathematics in a way that drives out the common fear of learning any mathematical subject.

Although there are several topics that could be included in this text, however, only those have been selected which are important and mostly taught in a fundamental course on mathematics. The ordering of the chapters reflects the most desirable sequence of topic coverage, while at the same time allowing for flexibility in the choice of topics.

The text is divided into two parts. Part I is devoted to topics on Business Mathematics, and Part-II contains topics on Linear Programming. There are 23 chapters and in each of them the concepts and methods are properly supported by illustrations followed by numerous and varied types of *solved examples* to provide students an integrated view of theory and applications of mathematics. A large number of *self practice problems with hints and answers* have been added in each chapter to enable students to learn at their own pace. Most of the questions conform to the trend questions appearing in the university and professional examinations.

I gratefully acknowledge the inspiration, encouragement and valuable suggestions received from my students and many teachers who have taught this course to undergraduate and postgraduate classes of several universities and schools of management.

Finally, I would like to thank my wife and children for their patience, understanding, love and assistance in making this book a reality. It is to them I dedicate this book.

I have tried my best to make this book free from errors, omissions and discrepancies. If you have any suggestions, or material requiring clarification or should you found potential errors, please write to me. Your suggestions and comments for the improvement of this book will be thankfully received and duly incorporated in the subsequent editions.

J K Sharma

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Contents

Preface

v

PART I : BUSINESS MATHEMATICS

Chapter 1: Set Theory and Binary Operations

1 – 52

- Introduction, 1
- Sets and Its Elements, 2
- Representation of a Set, 3
 - *Roster Method*, • *Set Selection Method*
- Some Standard Sets, 3
- Types of Sets, 4
- Venn-Euler Diagrams, 7
 - Self Practice Problems A*, 8
 - Hints and Answers*, 8
- Operations on Sets, 8
 - *Union of Sets*, • *Properties of Union of Sets*,
 - *Intersection of Sets*, • *Properties of Intersection of Sets*,
 - *Disjoint Sets*, • *Difference of Two Sets*,
 - *Complement of a Set*, • *Distributive Laws*,
- De-Morgan's Laws, 15
 - Self Practice Problems B*, 21
 - Hints and Answers*, 22
- Symmetric Difference of Sets, 22
 - *Properties of Symmetric Difference*
- Cardinal Number of a Set, 22
 - Self Practice Problems C*, 27
 - Hints and Answers*, 28
- Ordered Pairs and Cartesian Product of two Sets, 29
 - Self Practice Problems D*, 32
 - Hints and Answers*, 33
- Functions or Mappings, 33
 - *Range and Domain of a Function*, • *Function as Sets of Ordered Pair*,
 - *Difference between a Function and its Value*
- Types of Functions or Mappings, 35
- Inverse Function or Mapping, 36
- Product (or Composite) Mapping, 39

Self Practice Problems E, 40

Hints and Answers, 41

Relations, 41

- *Binary Relation in a Set*, • *Domain and Range of Relation*,
- *Difference between Relation and Function*

Types of Relations, 43

Equivalence Relation, 44

Equivalence Classes (or Sets), 46

Partition of a Set, 47

Review Questions, 48

Self Practice Problems F, 48

Hints and Answers, 49

Binary Operations, 49

Self Practice Problems G, 52

Hints and Answers, 52

Chapter 2: Functions and their Applications

53 – 72

Introduction, 53

The Concept of a Function, 54

Self Practice Problems A, 57

Types of Functions, 58

- *Polynomial Function*, • *Absolute Value Function*,
- *Step-Function*, • *Convex and Concave Function*,
- *Inverse Function*, • *Rational Function*,
- *Algebraic Function*, • *Transcendental Function*,
- *Even and Odd Functions* • *Periodic Function*

Self Practice Problems B, 62

Zeros (or Roots) of a Function, 63

- *Relationship between the Roots and the Coefficients*
- of a Quadratic Equation*

Self Practice Problems C, 64

Some Useful Functions in Business and Economics, 65

Self Practice Problems D, 67

Equilibrium of an Economic System, 68

Break-Even Analysis, 68

Self Practice Problems E, 71

Hints and Answers, 72

Chapter 3: Limit and Continuity

73 – 98

Introduction, 73

Limit of a Variable, 74

Limit of a Function, 74

Left Hand and Right Hand Limits, 76

Limit of a Sequence, 77

Important Results on Limit of Functions, 78

Distinction between Limit and Value of a Function, 79

Methods of Evaluation of Limits, 80

<i>Self Practice Problems A</i> ,	86
<i>Hints and Answers</i> ,	88
Continuity,	89
<i>Self Practice Problems B</i> ,	94
<i>Hints and Answers</i> ,	97

Chapter 4: Differentiation : Basic Concepts

99 – 128

Introduction,	99
Concept of Slope and Rate of Change,	100
• <i>Slope of a Straight Line</i> ,	
• <i>Positive or Negative Slope</i> ,	
• <i>Slope of a Curve (at a Point)</i>	
Concept of Derivative,	101
• <i>Existence of Derivative</i>	
Some Standard Derivatives,	103
General Rules for Differentiation,	106
<i>Self Practice Problems A</i> ,	109
<i>Hints and Answers</i> ,	110
Derivative of Composite Function	
<i>Self Practice Problems B</i> ,	112
<i>Hints and Answers</i> ,	112
Derivative of Exponential Functions	
Derivative of Logarithmic Functions	
<i>Self Practice Problems C</i> ,	116
<i>Hints and Answers</i> ,	117
Derivative of Inverse Functions,	
Derivative of a Function Defined Parametrically	
<i>Self Practice Problems D</i> ,	118
Derivative of an Implicit Function	
<i>Self Practice Problems E</i> ,	121
<i>Hints and Answers</i> ,	122
Successive (or Repeated) Differentiation,	122
<i>Self Practice Problems F</i> ,	125
<i>Hints and Answers</i> ,	126
The n th Derivative : Standard Results,	126
<i>Self Practice Problems G</i> ,	128

Chapter 5: Applications of Derivative

129 – 156

Introduction,	129
Average and Marginal Cost,	130
Relationship Among Total Cost, Average Cost and Marginal Cost Curves,	131
Total Revenue, Marginal Revenue and Average Revenue,	134
<i>Self Practice Problems A</i> ,	137
<i>Hints and Answers</i> ,	139
Growth and Decay,	139
Point of Elasticity of a Function,	141
Marginal Revenue and Elasticity of Demand,	147

Self Practice Problems B, 149

Hints and Answers, 151

Market Model, 153

National Income, Consumption and Savings Model, 154

Self Practice Problems C, 156

Hints and Answers, 156

Chapter 6: Maxima and Minima

157 – 189

The sign and Magnitude of the Derivative, 157

Maximum and Minimum Values, 158

Concavity, Convexity and Point of Inflexion, 162

Self Practice Problems A, 163

Hints and Answers, 165

Applications of Maxima and Minima, 166

Areas and Volumes

Self Practice Problems B, 169

Hints and Answers, 170

Revenue Function, Profit Function, and Inventory Control

Self Practice Problems C, 177

Hints and Answers, 182

Effect of Taxation and Subsidy on Monopoly, 184

Self Practice Problems D, 187

Hints and Answers, 188

Chapter 7: Partial Differentiation

190 – 227

Introduction, 190

Continuity of two Variables Function, 192

Partial Derivatives of Higher Order, 193

Self Practice Problems A, 195

Hints and Answers, 196

Total Differential, 196

Total Derivative of a Composite Function, 197

Self Practice Problems B, 197

Hints and Answers, 198

Homogeneous Function, 198

- Euler's Theorem on Homogeneous Function

Self Practice Problems C, 202

Hints and Answers, 203

Applications in Business and Economics, 203

- Utility Function, • Marginal Rate of Substitution,
- Constrained Utility Maximization,
- Lagrange's Multiplier Method to Maximize Utility
- Production Function

Laws of Returns and Returns to Scale, 208

Properties of Cobb-Douglas Production Function, 208

Self Practice Problems D, 210

Hints and Answers, 211

Partial Elasticities of Demand,	213
Demand Analysis,	
<i>Self Practice Problems E,</i>	217
Elasticity of Substitution,	219
<i>Self Practice Problems F,</i>	221
<i>Hints and Answers,</i>	222
Maxima, Minima and Saddle Points of a Function of Two Variables,	222
<i>Self Practice Problems G,</i>	226
<i>Hints and Answers,</i>	227

Chapter 8: Integral Calculus

228 – 279

Introduction,	228
Indefinite Integral,	229
Fundamental Formulae of Integration,	230
Rules of Integration,	230
<i>Self Practice Problems A,</i>	232
<i>Hints and Answers,</i>	233
Methods of Integration,	234
Integration by Substitution,	234
<i>Self Practice Problems B,</i>	237
<i>Hints and Answers,</i>	237
Integral of Functions Involving $a^2 \pm x^2$	
Special Integrals Reducible to the Form $x^2 \pm a^2$	
<i>Self Practice Problems C,</i>	241
<i>Hints and Answers,</i>	242
Integration by Parts,	242
<i>Self Practice Problems D,</i>	244
<i>Hints and Answers,</i>	245
Integration by Partial Fractions,	245
<i>Self Practice Problems E,</i>	247
<i>Hints and Answers,</i>	248
Definite Integral,	249
<i>Self Practice Problems F,</i>	251
<i>Hints and Answers,</i>	252
Applications of Indefinite Integration,	252
<i>Self Practice Problems G,</i>	258
<i>Hints and Answers,</i>	261
Applications of Definite Integration,	264
Relation between Profit, Marginal Cost and Revenue,	266
<i>Self Practice Problems H,</i>	267
<i>Hints and Answers,</i>	268
Consumer's Surplus,	268
Producer's Risk,	271
<i>Self Practice Problems I,</i>	272
<i>Hints and Answers,</i>	273

- The Learning Curve, 274
 Present Value of Stream of Cash Flows, 275
Self Practice Problems J, 276
Hints and Answers, 277

Chapter 9: Vector Algebra

280 – 301

- Introduction, 280
 Representation and Notation of Vectors, 281
 Types of Vectors, 281
 Important Definitions, 282
 Addition and Subtraction of Vectors, 285
 • *Properties of Vector Addition*
 Multiplication of Vector by a Scalar, 286
 • *Properties of Scalar Multiplication*
 Direction Cosines and Modulus of a Vector, 286
 Product of Two Vectors, 288
 Self Practice Problems A, 292
 Hints and Answers, 293
 Vector (or Cross) Product, 294
 Vector Triple Products, 299
 Self Practice Problems B, 300
 Hints and Answers, 301

Chapter 10: Matrix and Determinant

302 – 373

- Introduction, 302
 Definition and Notations, 303
 Types of Matrices, 303
 Algebra of Matrices, 306
 • *Addition and Subtraction of Matrices*
 • *Scalar Multiplication of Matrices*
 Self Practice Problems A, 310
 Multiplication of Matrices, 311
 • *Post-multiplication and Pre-multiplication of Matrices*
 • *Properties of Matrix Multiplication*
 Self Practice Problems B, 322
 Hints and Answers, 326
 Transpose of a Matrix, 327
 Self Practice Problems C, 329
 Hints and Answers, 330
 Determinants, 330
 • *Rule of Expansion of Determinant*, • *Minors and Cofactors*
 • *Properties of Determinants*
 Self Practice Problems D, 338
 Hints and Answers, 340
 Adjoint of a Square Matrix, 341
 Inverse of a Square Matrix, 342

Methods of Computing the Inverse of a Matrix, 343

Self Practice Problems E, 345

Hints and Answers, 347

Solution of Simultaneous Linear Equations, 347

Methods of Solving Non-Homogeneous Linear Equations, 348

• *Matrix Inverse Method*, • *Gauss Elimination Method*

• *Cramer's Rule*

Self Practice Problems F, 360

Hints and Answers, 365

Solution of Homogeneous Simultaneous Linear Equations, 370

Self Practice Problems G, 371

Hints and Answers, 373

Chapter 11: Mathematics for Finance

374 – 415

Introduction, 374

Types of Interest, 375

Nominal and Effective Rates of Interest, 381

Continuous Compounding, 383

Equation of Value of Money, 384

Simple, Compound and Continuous Discounting, 387

Depreciation, 390

Self Practice Problems A, 392

Present Value of an Annuity, 394

• *Present Value of an Annuity Due*, • *Present Value of Deferred Annuity*

Amount of Annuity, 396

• *Amount of an Annuity Due*, • *Amount of Deferred Annuity*

Self Practice Problems B, 397

Hints and Answers, 406

Amortization and Sinking Funds, 406

Continuous Compounding, 409

Self Practice Problems C, 412

Leasing, Capital Expenditure, Bonds, 412

• *Capital Expenditure Analysis*, • *Corporate Bonds*

Self Practice Problems D, 414

Hints and Answers, 415

Chapter 12: Fundamentals of Probability

416 – 442

Introduction, 416

Concepts of Probability, 417

• *Statistical Experiment*, • *Sample Space*,

• *Event Types*

Definition of Probability, 418

• *The Classical Approach*, • *The Relative Frequency Approach*

• *The Subjective Approach*, • *Modern Approach*

Rules of Probability and Algebra of Events, 421

• *Addition Rules*, • *Multiplication Rules*

Probability Tree Diagram, 430

The Baye's Theorem, 431

Review Questions, 433

Self Practice Problems, 434

Hints and Answers, 437

Chapter 13: Probability Distributions

443 – 471

Introduction, 443

Probability Density Function, 444

Cumulative Probability Density Function, 445

Expected Value and Variance of a Random Variable, 446

Self Practice Problems A, 449

Discrete Probability Distributions, 451

- Binomial Distribution, • Poisson Distribution

Continuous Probability Distributions, 457

- Normal Distribution, • Uniform (Rectangular) Distribution
- Gamma (or Erlang) Distribution, • Beta Distribution

Review Questions, 465

Self Practice Problems, 465

Hints and Answers, 468

Chapter 14: Progressions (AP, GP, HP)

472 – 508

Introduction, 472

Arithmetic Progression, 473

- General Form of an AP, • The n th Term of an AP,
- Sum of First n Terms of an AP

Arithmetic Mean (AM), 480

- Insertion of AM's between Extreme Values

Number of Terms in A.P.

Self Practice Problems A, 486

Hints and Answers, 488

Geometric Progression, 489

- General Form of a GP, • The n th Term of GP

Sum of an Infinite GP, 492

Representation of Terms in GP, 496

Geometric Mean (GM), 498

- Insertion of GM's between Two Quantities
- Product of n -Geometric Means

Applications of A.P. and G.P., 500

Self Practice Problems B, 501

Hints and Answers, 504

Harmonic Progression, 505

- The n th Term of HP, • Harmonic Mean between Two Quantities

Self Practice Problems C, 507

Hints and Answers, 508

Chapter 15: Analytical Geometry**509 – 545**

- Introduction, 510
- Abscissa and Ordinate of a Point, 510
- Rectangular Coordinates, 510
- Distance between Two Points, 511
- Division (or Section) Formula, 512
- Centroid of a Triangle, 512
- Area of a Triangle, 514
- Polar Coordinates, 516
 - Self Practice Problems A*, 517
 - Hints and Answers*, 518
- Locus of a Moving Point and its Equation, 518
- The Straight Line and Standard Form of its Equations, 519
- General Equation of a Straight Line, 523
- Intersection of Two Straight Lines, 525
- Angle between Two Straight Lines, 527
 - Self Practice Problems B*, 529
 - Hints and Answers*, 531
- Circles and Equations of Circle, 531
- Standard Results on Equation of Circle, 532
- General Equation of the Circle, 533
- Conditions for a Circle, 533
 - Self Practice Problems C*, 537
 - Hints and Answers*, 538
- Conic, 538
- Parabola, 539
 - Equation of a Parabola, • Different Forms of a Parabola
 - Self Practice Problems D*, 543
 - Hints and Answers*, 545

Chapter 16: Theory of Equations**546 – 594**

- Introduction, 546
- Degree of an Equation, 547
- Solution of an Equation, 548
- Linear Equation in Two Variables, 548
- Applications of Linear Equation, 549
- Quadratic Equations, 552
- Graphical Solution of Linear Equations, 553
- Graphical Solution of Quadratic Equations, 554
- Solution of a General Quadratic Equation, 557
 - Extraneous Roots, • Irrational Equations Reducible to Quadratic Form,
 - Reciprocal Equations
 - Self Practice Problems A*, 564
 - Hints and Answers*, 566
- Nature of Roots of a Quadratic Equation, 566
 - Relation between the Roots and Coefficients of a Quadratic Equation
 - Formation of an Equation with Given Roots

- Applications of Quadratic Equation, 574
 - Self Practice Problems B*, 575
 - Hints and Answers*, 576
- Linear Simultaneous Equations with two Variables, 577
 - *Solution Methods of Linear Simultaneous Equations*
- Quadratic Simultaneous Equations, 580
- Linear Simultaneous Equations with Three Variables, 583
 - Self Practice Problems C*, 584
 - Hints and Answers*, 585
- Cubic and Bi-quadratic Equations, 586
 - *Relation between Roots and Coefficients*
 - *Symmetric Functions of the Roots*
- Self Practice Problems D*, 593
- Hints and Answers*, 594

Chapter 17: Differential Equations and their Applications 595 – 644

- Introduction, 595
- Order and Degree of a Differential Equation, 596
- Solution of Differential Equation, 596
- Formation of Differential Equation, 597
 - Self Practice Problems A*, 598
 - Hints and Answers*, 599
- Solution of First Order and First Degree Differential Equation, 600
 - Self Practice Problems B*, 607
 - Hints and Answers*, 608
- Homogeneous Differential Equations
 - Self Practice Problems C*, 615
 - Hints and Answers*, 615
- Equations Reducible to Homogeneous Form
 - Self Practice Problems D*, 620
 - Hints and Answers*, 620
- Linear Differential Equations
 - Self Practice Problems E*, 624
 - Hints and Answers*, 624
- Exact Differential Equations
 - Equations Reducible to Exact Differential Equation Form
 - Self Practice Problems F*, 632
 - Hints and Answers*, 632
 - Equations Reducible to Linear Form (Bernoulli's Differential Equations)
 - Self Practice Problems G*, 636
 - Hints and Answers*, 636
- First Order Linear Differential Equations with Constant Coefficients, 636
- Second Order Linear Differential Equations with Constant Coefficients, 638
 - Self Practice Problems H*, 641
 - Hints and Answers*, 641
- Applications of Differential Equations, 641
 - Self Practice Problems I*, 644

Chapter 18: Input-Output Analysis 645 – 661

- Introduction, 645
- Closed and Open Input-Output Models, 646
- Coefficient Matrix, 646
- Hawkins-Simon Conditions for the Viability of the System, 648
- Technology Matrix in Value Terms, 651
- Determination of Equilibrium Prices, 652
- Self Practice Problems*, 656
- Hints and Answers*, 659

PART II : LINEAR PROGRAMMING**Chapter 19: Linear Programming : Applications and Model Formulation 662 – 693**

- Introduction 662
- Structure of Linear Programming Model 663
 - *General Structure of LP Model* 34
 - *Assumptions of LP Model* 35
- Advantages of Using Linear Programming 664
- Limitations of Linear Programming 665
- Application Areas of Linear Programming 665
- General Mathematical Model of Linear Programming Problem 667
- Guidelines on Linear Programming Model Formulation 668
- Examples of LP Model Formulation 669
 - *Examples on Production* 46 • *Examples on Marketing* 56
 - *Examples on Finance* 60 • *Examples on Agriculture* 68
 - *Examples on Transportation* 71 • *Examples on personnel* 74
- Review Questions* 682
- Self Practice Problems* 682
- Hints and Answers* 689

Chapter 20: Graphical Method 694–721

- Introduction 694
- Important Definitions 695
- Graphical Solution Methods of LP Problems 695
 - *Extreme Point Solution Method* 696
 - *Examples on Maximization LP Problem*
 - *Examples on Minimization LP Problem*
 - *Examples on Mixed Constraints LP Problem*
 - *Iso-profit (Cost) Function Line Method* 708
 - *Comparison of Two Graphical Solutions Methods* 709

Special Cases in Linear programming 710

- *Alternative (or Multiple) Optimal Solutions* 710
- *Unbounded Solution* 711
- *Infeasible Solution* 712 • *Redundancy* 713

Review Questions 713

Self Practice Problems 714

Hints and Answers 719

Chapter 21: Simplex Method

722 – 771

Introduction 722

Standard form of an LP Problem 723

Simplex Algorithm (Maximization Case) 724

Simplex Algorithm (Minimization Case) 734

Self Practice problems A 749

Hints and Answers 753

Some Complications and Their Resolution 756

- *Unrestricted Variables* 756
- *Tie for Entering Basic Variable (Key Column)* 759
- *Tie for Leaving Basic Variable (Key Row)—Degeneracy* 759

Types of Linear Programming Solutions 761

- *Alternative (Multiple) Optimal Solutions* 761
- *Unbounded Solution* 763
- *Infeasible Solution* 765

Review Questions 766

Self Practice Problems B 767

Hints and Answers 770

Chapter 22: Duality in Linear Programming

772 – 784

Introduction 772

Formulation of Dual Linear Programming Problem 773

- *Symmetrical Form*, 773 • *Economic Interpretation of Dual Variables* 774
- *Economic Interpretation of Dual Constraints* 775
- *Rules for Constructing the Dual from Primal* 776

Standard Results on Duality 780

- *Principle of complementary slackness* 781

Managerial Significance of Duality 781

Advantages of Duality 782

Review Questions 782

Self Practice Problems 783

Hints and Answers 783