DEMANA WAITS CLEMENS

PRECALCULUS MATHEMATICS

A GRAPHING APPROACH

THIRD EDITION

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PRECALCULUS MATHEMATICS

A Graphing Approach — Third Edition

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Preface

Precalculus Mathematics: A Graphing Approach, grew out of our strong conviction that incorporating graphing technology into the precalculus curriculum better prepares students for further study in mathematics and science. Our own research at The Ohio State University and at dozens of other high school and college test sites shows that the use of a calculator- or computer-based graphing approach dramatically changes results in the classroom. Instead of being bored and discouraged by conventional contrived problems, students suddenly grow excited by their ability to explore problems that arise from real world situations and learn from their experiences. The mathematics classroom is transformed into a mathematics laboratory, with a new interactive instructional approach that focuses on problem solving. As a natural outgrowth of this excitement, students complete the course with a better understanding of mathematics and a solid intuitive foundation for calculus.

The Graphing Approach

As in the previous editions, this text is designed to be used in a one or two semester precalculus course. We take advantage of the power and speed of modern technology to apply a graphing approach to the course. The characteristics of this approach are described below.

Integration of Technology Use of a graphing utility—whether a hand-held graphing calculator or computer graphing software—is not optional. Technology allows the focus of the course to be on problem solving and exploration, while building a deeper understanding of algebraic techniques. Students are expected to have regular and frequent access to a graphing utility for class activities as well as homework.

Problem Solving The ultimate power of mathematics is that it can be used to solve problems. Technology removes the need for contrived problems and opens the door for realistic and interesting applications. Throughout this text, we focus on what we call problem situations—situations from the physical world, from our social environment, or from the quantitative world of mathematics. Using real life situations makes the math understandable to the students, and students come to value mathematics because they appreciate its power.

Throughout this text, we use a three step problem solving process. Students will be asked to:

1. Find an algebraic representation of the problem;
2. Find a complete graph of the algebraic representation; and
3. Find a complete graph of the problem solving situation.

These three steps prepare the student to find either a graphical or an algebraic solution to the problem. Problem situations are highlighted in the exercise sets, and we encourage students to complete all the exercises which deal with that problem. See page 74, exercises 143–147.

Multiple Representations A quantitative mathematical problem can often be approached using multiple representations. In a traditional precalculus course, problems are analyzed using an algebraic representation, and perhaps a numerical representation. However, modern technology allows us to take full advantage of a graphical, or geometric, representation of a problem. Our understanding of the problem is enriched by exploring it numerically, algebraically, and graphically. See pages 84 and 85.

Exploration We believe that a technology-based approach enriches the student's mathematical intuition through exploration. With modern technology, accurate graphs can be obtained quickly and used to study the properties of functions. Students learn to decide for themselves what technique should be used. The speed and power of graphing technology allows an emphasis on exploration. See page 45.

Geometric Transformations The exploratory nature of graphing helps students learn how to transform a graph geometrically by horizontal or vertical shifts, horizontal or vertical stretches and shrinks, and reflection with respect to the axes. This develops students' abilities so that they can sketch graphs of functions quickly and understand the behavior of graphs. See page 152.

Foreshadowing Calculus We foreshadow important concepts of calculus through an emphasis on graphs. Using graphs, students can find maxima and minima of functions, and intervals where functions are increasing or decreasing and limiting behavior of functions are determined graphically. We do not borrow the techniques of calculus—rather we lay the foundation for the later study by providing students with rich intuitions about functions and graphs. See page 147.

Approximate Answers Technology allows a proper balance between exact answers that are rarely needed in the real world and accurate approximations. Graphing techniques such as zoom-in provide an excellent geometric vehicle for discussion about error in answers. Students can read answers from graphs with accuracy up to the limits of machine precision. See page 80.

Visualization Graphing helps students to gain an understanding of the properties of graphs and makes the addition of geometric representations to the usual numeric and algebraic representations very natural. Exploring the connections between graphical representations and problem situations deepens student understanding about mathematical concepts and helps them appreciate the role of mathematics.

About the Third Edition

This third edition of *Precalculus Mathematics: A Graphing Approach* grew out of the experiences of hundreds of classrooms. We have carefully listened to comments and suggestions of both teachers and students, and incorporated them fully into the text. The entire text has been extensively revised and rewritten.

Development of Functions In this edition a presentation of functions including operations on functions, composition of functions, graphs of functions, and transformations applied to graphs of functions is the major focus of Chapter 1. Finding zeros of functions as a way of solving equations and solving applied problems is the focus of Chapter 2. Parametric equations are also introduced in Chapter 2 as a way of defining a relation. Parametric equations are then used to define inverse functions as a special type of relation. Polynomial functions and their graphs remain in a separate chapter (Chapter 3).

Rational Functions Coverage of rational functions and functions involving radicals (Chapter 4) has been streamlined from seven sections to four.

Trigonometry Chapters The three chapters on trigonometry (Chapters 6–8) have been developed to make concepts even more accessible, and now contain an even greater emphasis on graphing. The trigonometric functions are introduced in terms of the unit circle rather than in terms of right triangles as in the previous editions. A more complete development of trigonometric identities and solving trigonometric equations has been included since students need to practice these skills to be successful in calculus.

Parametric Equations and Polar Coordinates These topics are now covered in a separate chapter (Chapter 9) together with conic sections. The material on conics is treated in two sections. Topics from this chapter may be incorporated earlier if the instructor chooses.

Systems of Equations Sections on solving systems of equations algebraically and graphically are now combined with material on matrices. The concept of solving systems of equations graphically is first introduced in Chapter 3. Chapter 10 focuses on systems of equations and the use of matrices to solve them. Also in

Chapter 10 matrices are used to describe rotations and rotation images of conic sections.

Topics in Discrete Mathematics Chapter 11 focuses on a variety of topics from discrete mathematics-sequences and series, the binomial theorem, mathematical induction, and permutations and combinations. It also includes a discussion of some topics from three dimensional geometry.

Topics in Statistics This third edition includes new coverage on a variety of statistics topics. Chapter 12 includes a discussion of probability, stem-and-leaf tables, histograms, box-and-whisker plots, scatter plots, and line graphs, mean, mode, median, variance, and standard deviation, and curves of best fit.

Algebra Skills A new appendix was added that provides a brief review of concepts from intermediate algebra. This algebra review illustrates how a grapher can be used to support algebraic skills.

Features

New pedagogical features have been incorporated into this text. It is our hope that these features will make the text a stronger teaching and learning tool. The pedagogy now includes:

Explore with a Graphing Utility This recurring box places the student in the role of participant in the development of the mathematics. By introducing topics through this experience-based process, the book literally interacts with the student. Students develop their critical thinking skills, and form generalizations about the behavior of functions.

Sidelight Boxes Shaded boxes placed in the margin provide commentary on the mathematical development, and include problem solving tips, calculator hints, and reminders.

Color A functional use of color has been introduced to help the reader better navigate through the text. In addition to using color to mark beginnings and ends of examples, and to identify definitions and theorems, the text uses color in the artwork to help the student correctly identify the concept being illustrated.

Artwork We have made a visual distinction between graphs generated with a graphing utility and hand-sketched art. Art which has been derived from a grapher is outlined with a colored box; while the graphs have been drawn more smoothly we still try to emulate what the student sees on the grapher. Traditional art uses color within the graphs but is not boxed. The distinction between types of artwork underscores the difference between a sketch and a grapher-drawn complete graph.

Examples As in the previous editions, we have included many examples to develop the concepts. Titled examples help the student focus on the purpose of the example.

Exercises We have closely focused on correlating end of section exercises to examples, and added many new exercises. Writing to Learn and Discussion exercises have also been included in nearly every exercise set.

Acknowledgments

We would like to thank the many wonderful teachers who participated in the development of this text. Their dedication and enthusiasm made this revision possible. Creative suggestions for this edition came from so many of our family of teachers that it would be impossible to name them all here. However, we particularly want to thank the reviewers of this edition and the participants of focus groups:

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CHAPTER 1

Functions and Graphs

1.1 Real Numbers and the Coordinate Plane

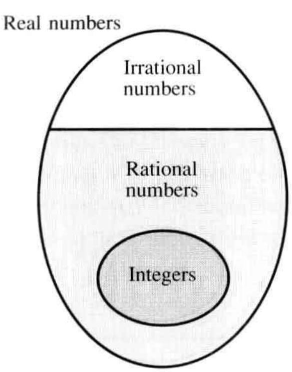


Figure 1.1 All integers are rational numbers, and all rational numbers are real numbers. A real number is either rational or irrational but not both.

The set of numbers used most frequently in algebra is known as the **real numbers**. Real numbers are either **rational** or **irrational**. The set of **integers** is a subset of the set of rational numbers, since for each integer a, $a = a/1$ (see Fig. 1.1). Following are some examples:

$$\text{integers:} \quad 3, \quad -5, \quad 48$$

$$\text{rational numbers:} \quad \frac{3}{8}, \quad -\frac{2}{3}, \quad \frac{22}{7}$$

$$\text{irrational numbers:} \quad \pi, \quad \sqrt{2}, \quad \sqrt{17}$$

When represented as decimal numbers, integers have all zeros to the right of the decimal point, rational numbers always have a block of digits that repeat, and irrational numbers have no repeating blocks of digits. Some examples of rational and irrational numbers are

$$\pi = 3.141592654\cdots, \quad \tfrac{5}{8} = 0.625, \quad \sqrt{2} = 1.414213562\cdots, \quad \tfrac{1}{3} = 0.33333\cdots.$$

Because a calculator or computer display of a decimal number can show only a finite number of digits, usually 7 to 10, many displays represent only approximations to the number keyed in. However, they are very accurate approximations (see Fig. 1.2).

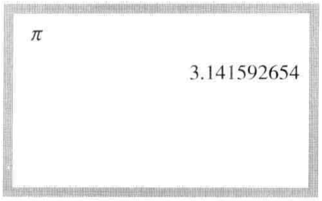


Figure 1.2 When an irrational number is keyed into a calculator, the digits to the right of the ten digits displayed and the three additional check digits are usually interpreted to be zero. Thus the number displayed is a rational number approximation to the exact value of the number. What does your calculator give for π?

Arithmetic Operations

There are four binary operations for numbers: addition, subtraction, multiplication, and division, represented by the symbols $+$, $-$, $\times$ (or $\cdot$), $\div$, respectively.

Addition and multiplication satisfy a set of properties that can be used to change the form of a mathematical expression into an equivalent form. These properties are summarized as follows.

Real-number Properties of Addition and Multiplication

Let a, b, and c represent real numbers. Then all of the following are true:

	Addition	*Multiplication*
Closure:	$a+b$ is real.	$a \cdot b$ is real.
Commutative:	$a+b=b+a$	$a \cdot b = b \cdot a$
Associative:	$a+(b+c)=(a+b)+c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Identity:	$a+0=0+a=a$	$a \cdot 1 = 1 \cdot a = a$
Inverse:	$a+(-a)=(-a)+a=0$	$a \cdot \left(\frac{1}{a}\right) = \left(\frac{1}{a}\right) \cdot a = 1 \quad (a \neq 0)$
Distributive:	$a(b+c)=ab+ac$	

An **equation** is a statement of equality between two expressions. For example, $3+4$ and $2+5$ both represent 7. Therefore we can write the equation $3+4=2+5$. We can use the commutative property of addition to conclude that for any real number x, $x+5=5+x$. In solving problems involving equations, three important properties of equality will be useful.

Real-number Properties of Equality

Let a, b, and c be real numbers. Then the following properties are true:

Reflexive: $a=a$.
Symmetric: If $a=b$, then $b=a$.
Transitive: If $a=b$ and $b=c$, then $a=c$.

EXAMPLE 1 Using Properties of Real Numbers

If x is any real number, show that $2 \cdot (x + 3) = 6 + 2 \cdot x$.

Solution

	$2 \cdot (x + 3) = 2 \cdot x + 2 \cdot 3$	Distributive property
	$2 \cdot x + 2 \cdot 3 = 6 + 2 \cdot x$	Commutative property
so	$2 \cdot (x + 3) = 6 + 2 \cdot x.$	Transitive property of equality

Real-number Line

The set of all real numbers is often represented as points on a line (see Fig. 1.3). To construct a **coordinate system** on a line, draw a line and label one point 0. This point is called the **origin**. Then mark equally spaced points on each side of 0. Label points to the right of zero 1, 2, 3, ... and to the left of zero −1, −2, −3, ... The properties of *order* in Definition 1.1 describe the placement of all other numbers on the line. Figure 1.3 is called a **real-number line**.

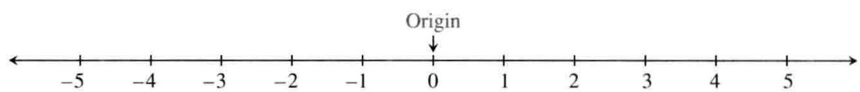


Figure 1.3 Each real number corresponds to one and only one point on the number line, and each point on the number line corresponds to one and only one real number.

The number associated with a point P is called the **coordinate** of point P.

Figure 1.4 a is less than b.

Definition 1.1 Order on the Real-number Line

If a and b are any two real numbers, then **a is less than b** if $b - a$ is a positive number. In this case, a is to the left of b on the real-number line (see Fig. 1.4). This order relation is denoted by the **inequality $a < b$**. In all, there are four inequality symbols that express order relationships, as follows:

$a < b$	a is less than b.
$a \leq b$	a is less than or equal to b.
$a > b$	a is greater than b.
$a \geq b$	a is greater than or equal to b

EXAMPLE 2 Solving Inequalities

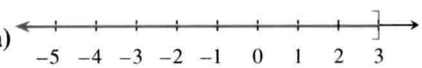
On a real-number line, draw all numbers that are solutions to the following inequalities:

a) $x \leq 3$

b) $-3 \leq x < 2$

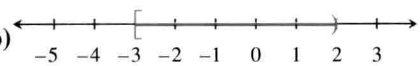
Solution

a)



Shade all those points whose coordinates are less than or equal to 3.

b)



Shade all those points whose coordinates are greater than or equal to −3 and less than 2. ∎

Note in the number lines above the use of a square bracket to show inclusions and a rounded parenthesis to show exclusions of particular endpoints.

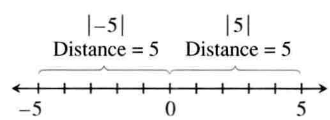


Figure 1.5 Both −5 and 5 are a distance of 5 from 0.

The numbers 5 and −5 are the same distance from zero (0) on the number line (see Fig. 1.5). So are π and $-\pi$ and $\sqrt{2}$ and $-\sqrt{2}$. To communicate this idea of distance from the origin of the coordinate line, we introduce a symbol. The **absolute value** of a number c, denoted $|c|$, represents the distance of c from zero on the real-number line. For example, $|-3| = 3$. A more formal definition follows.

> **Definition 1.2 Absolute Value**
>
> If a is a real number, then the **absolute value of a** is given by
>
> $$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$$

EXAMPLE 3 Evaluating Absolute Value

Write the expression $|\sqrt{3} - 2|$ without using absolute value notation.

Solution

$$|\sqrt{3} - 2| = -(\sqrt{3} - 2) \quad \text{Because } \sqrt{3} \text{ is less than 2, } \sqrt{3} - 2 \text{ is negative.}$$

We can use absolute value notation to describe the distance between any two points on the real-number line. Notice that the distance between the points with coordinates 3 and 8 is $|3 - 8|$.

> **Definition 1.3 Distance between Points on a Line**
>
> Suppose A and B are two points on a real-number line with coordinates a and b, respectively. The **distance between A and B**, denoted $d(A, B)$, is given by
>
> $$d(A, B) = |a - b|.$$

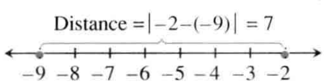


Figure 1.6 Distance between −2 and −9.

EXAMPLE 4 Finding the Distance between Points on a Real-number Line

Find the distance between the points with the coordinates −2 and −9.

Solution (See Fig. 1.6.)

$$d(-2, -9) = |-2 - (-9)| = |7| = 7 \quad \text{Notice that } d(-2, -9) = d(-9, -2).$$

Notice in the solution to Example 4 that −2 and −9 are used as both the coordinates and the names of points on the number line. It is common to simplify notation this way.

Cartesian Coordinate System

Just as each point on a real-number line is associated with a real number, each point in a plane is associated with an ordered pair of real numbers. To determine which pairs of numbers get associated with which points, we use the **Cartesian coordinate system** (also called the **rectangular coordinate system**). To construct a coordinate system, draw a pair of perpendicular real-number lines, one horizontal and the other vertical, with the lines intersecting at their respective origins (see Fig. 1.7). The horizontal line is usually called the ***x*-axis** and the vertical line is usually called the ***y*-axis**. The positive direction on the x-axis is to the right, and the positive direction on the y-axis is up.