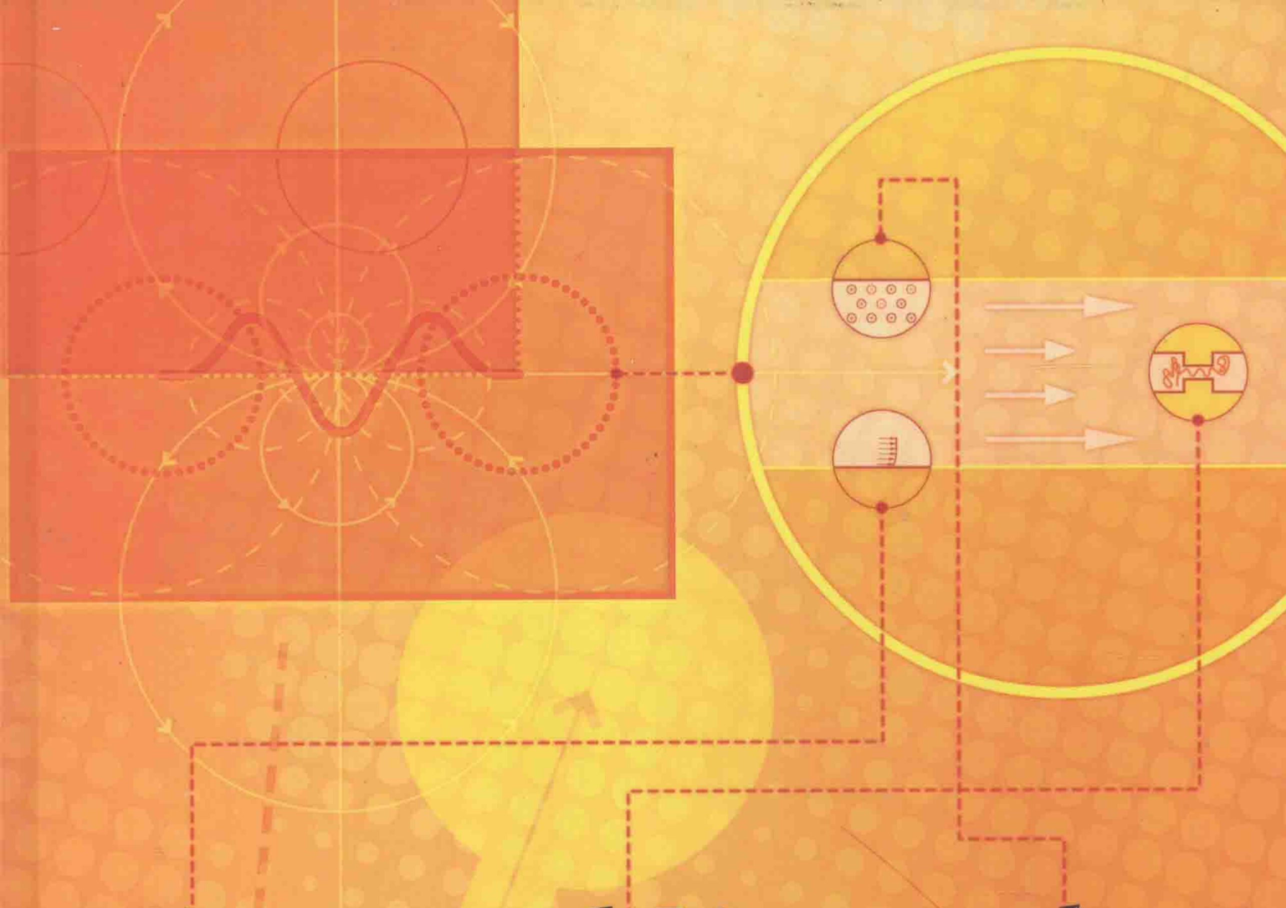
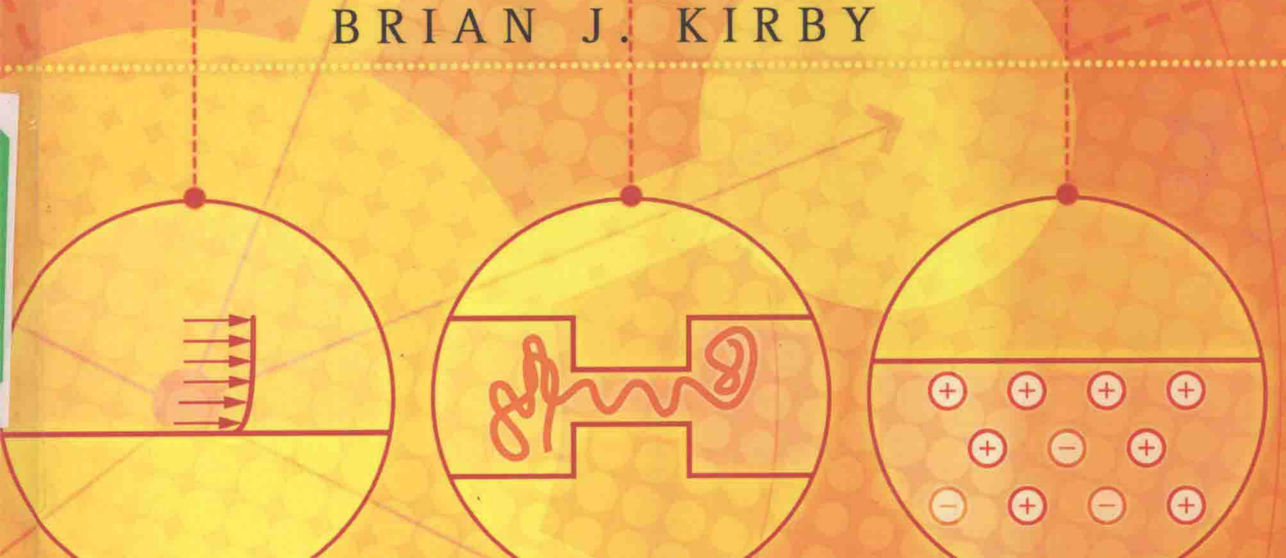


Micro- and Nanoscale Fluid Mechanics

Transport in Microfluidic Devices

BRIAN J. KIRBY



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TRANSPORT IN MICROFLUIDIC DEVICES

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MICRO- AND NANOSCALE FLUID MECHANICS: TRANSPORT IN MICROFLUIDIC DEVICES

This text describes the physics of fluid transport in microfabricated and nanofabricated liquid-phase systems, with consideration of particles and macromolecules. This text brings together fluid mechanics, electrodynamics, and interface science with a focused goal of preparing the modern microfluidics researcher to analyze and model continuum fluid mechanical systems encountered when working with micro- and nanofabricated devices. This text is designed for classroom instruction and also serves as a useful reference for practicing researchers. Worked sample problems are inserted throughout to assist the student, and exercises are included at the end of each chapter to facilitate use in classes.

Brian J. Kirby currently directs the Micro/Nanofluidics Laboratory in the Sibley School of Mechanical and Aerospace Engineering at Cornell University. He joined the school in August 2004. Previously, he was a Senior Member of the Technical Staff in the Microfluidics Department at Sandia National Laboratories in Livermore, California. He was educated at Stanford University and The University of Michigan. Professor Kirby has received numerous research and teaching awards, including the Presidential Early Career Award for Scientists and Engineers (PECASE) and the Mr. and Mrs. Robert F. Tucker Excellence in Teaching Award.

Preface

This text focuses on the physics of liquid transport in micro- and nanofabricated systems. It evolved from a graduate course I have taught at Cornell University since 2005, titled "Physics of Micro- and Nanoscale Fluid Mechanics," housed primarily in the Mechanical and Aerospace Engineering Department but attracting students from Physics, Applied Physics, Chemical Engineering, Materials Science, and Biological Engineering. This text was designed with the goal of bringing together several areas that are often taught separately – namely, fluid mechanics, electrodynamics, and interfacial chemistry and electrochemistry – with a focused goal of preparing the modern microfluidics researcher to analyze and model continuum fluid-mechanical systems encountered when working with micro- and nanofabricated devices. It omits many standard topics found in other texts – turbulent and transitional flows, rheology, transport in gel phase, Van der Waals forces, electrode kinetics, colloid stability, and electrode potentials are just a few of countless examples of fascinating and useful topics that are found in other texts, but are omitted here as they are not central to the fluid flows I wish to discuss.

Although I hope that this text may also serve as a useful reference for practicing researchers, it has been designed primarily for classroom instruction. It is thus occasionally repetitive and discursive (where others might state results succinctly and only once) when this is deemed useful for instruction. Worked sample problems are inserted throughout to assist the student, and exercises are included at the end of each chapter to facilitate use in classes. Solutions for qualified instructors are available from the publisher at http://www.cambridge.org/kirby. This text is *not* a summary of current research in the field and omits any discussion of microfabrication techniques or any attempt to summarize the technological state of the art.

The text considers, in turn, (a) low-Reynolds-number fluid mechanics and hydraulic circuits; (b) outer solutions for microscale flow, focusing primarily on the unique aspects of electroosmotic flow outside the electrical double layer; (c) inner solutions for microscale flow, focusing on sources of interfacial charge and modeling of electrical double layers; and (d) unsteady and nonequilibrium solutions, focusing on nonlinear electrokinetics, dynamics of electrical double layers, electrowetting, and related phenomena. In each case, several applications are selected to motivate the presentation, including microfluidic mixing, DNA and protein separations, microscale fluid velocity measurements, dielectrophoretic particle manipulation, electrokinetic pumps, and the like.

I select notation with the goal of helping students new to the field and with the understanding that this (on occasion) leads to redundant or unwieldy results. I minimize use of one symbol for multiple different variables, so the radius in spherical coordinates (r) is typeset with a symbol different from the radius in cylindrical coordinates ($\mathscr{r}$) and the colatitudinal angle ϑ in spherical coordinates is distinguished from the polar coordinate in cylindrical coordinates (θ). Because I teach from this text using a chalkboard, I use symbols that I can reproduce on a chalkboard – thus I avoid the use of the Greek letter ν for the kinematic viscosity $\nu = \eta/\rho$, because I am utterly unable to make it distinguishable from the y velocity v. Vectors, though they are placed in boldface to make them stand out, are also written with (admittedly redundant) superscripted arrows to match the chalkboard presentation.

This material is used for a semester-long graduate course at Cornell. Chapters 1, 2, 5, 7, and 8, as well as the appendices, are not covered in class as they are considered review or supplementary material. The remainder of the text is covered in approximately forty-two 50-minute classroom sessions.

I would like to acknowledge a number of people who helped with various aspects of this text. In particular, Dr. Elizabeth Strychalski and Professors Stephen Pope and Claude Cohen at Cornell, Professor Shelley Anna of Carnegie-Mellon University, Professor Kevin Dorfman of the University of Minnesota, Professor Nicolas Green of the University of Southampton, Donald Aubrecht of Harvard University, Professor Sumita Pennathur of UCSB, and Professor Aaron Wheeler of the University of Toronto were kind enough to offer useful suggestions. Professor Amy Herr of the University of California, Berkeley, used a draft of this text for her class during spring 2009; her insight and the feedback from her students were both immensely helpful. Professor Martin Bazant of the Massachusetts Institute of Technology provided materials helpful in completing the bibliography for several of the chapters. The students that have taken my class since 2004 have all contributed to this text in some way, but I would like to thank my student researchers Alex Barbati, Ben Hawkins, Sowmya Kondapalli, and Vishal Tandon in particular for their input, and my student Michael Allen for careful proofreading. Ben Hawkins and Dr. Jason Gleghorn contributed a number of the figures and helped to write material that was included in the chapters on Stokes flow and dielectrophoresis. David J. Griffiths (Reed College) provided files that assisted with typesetting. Gabe Terrizzi created many of the figures; his contributions were immensely helpful. Greg Parker (gparker@chorus.net) designed the cover.

Although many people assisted with review of this text, I am solely responsible for any errors, and I hope that readers will notify me or the publisher of those that they find. Errata will be maintained at http://www.cambridge.org/kirby.

Brian J. Kirby
Ithaca, NY
May 2010

Nomenclature

Symbol	Meaning	Page of first use or definition
A	area	61
$\mathcal{A}$	Helmholtz free energy	324
α	coefficient	112
α	phase lag angle	69
α	rotation angle	158
α	thermal diffusivity	80
a	acceleration	255
a	particle radius	171
a_i	activity	413
β	compressibility	75
β	coefficient	236
b	slip length	31
$\vec{\boldsymbol{B}}$	applied magnetic field	391
$\mathcal{B}$	Brillouin function	104
c_i	species molar concentration	407
c_p	specific heat	80
c	passive scalar	80
C	capacitance	117
C	constant of integration	43
C_{h}	compliance	66
$\underset{\sim}{C}$	complex number	465
C_{D}	drag coefficient	188
Γ	2D vortex strength	163
Γ	circulation	13
Γ	surface chemical site density	229
Γ	magnitude of injected sample	90
γ	surface tension	20
γ_i	natural logarithm of species concentration	259
χ	electrokinetic coupling matrix	65
χ_{e}	electric susceptibility	100
χ_{m}	magnetic susceptibility	98
d	depth	140
d	diameter	22
D	scalar diffusivity	80
D_i	species diffusivity	252
$\vec{\boldsymbol{D}}$	electric displacement	100

Symbol	Meaning	Page of first use or definition
Du	Dukhin number	263
δ	Dirac delta function	458
$\vec{\vec{\delta}}$	identity tensor	16
∇	del operator	426
e	eccentricity	188
e	fundamental charge	201
e_1	singlet potential	475
e_2	pair potential	476
e_{mf}	potential of mean force	227
$\vec{E}$	electric field	97
ε	electrical permittivity	98
$\underset{\sim}{\varepsilon}$	complex electrical permittivity for sinusoidal fields	113
ε_S	Stern layer permittivity	360
ε_0	electrical permittivity of free space	100
ε_r	relative permittivity, i.e., dielectric constant	101
ε'	reactive permittivity	115
ε''	dissipative permittivity	115
ε_{LJ}	potential well depth	477
$\vec{\vec{\varepsilon}}$	strain rate tensor	10
$\frac{\partial \varepsilon}{\partial c}$	dielectric increment	413
F	Faraday constant	99
$\vec{F}$	force	108
$\vec{f}$	force per unit volume	6
$\underset{\sim}{f}_{CM}$	Clausius–Mossotti factor	393
f_{ad}	adjusted distribution function	480
f_d	distribution function	217
f_{dc}	direct correlation function	482
f_{tc}	total correlation function	482
f_M	Mayer f function	480
f_0	Henry's function	288
f	electrophoretic correction factor	287
ϕ	electric potential	97
φ	electric potential difference from bulk	133
φ_0	total potential drop across the double layer	133
ϕ_v	velocity potential	153
$\underset{\sim}{\phi_v}$	complex velocity potential	158
φ	azimuthal coordinate	419
Φ	cross-correlation	189
ζ	zeta potential	139
G	Gibbs free energy	20
G	electrical conductance	117
G_s	excess surface conductance	262
$\vec{g}$	gravitational acceleration	6
g_i	chemical potential	227
$\overline{g_i}$	electrochemical potential	227

Symbol	Meaning	Page of first use or definition
$\vec{\vec{G}}$	hydrodynamic interaction tensor	187
$\vec{\vec{G}}_0$	Oseen–Burgers tensor	187
H	capillary height	23
$\vec{H}$	induced magnetic field	98
h	height	43
η	dynamic viscosity	17
$\vec{i}$	current density	110
i_0	exchange current density	112
I	current	64
I	second moment of area	309
I_c	ionic strength	408
j	square root of minus 1	157
$\vec{j}$	scalar flux density	80
$\mathcal{J}$	Joukowski transform	171
k	spring constant	325
k	chemical reaction rate	409
k_{ve}	viscoelectric coefficient	235
k_B	Boltzmann constant	104
K_a	acid dissociation constant	409
K_{eq}	equilibrium constant	409
K_{sp}	solubility product	412
κ	2D doublet strength	165
κ	Debye screening parameter	288
Λ	molar conductivity	256
Λ	2D source strength	160
λ_B	Bjerrum length	478
λ_D	Debye length	202
λ_{HS}	hard-sphere packing length	213
λ_S	Stern layer thickness	360
ℓ_c	polymer contour length	301
ℓ_e	polymer end-to-end length	303
ℓ_K	polymer Kuhn length	312
ℓ_p	polymer persistence length	299
L	length	61
L	electrical inductance	117
L	depolarization factor	384
m	mass	184
$\vec{M}$	magnetization	98
μ	viscous mobility	252
μ_{DEP}	dielectrophoretic mobility	374
μ_{EK}	electrokinetic mobility	265
μ_{EO}	electroosmotic mobility	138
μ_{EP}	electrophoretic mobility	252
μ_{mag}	magnetic permeability	98
$\mu_{mag,0}$	magnetic permeability of free space	98
N_A	Avogadro's number	112

Symbol	Meaning	Page of first use or definition
N_{bp}	number of base pairs in DNA molecule	301
n	normal coordinate	106
p	pressure	6
$\vec{\boldsymbol{p}}$	dipole moment	104
$\mathrm{p}K_a$	negative logarithm of acid dissociation constant	410
pH	negative logarithm of molar proton concentration	410
pOH	negative logarithm of molar hydroxyl ion concentration	411
pzc	point of zero charge	230
$\mathcal{P}$	perimeter	63
$\mathcal{P}$	probability density function	313
Pe	mass transfer Peclet number	79
ϖ	dummy frequency integration variable	115
ψ	stream function	8
ψ_S	Stokes stream function	9
ψ_e	electric stream function	469
$\vec{\boldsymbol{P}}$	electric polarization	100
$\vec{\vec{\boldsymbol{P}}}$	pressure interaction tensor	187
Q	volumetric flow rate	60
q	electric charge	97
q''	electric areal charge density	359
ρ	fluid density	6
ρ_E	net charge density	99
r	radial coordinate – spherical coordinates	418
r_h	hydraulic radius	63
Δr	radial distance – spherical coordinates	98
$\vec{\boldsymbol{\Delta r}}$	distance vector	98
$\imath$	radial coordinate – cylindrical coordinates	418
$\Delta \imath$	radial distance – cylindrical coordinates	157
Re	Reynolds number	442
R	universal gas constant	112
R	electrical resistance	117
R	radius of channel	47
R	radius of curvature	21
R	separation resolution	267
R_h	hydraulic resistance	61
$\langle r_g \rangle$	radius of gyration	303
s	arc length	302
$\mathcal{S}$	entropy	324
$\mathcal{S}$	Schwarz–Christoffel transform	473
σ	conductivity	110
σ_{LJ}	Lennard–Jones "bond length"	477
$\underset{\sim}{\sigma}$	complex electrical conductivity	114
σ_s	effective surface conductivity	210

Symbol	Meaning	Page of first use or definition
Sk	Stokes number	186
St	Strouhal number	442
t	time	7
T	Kelvin temperature	20
$\vec{\boldsymbol{T}}$	torque	109
$\vec{\vec{\boldsymbol{T}}}$	Maxwell stress tensor	107
$\vec{\vec{\boldsymbol{\tau}}}$	stress tensor	15
τ	characteristic time	103
θ	polar coordinate – cylindrical coordinates	418
θ	contact angle	21
θ_0	corner angle	170
ϑ	colatitude coordinate – spherical coordinates	418
$\Delta\theta$	polar coordinate of distance vector	157
$\vec{\boldsymbol{u}}$	velocity vector	7
$\underset{\sim}{u}$	complex velocity	159
u_{EK}	electrokinetic velocity	269
u_{EO}	electroosmotic velocity	140
u_{EP}	electrophoretic velocity	255
$u_{\mathscr{r}}$	radial velocity – cylindrical coordinates	8
u_r	radial velocity – spherical coordinates	9
u_θ	circumferential velocity – cylindrical coordinates	8
u_ϑ	circumferential velocity – spherical coordinates	9
$\mathcal{U}$	molecular internal energy	324
V	voltage	106
$\mathcal{V}$	volume	66
ω	angular frequency	49
$\vec{\boldsymbol{\omega}}$	vorticity	12
$\vec{\vec{\boldsymbol{\omega}}}$	rotation rate tensor	11
w	width	90
x	x coordinate	418
ξ	hard-sphere packing parameter	215
ξ	thermodynamic efficiency	143
y	y coordinate	418
Y	Young's modulus	309
z	z coordinate	418
z	valence magnitude for symmetric electrolytes	203
z_i	species valence	99
Z	partition function	326
$\underset{\sim}{Z}$	impedance	119
$\underset{\sim}{Z}_{\mathfrak{h}}$	hydraulic impedance	69

Subscript	Example	Meaning
0	p_0	phasor or sinusoid magnitude
0	w_0	value at reference state
∞	$c_{i,\infty}$	value in freestream or in bulk
bend	$\mathcal{U}_{\text{bend}}$	bending
conv	$\vec{\boldsymbol{j}}_{\text{conv},i}$	convective
diff	$\vec{\boldsymbol{j}}_{\text{diff},i}$	diffusive
edl	q''_{edl}	electrical double layer
eff	ζ_{eff}	effective
ext	$\vec{\boldsymbol{E}}_{\text{ext}}$	extrinsic
H	u_H	high
L	u_L	low
m	$\underset{\sim}{\varepsilon}_{\text{m}}$	suspending medium
n	E_n	normal
p	ρ_{p}	particle
pre	$\vec{\vec{\boldsymbol{\tau}}}_{\text{pre}}$	isotropic (pressure) components
str	I_{str}	streaming
t	u_{t}	tangential
visc	$\vec{\vec{\boldsymbol{\tau}}}_{\text{visc}}$	deviatoric (viscous) components
w	ρ_{w}	water

Superscript, accent	Example(s)	Meaning
$\circ$	g_i°	value at reference condition
$'$	φ', y'	dummy integration variable
$'$	F', I'	per unit length
$''$	F'', q''	per unit area
$'$, $''$	f', f''	derivatives of functions
$'$	ε'	reactive component
$''$	ε''	dissipative component
$-$	$\overline{u}$	spatially averaged
$\sim$	$\underset{\sim}{Z}$	analytic representation of real parameters
$\vec{}$	$\vec{\boldsymbol{u}}$, $\vec{\boldsymbol{T}}$	vector or pseudovector
$\vec{\vec{}}$	$\vec{\vec{\boldsymbol{\tau}}}$, $\vec{\vec{\boldsymbol{\varepsilon}}}$	rank 2 tensor
$\hat{}$	$\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{\vartheta}}$	unit vector
$\hat{}$	$\hat{e}_1$	molar value
$*$	d^*, p^*	nondimensionalized quantity
$\langle\rangle$	$\langle \ell_{\text{e}} \rangle$, $\langle r_{\text{g}} \rangle$	time- or ensemble-averaged property
Δ	Δp, Δx	difference in property

Contents