VOLUME 5

HANDBOOK OF ALGEBRA

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EDITOR

N·H
NORTH-HOLLAND

HANDBOOK OF ALGEBRA

Volume 5

edited by
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Preface

Basic philosophy

Algebra, as we know it today (2007), consists of a great many ideas, concepts and results. And this was also the case in 1995 when this Handbook started (which does not mean that nothing has happened in those 12 years; on the contrary, the field of algebra and its applications has developed at a furious pace.)

A reasonable estimate of the number of all the various different concepts, ideas, definitions, constructions, results, ... would be somewhere between 50 000 and 200 000. Many of these have been named and many more could (and perhaps should) have a "name", or other convenient designation. Even a nonspecialist is quite likely to encounter most of these, either somewhere in the published literature in the form of an idea, definition, theorem, algorithm, ... somewhere, or to hear about them, often in somewhat vague terms, and to feel the need for more information. In such a case, if the concept relates to algebra, then one should be able to find something in this handbook; at least enough to judge whether it is worth the trouble to try to find out more. In addition to the primary information the numerous references to important articles, books, or lecture notes should help the reader find out as much as desired.

As a further tool the index is perhaps more extensive than usual, and is definitely not limited to definitions, (famous) named theorems and the like.

For the purposes of this Handbook, "algebra" is more or less defined as the union of the following areas of the Mathematics Subject Classification Scheme:

- 20 (Group theory)
- 19 (K-theory; this will be treated at an intermediate level; a separate Handbook of K-theory which goes into far more detail than the section planned for this Handbook of Algebra is under consideration)
- 18 (Category theory and homological algebra; including some of the uses of categories in computer science, often classified somewhere in section 68)
- 17 (Nonassociative rings and algebras; especially Lie algebras)
- 16 (Associative rings and algebras)
- 15 (Linear and multilinear algebra. Matrix theory)
- 13 (Commutative rings and algebras; here there is a fine line to tread between commutative algebras and algebraic geometry; algebraic geometry is definitely not a topic that will be dealt with in any detail in this Handbook; there will, hopefully, one day be a separate Handbook on that topic)
- 12 (Field theory and polynomials)
- 11 (Number theory, the part that also used to be classified under 12 (Algebraic number theory))

– 08 (General algebraic systems)
– 06 (Order, lattices, ordered algebraic structures; certain parts; but not topics specific to Boolean algebras as there is a separate three-volume Handbook of Boolean Algebras)

Planning

Originally (1992), we expected to cover the whole field in a systematic way. Volume 1 would be devoted to what is now called Section 1 (see below), Volume 2 to Section 2, and so on. A quite detailed and comprehensive plan was made in terms of topics that needed to be covered and authors to be invited. That turned out to be an inefficient approach. Different authors have different priorities and to wait for the last contribution to a volume, as planned originally, would have resulted in long delays. Instead there is now a dynamic evolving plan. This also permits to take new developments into account.

Chapters are still by invitation only according to the then current version of the plan, but the various chapters are published as they arrive, allowing for faster publication. Thus in this Volume 5 of the Handbook of Algebra the reader will find contributions from 5 sections.

As the plan is dynamic, suggestions from users, both as to topics that could or should be covered, and authors, are most welcome and will be given serious consideration by the board and editor.

The list of sections looks as follows:

Section 1: Linear algebra. Fields. Algebraic number theory
Section 2: Category theory. Homological and homotopical algebra. Methods from logic (algebraic model theory)
Section 3: Commutative and associative rings and algebras
Section 4: Other algebraic structures. Nonassociative rings and algebras. Commutative and associative rings and algebras with extra structure
Section 5: Groups and semigroups
Section 6: Representations and invariant theory
Section 7: Machine computation. Algorithms. Tables
Section 8: Applied algebra
Section 9: History of algebra

For the detailed plan (2007 version), the reader is referred to the Outline of the Series following this preface.

The individual chapters

It is not the intention that the handbook as a whole can also be a substitute undergraduate or even graduate, textbook. Indeed, the treatments of the various topics will be much too dense and professional for that. Basically, the level should be graduate and up, and such material as can be found in P.M. Cohn's three volume textbook 'Algebra' (Wiley) should, as a rule, be assumed known. The most important function of the chapters in this Handbook is to provide professional mathematicians working in a different area with a sufficiency of information on the topic in question if and when it is needed.

Each of the chapters combines some of the features of both a graduate level textbook and a research-level survey. Not all of the ingredients mentioned below will be appropriate in each case, but authors have been asked to include the following:

- Introduction (including motivation and historical remarks)
- Outline of the chapter
- Basic concepts, definitions, and results. (These may be accompanied by proofs or (usually better) ideas/sketches of the proofs when space permits)
- Comments on the relevance of the results, relations to other results, and applications
- Review of the relevant literature; possibly complete with the opinions of the author on recent developments and future directions
- Extensive bibliography (several hundred items will not be exceptional)

The present

Volume 1 appeared in 1995 (copyright 1996), Volume 2 in 2000, Volume 3 in 2003, Volume 4 in 2005 (copyright 2006). Volume 6 is planned for 2008. Thereafter, we aim at one volume every two years (or better).

The future

Of course, ideally, a comprehensive series of books like this should be interactive and have a hypertext structure to make finding material and navigation through it immediate and intuitive. It should also incorporate the various algorithms in implemented form as well as permit a certain amount of dialogue with the reader. Plans for such an interactive, hypertext, CDROM (DVD)-based (or web-based) version certainly exist but the realization is still a non-trivial number of years in the future.

Kvoseliai, August 2007 Michiel Hazewinkel

Kaum nennt man die Dinge beim richtigen Namen
so verlieren sie ihren gefährlichen Zauber

(You have but to know an object by its proper name
for it to lose its dangerous magic)

Elias Canetti

Outline of the Series

(as of June 2007)

Philosophy and principles of the Handbook of Algebra

Compared to the outline in Volume 1 this version differs in several aspects.

First there is a major shift in emphasis away from completeness as far as the more elementary material is concerned and towards more emphasis on recent developments and active areas. Second the plan is now more dynamic in that there is no longer a fixed list of topics to be covered, determined long in advance. Instead there is a more flexible nonrigid list that can and does change in response to new developments and availability of authors.

The new policy, starting with Volume 2, is to work with a dynamic list of topics that should be covered, to arrange these in sections and larger groups according to the major divisions into which algebra falls, and to publish collections of contributions (i.e. chapters) as they become available from the invited authors.

The coding below is by style and is as follows.

- **Author(s) in bold**, followed by chapter title: articles (chapters) that have been received and are published or are being published in this volume.
- *Chapter* title in *italic*: chapters that are being written.
- Chapter title in plain text: topics that should be covered but for which no author has yet been definitely contracted.

Chapters that are included in Volumes 1–5 have a (x; yy pp.) after them, where ‘x’ is the volume number and ‘yy’ is the number of pages.

Compared to the plan that appeared in Volume 1 the section on “Representation and invariant theory” has been thoroughly revised from Volume 2 on.

Compared to the plan that appeared in Volume 4, Section 4H (Rings and algebras with additional structure) has been split into two parts: 4H (Hopf algebras and related structures) and 4I (Other rings and algebras with additional structure). The old Section 4I (Witt vectors) has been absorbed into the section on Hopf algebras.

There also a few more changes; mostly addition of some more topics.

Editorial set-up

Managing editor: M Hazewinkel.

Editorial board: M. Artin, M. Nagata, C. Procesi, O. Tausky-Todd†, R.G. Swan, P.M. Cohn, A. Dress, J. Tits, N.J.A. Sloane, C. Faith, S.I. A'dyan, Y. Ihara, L. Small, E. Manes, I.G. Macdonald, M. Marcus, L.A. Bokut', Eliezer (Louis Halle) Rowen, John S. Wilson, Vlastimil Dlab. Note that three editors have been added starting with Volume 5.

Planned publishing schedule (as of July 2007)

1996: Volume 1 (published)
2001: Volume 2 (published)
2003: Volume 3 (published)
2005: Volume 4 (published)
2007: Volume 5 (last quarter)
Further volumes at the rate of one every year.

Section 1. Linear algebra. Fields. Algebraic number theory

A. Linear Algebra

G.P. Egorychev, Van der Waerden conjecture and applications (1; 22 pp.)
V.L. Girko, Random matrices (1; 52 pp.)
A.N. Malyshev, Matrix equations. Factorization of matrices (1; 38 pp.)
L. Rodman, Matrix functions (1; 38 pp.)
Correction to the chapter by **L. Rodman**, Matrix functions (3; 1 p.)
J.A. Hermida-Alonso, Linear algebra over commutative rings (3; 49 pp.)
Linear inequalities (also involving matrices)
Orderings (partial and total) on vectors and matrices
Positive matrices
Structured matrices such as Toeplitz and Hankel
Integral matrices. Matrices over other rings and fields.
Quasideterminants, and determinants over noncommutative fields.
Nonnegative matrices, positive definite matrices, and doubly nonnegative matrices.
Linear algebra over skew fields

B. Linear (In)dependence

J.P.S. Kung, Matroids (1; 28 pp.)

C. Algebras Arising from Vector Spaces

Clifford algebras, related algebras, and applications
Other algebras arising from vector spaces (working title only)

D. Fields, Galois Theory, and Algebraic Number Theory

(There is also a chapter on ordered fields in Section 4)
J.K. Deveney, **J.N. Mordeson**, Higher derivation Galois theory of inseparable field extensions (1; 34 pp.)
I. Fesenko, Complete discrete valuation fields. Abelian local class field theories (1; 48 pp.)
M. Jarden, Infinite Galois theory (1; 52 pp.)
R. Lidl, **H. Niederreiter**, Finite fields and their applications (1; 44 pp.)
W. Narkiewicz, Global class field theory (1; 30 pp.)
H. van Tilborg, Finite fields and error correcting codes (1; 28 pp.)
Skew fields and division rings. Brauer group

Topological and valued fields. Valuation theory
Zeta and L-functions of fields and related topics
Structure of Galois modules
Constructive Galois theory (realizations of groups as Galois groups)
Dessins d'enfants
Hopf Galois theory
T. Albu, From field theoretic to abstract co-Galois theory (5; 81 pp.)

E. Nonabelian Class Field Theory and the Langlands Program

(to be arranged in several chapters by Y. Ihara)

F. Generalizations of Fields and Related Objects

U. Hebisch, **H.J. Weinert**, Semi-rings and semi-fields (1; 38 pp.)
G. Pilz, Near rings and near fields (1; 36 pp.)

Section 2. Category theory. Homological and homotopical algebra. Methods from logic

A. Category Theory

S. MacLane, **I. Moerdijk**, Topos theory (1; 28 pp.)
R. Street, Categorical structures (1; 50 pp.)
B.I. Plotkin, Algebra, categories and databases (2; 71 pp.)
P.S. Scott, Some aspects of categories in computer science (2; 71 pp.)
E. Manes, Monads of sets (3; 48 pp.)
M. Markl, Operads and PROPs (5; 54 pp.)

B. Homological Algebra. Cohomology. Cohomological Methods in Algebra. Homotopical Algebra

J.F. Carlson, The cohomology of groups (1; 30 pp.)
A. Generalov, Relative homological algebra. Cohomology of categories, posets, and coalgebras (1; 28 pp.)
J.F. Jardine, Homotopy and homotopical algebra (1; 32 pp.)
B. Keller, Derived categories and their uses (1; 32 pp.)
A.Ya. Helemskii, Homology for the algebras of analysis (2; 143 pp.))
Galois cohomology
Cohomology of commutative and associative algebras
Cohomology of Lie algebras
Cohomology of group schemes
V. Lyubashenko, **O. Manzyuk**, $A_{\{\infty\}}$-algebras, $A_{\{\infty\}}$-categories, and $A_{\{\infty\}}$-functors (5; 46 pp.)
B.V. Novikov, 0-Cohomology of semigroups (5; 21 pp.)

C. Algebraic K-theory

A. Kuku, Classical algebraic K-theory: the functors K_0, K_1, K_2 (3; 55 pp.)

A. Kuku, Algebraic K-theory: the higher K-functors (4; 122 pp.)
Grothendieck groups
K_2 and symbols
KK-theory and EXT
Hilbert C^*-modules
Index theory for elliptic operators over C^* algebras
Simplicial algebraic K-theory
Chern character in algebraic K-theory
Noncommutative differential geometry
K-theory of noncommutative rings
Algebraic L-theory
Cyclic cohomology
Asymptotic morphisms and E-theory
Hirzebruch formulae

D. Model Theoretic Algebra

(See also P.C. Eklof, Whitehead modules, in Section 3B)
M. Prest, Model theory for algebra (3; 31 pp.)
M. Prest, Model theory and modules (3; 34 pp.)
Logical properties of fields and applications
Recursive algebras
Logical properties of Boolean algebras
F.O. Wagner, Stable groups (2; 36 pp.)
The Ax–Ershov–Kochen theorem and its relatives and applications

E. Rings up to Homotopy

Rings up to homotopy
Simplicial algebras

Section 3. Commutative and associative rings and algebras

A. Commutative Rings and Algebras

(See also C. Faith, Coherent rings and annihilator conditions in matrix and polynomial rings, in Section 3B)
(See also Freeness theorems for group rings and Lie algebras in Section 5A)
J.P. Lafon, Ideals and modules (1; 24 pp.)
General theory. Radicals, prime ideals, etc. Local rings (general). Finiteness and chain conditions
Extensions. Galois theory of rings
Modules with quadratic form
Homological algebra and commutative rings. Ext, Tor, etc. Special properties (p.i.d., factorial, Gorenstein, Cohen–Macauley, Bezout, Fatou, Japanese, excellent, Ore, Prüfer, Dedekind, ... and their interrelations)
D. Popescu, Artin approximation (2; 45 pp.)

Finite commutative rings and algebras. (Absorbed in the Chapter A.A. Nechaev, Finite rings with applications, in Section 3B)
Localization. Local–global theory
Rings associated to combinatorial and partial order structures (straightening laws, Hodge algebras, shellability, ...)
Witt rings, real spectra
R.H. Villareal, Monomial algebras and polyhedral geometry (3; 62 pp.)

B. Associative Rings and Algebras

P.M. Cohn, Polynomial and power series rings. Free algebras, firs and semifirs (1; 30 pp.)
Classification of Artinian algebras and rings
V.K. Kharchenko, Simple, prime, and semi-prime rings (1; 52 pp.)
A. van den Essen, Algebraic microlocalization and modules with regular singularities over filtered rings (1; 28 pp.)
F. Van Oystaeyen, Separable algebras (2; 66 pp.)
K. Yamagata, Frobenius rings (1; 48 pp.)
V.K. Kharchenko, Fixed rings and noncommutative invariant theory (2; 27 pp.)
General theory of associative rings and algebras
Rings of quotients. Noncommutative localization. Torsion theories
von Neumann regular rings
Semi-regular and pi-regular rings
Lattices of submodules
A.A. Tuganbaev, Modules with distributive submodule lattice (2; 25 pp.)
A.A. Tuganbaev, Serial and distributive modules and rings (2; 25 pp.)
PI rings
Generalized identities
Endomorphism rings, rings of linear transformations, matrix rings
Homological classification of (noncommutative) rings
S.K. Sehgal, Group rings and algebras (3; 96 pp.)
Dimension theory
V.V. Bavula, Filter dimension (4; 28 pp.)
A. Facchini, The Krull–Schmidt theorem (3; 42 pp.)
Duality. Morita-duality
Commutants of differential operators
E.E. Enochs, Flat covers (3; 21 pp.)
C. Faith, Coherent rings and annihilation conditions in matrix and polynomial rings (3; 31 pp.)
Rings of differential operators
Graded and filtered rings and modules (also commutative)
P.C. Eklof, Whitehead modules (3; 23 pp.)
Goldie's theorem, Noetherian rings and related rings
Sheaves in ring theory
A.A. Tuganbaev, Modules with the exchange property and exchange rings (2; 25 pp.)

A.A. Nechaev, Finite rings with applications (5; 116 pp.)
T.Y. Lam, Hamilton's quaternions (3; 26 pp.)
Koszul algebras
A.A. Tuganbaev, Semiregular, weakly regular, and π-regular rings (3; 25 pp.)
Hamiltonian algebras
A.A. Tuganbaev, Max rings and V-rings (3; 23 pp.)
Algebraic asymptotics
Anti-automorphisms

C. Coalgebras

W. Michaelis, Coalgebras (3; 120 pp.)
Co-Lie-algebras
Comodules and corings

D. Deformation Theory of Rings and Algebras (Including Lie Algebras)

Deformation theory of rings and algebras (general)
Yu. Khakimdzanov, Varieties of Lie algebras (2; 35 pp.)
Deformation theoretic quantization

Section 4. Other algebraic structures. Nonassociative rings and algebras. Commutative and associative algebras with extra structure

A. Lattices and Partially Ordered Sets

Lattices and partially ordered sets
A. Pultr, Frames (3; 63 pp.)
D. Kruml, **J. Paseka**, Algebraic and categorical aspects of quantales (5; 40 pp.)

B. Boolean Algebras

C. Universal Algebra

Universal algebra

D. Varieties of Algebras, Groups, . . .

(See also Yu. Khakimdzanov, Varieties of Lie algebras, in Section 3D)
V.A. Artamonov, Varieties of algebras (2; 30 pp.)
Varieties of groups
V.A. Artamonov, Quasi-varieties (3; 19 pp.)
Varieties of semigroups

E. Lie Algebras

Yu.A. Bahturin, **M.V. Zaitsev**, **A.A. Mikhailov**, Infinite-dimensional Lie superalgebras (2; 30 pp.)
General structure theory
Ch. Reutenauer, Free Lie algebras (3; 21 pp.)
Classification theory of semisimple Lie algebras over **R** and **C**

The exceptional Lie algebras
M. Goze, **Y. Khakimdjanov**, Nilpotent and solvable Lie algebras (2; 51 pp.)
Universal enveloping algebras
Modular (ss) Lie algebras (including classification)
Infinite-dimensional Lie algebras (general)
Kac–Moody Lie algebras
Affine Lie algebras and Lie super algebras and their representations
Finitary Lie algebras
Standard bases
A.I. Molev, Gelfand–Tsetlin bases for classical Lie algebras (4; 69 pp.)
Kostka polynomials

F. Jordan Algebras (finite and infinite-dimensional and including their cohomology theory)

G. Other Nonassociative Algebras (Mal'tsev, alternative, Lie admissible, ...)

Mal'tsev algebras
Alternative algebras

H. Hopf Algebras and Related Structures

(See also "Hopf-Galois theory" in Section 1D)
(See also "Co-Galois theory" in Section 1D)
(See also "Algebraic structures on braided categories" in Section 2A)
(See also "Representation theory of semi-simple Hopf algebras" in Section 6D)
M. Cohen, **S. Gelakov**, **S. Westreich**, Hopf algebras (4; 87 pp.)
Classification of pointed Hopf algebras
Recursive sequences from the Hopf algebra and coalgebra points of view
Quantum groups (general)
Crystal bases
A.I. Molev, Yangians and their applications (3; 54 pp.)
Formal groups
p-divisible groups
Combinatorial Hopf algebras
Symmetric functions
Special functions and q-special functions, one and two variable case
Quantum groups and multiparameter q-special functions
D. Manchon, Hopf algebras in renormalisation (5; 63 pp.)
Noncommutative geometry à la Connes
Noncommutative geometry from the algebraic point of view
Noncommutative geometry from the categorical point of view
Hopf algebras and operads
Noncommutative symmetric functions and quasi-symmetric functions
Solomon descent algebras
Witt vectors and symmetric function
Picard–Vessiot theory and Hopf algebras

Hopf-algebroids
Trees, dendriform algebras and dialgebras
A. Masuoka, Classification of semisimple Hopf algebras (5; 27 pp.)
Quantum differential geometry, quantum calculus and the quantum approach to noncommutative geometry
Connes–Baum theory

I. Other Rings and Algebras with Additional Structure

Graded and super algebras (commutative, associative; for Lie superalgebras, see Section 4E)
Topological rings
F. Patras, Lambda-rings (3; 34 pp.)
Ordered and lattice-ordered groups, rings and algebras
Rings and algebras with involution. C^*-algebras
A. Levin, Difference algebra (4; 100 pp.)
Differential algebra
Ordered fields
Hypergroups
Stratified algebras

Section 5. Groups and semigroups

A. Groups

(See also "Groups and semigroups of automata transformations" in Section 5B)
A.V. Mikhalev, **A.P. Mishina**, Infinite Abelian groups: Methods and results (2; 48 pp.)
Simple groups, sporadic groups
Representations of the finite simple groups
Diagram methods in group theory
Abstract (finite) groups. Structure theory. Special subgroups. Extensions and decompositions.
Solvable groups, nilpotent groups, p-groups
Infinite soluble groups
Word problems
Burnside problem
Combinatorial group theory
Free groups (including actions on trees)
Formations
Infinite groups. Local properties
Algebraic groups. The classical groups. Chevalley groups
Chevalley groups over rings
The infinite-dimensional classical groups
Other groups of matrices. Discrete subgroups.
M. Geck, **G. Malle**, Reflection groups. Coxeter groups (4; 38 pp.)

M.C. Tamburini, **M. Vsemirnov**, Hurwitz groups and Hurwitz generation (4; 38 pp.)
Groups with BN-pair, Tits buildings, ...
Groups and (finite combinatorial) geometry
"Additive" group theory
Probabilistic techniques and results in group theory
V.V. Vershinin, Survey on braids (4; 24 pp.)
L. Bartholdi, **R.I. Grigorchuk**, **Z. Šuník**, Branch groups (3; 129 pp.)
Frobenius groups
Just infinite groups
V.I. Senashov, Groups with finiteness conditions (4, 27 pp.)
Automorphism groups of groups
Automorphism groups of algebras and rings
Freeness theorems in groups and rings and Lie algebras
Groups with prescribed systems of subgroups
Automatic groups
Groups with minimality and maximality conditions (school of Chernikov)
Lattice-ordered groups
Linearly and totally ordered groups
Finitary groups
Random groups
Hyperbolic groups
Probabilistic techniques in group theory
Infinite dimensional groups

B. Semigroups

(See also B.V. Novikov, 0-cohomology of semigroups, in Section 2B)
Semigroup theory. Ideals, radicals, structure theory
Semigroups and automata theory and linguistics
Groups and semigroups of automata transformations

C. Algebraic Formal Language Theory. Combinatorics of Words

D. Loops, Quasigroups, Heaps, ...

Quasigroups in combinatorics

E. Combinatorial Group Theory and Topology

(See also "Diagram methods in group theory" in Section 5A)

Section 6. Representation and invariant theory

A. Representation Theory. General

Representation theory of rings, groups, algebras (general)
Modular representation theory (general)
Representations of Lie groups and Lie algebras (general)
Multiplicity free representations