Macroeconomic Theory

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ECONOMIC THEORY, ECONOMETRICS, AND MATHEMATICAL ECONOMICS

MACROECONOMIC THEORY

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PREFACE

This book grew out of a series of lecture notes prepared for a first-year graduate course at the University of Minnesota. Part I is devoted to a presentation of some fairly standard nonstochastic macroeconomic analysis. Part II attempts to provide an introduction to some of the methods and issues in stochastic macroeconomics. This book does not purport to present a unified treatment of a single, widely received macroeconomic theory since the economics profession has not yet attached itself to any one such theory. On the contrary, one can concoct a large variety of plausible macroeconomic models at the level of rigor of the usual Keynesian model, models exhibiting very different responses to policy experiments. A partial aim of the first five chapters and the exercises given there is to exhibit the extent of this variety.

Part II uses a somewhat roundabout means of production and devotes some space to a treatment of some of the tools of modern macroeconomics: lag operators, linear least squares prediction, and stochastic difference equations. The chapter on stochastic difference equations is intended for browsing on the first reading, only parts of this chapter being required for understanding the sequel.

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CONTENTS

PART I

Nonstochastic Macroeconomics

INTRODUCTION

These pages present static and dynamic analyses of some standard macroeconomic models. By *static* analysis we mean the analysis of events assumed to occur at a point in time. In effect, statics studies the alternative point-in-time or momentary equilibrium values for a set of *endogenous variables* associated with alternative possible settings for the *exogenous variables* at the particular point in time under consideration. Endogenous variables are those determined by the model at hand, while exogenous variables are those given from outside the model.

The task of *dynamics* is to study the time paths of the endogenous variables associated with alternative possible time paths of the exogenous variables. Thus, in a dynamic analysis the behavior of a model is studied as time is permitted to pass. In contradistinction, in a static analysis attention is confined to events assumed to occur instantaneously, i.e., at a given moment.

A third kind of analysis, that of *stationary* states, is a limiting form of dynamic analysis, and is directed toward establishing the ultimate tendencies of certain endogenous variables, such as the capital–output ratio, as time passes without limit and as certain critical exogenous variables remain constant through time. Stationary analysis ought not to be confused with statics.

The distinguishing feature of a static analysis is that it is capable of determining alternative values of the endogenous variables, taking as given only the values of the exogenous variables at that point in time, which may include values of endogenous and exogenous variables that were determined in the past and are thus given or predetermined at the present moment. As we shall see, some models for which a dynamic analysis is possible simply cannot be subjected to static analysis. In order to perform static experiments it is necessary partly to divorce current events from future events so that what happens in the future does not affect what happens now. This requires restricting the way in which people are assumed to form expectations about the future, and in particular requires that people not possess perfect foresight.

Generally, our models will consist of *n structural* equations in *n* endogenous variables $y_i(t)$, $i = 1, \ldots, n$, and *m* exogenous variables $x_i(t)$, $i = 1, \ldots, m$:

$$g_i(y_1(t), y_2(t), \ldots, y_n(t), x_1(t), \ldots, x_m(t)) = 0, \qquad i = 1, \ldots, n. \qquad (1)$$

A *structural equation* summarizes behavior, an equilibrium condition, or an accounting identity, and constitutes a building block of the model. In general, more than one and possibly all n endogenous variables can appear in any given structural equation. The system of equations (1) will be thought of as holding at each point in time t. Time itself will be regarded as passing continuously, so that t may be regarded as taking all values along the (extended) real line.

The exogenous variables $x_i(t)$, $i = 1, \ldots, m$, are assumed to be right-continuous functions of time, and furthermore are assumed to possess right-hand time derivatives of at least first and sometimes higher order at all points in time. By right-continuity of the functions $x_i(t)$ we mean

$$\lim_{t \to \bar{t},\, t > \bar{t}} x_i(t) = x_i(\bar{t}),$$

so that $x_i(t)$ approaches $x_i(\bar{t})$ as t approaches $\bar{t}$ from above, i.e., from the future. However, the function $x_i(t)$ can jump at $\bar{t}$, so that we do *not* require

$$\lim_{t \to \bar{t},\, t < \bar{t}} x_i(t) = x_i(\bar{t}).$$

For example, consider the function

$$x_i(t) = \begin{cases} 0, & t < \bar{t} \\ 1, & t \geq \bar{t}, \end{cases}$$

which is graphed in Figure 1. It is right-continuous everywhere even though it jumps, i.e., is discontinuous, at $\bar{t}$.

The right-hand time derivative of $x_i(t)$, which is assumed to exist everywhere, is defined as

$$\frac{d}{dt} x_i(\bar{t}) \equiv \lim_{t \to \bar{t},\, t > \bar{t}} \frac{x_i(t) - x_i(\bar{t})}{t - \bar{t}}.$$

For the function graphed in Figure 1, the right-hand derivative is zero everywhere, even though the function jumps and hence is not differentiable at $t = \bar{t}$.

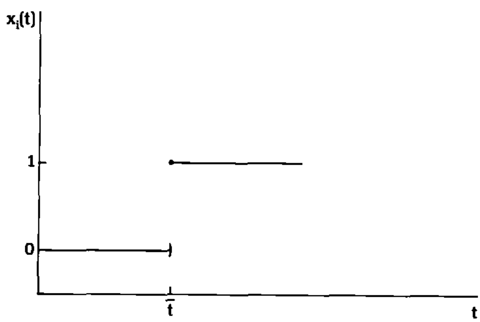

FIGURE 1

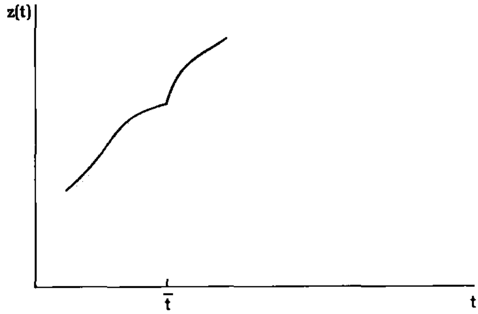

FIGURE 2 $z(t)$ is continuous, but the right-hand derivative jumps at $\bar{t}$.

A model is said to be in static *equilibrium* at a particular moment if the endogenous variables assume values that assure that equations (1) are all satisfied. Notice that it is *not* an implication of this definition of equilibrium that the values of the endogenous variables are unchanging through time. On the contrary, since the values of the exogenous variables will in general be changing at some nonzero rates per unit time, the endogenous variables will also be changing over time.

Static analysis is directed toward answering questions of the following form. Suppose that one of the exogenous variables $x_i(t)$ takes a (small) jump at time $\bar{t}$, so that

$$\lim_{t \to \bar{t},\, t < \bar{t}} x_i(t) \neq x_i(\bar{t}).$$

Then the question is to determine the responses of the endogenous variables at $\bar{t}$. The distinguishing characteristic of endogenous variables is that each of them is assumed to be able to jump discontinuously at any moment in time in order to guarantee that system (1) remains satisfied in the face of jumps in the $x_i(t)$. Thus, to be endogenous from the point of view of statics, a variable must be able to change instantaneously. Notice that it is possible for the right-hand time derivative of a variable to be endogenous, i.e., to be capable of jumping discontinuously, even though the variable itself must change continuously through time (Figure 2 gives an example). One way to view the difference between the classical and Keynesian models is that in the former the money wage is an endogenous variable in static experiments, while in the latter the right-hand time derivative of the money wage is an endogenous variable but the level of the money wage is exogenous.

To answer the typical question addressed in statics the *reduced form* equations corresponding to the system (1) must be found. The reduced form equations are a set of equations, each expressing one $y_i(t)$ as a function of only the $x_i(t)$:

$$y_i(t) = h_i(x_1(t), x_2(t), \ldots, x_m(t)), \qquad i = 1, \ldots, n. \tag{2}$$

We shall generally assume that the functions $g_i(\ )$ in the structural equations (1) are continuously differentiable in all directions, that the n structural equations were satisfied at all moments immediately preceding the moment we are studying, and that the Jacobian determinant

$$\left|\frac{\partial g}{\partial y}\right| = \begin{vmatrix} \partial g_1/\partial y_1 & \partial g_1/\partial y_2 & \cdots & \partial g_1/\partial y_n \\ \partial g_2/\partial y_1 & & \cdots & \partial g_2/\partial y_n \\ \vdots & & & \\ \partial g_n/\partial y_1 & & \cdots & \partial g_n/\partial y_n \end{vmatrix}$$

does not vanish when evaluated at the immediately preceding values of all variables. That is, we shall assume the hypotheses of the implicit function theorem.[1] Under these hypotheses there exist continuously differentiable functions of the reduced form (2) that hold for the $x_i(t)$ sufficiently close to the initial (prejump) values of the $x_i(t)$. If these equations (2) are satisfied, we are guaranteed that the structural equations (1) are satisfied. For jumps in $x_i(t)$ sufficiently small, i.e., within the neighborhood identified in the implicit function theorem, the equations (2) hold and can be used to answer the characteristic question posed in static analysis. In particular, the reduced form partial derivative

$$\frac{\partial y_i(t)}{\partial x_j(t)} = \frac{\partial h_i}{\partial x_j(t)}(x_1(t), \ldots, x_m(t)) \tag{3}$$

gives the response of $y_i(t)$ to a jump in $x_j(t)$ that occurs at t. We are generally interested in the sign of the partial derivative of the reduced form.

Rather than using the implicit function theorem directly to calculate the reduced form partial derivatives (3), it will be convenient to use the following alternative technique that always gives the correct answer. First, take the differential of all equations in (1) to obtain

$$\frac{\partial g_i}{\partial y_1} dy_1 + \cdots + \frac{\partial g_i}{\partial y_n} dy_n + \frac{\partial g_i}{\partial x_1} dx_1 + \cdots + \frac{\partial g_i}{\partial x_m} dx_m = 0, \qquad i = 1, \ldots, n, \tag{4}$$

all partial derivatives being evaluated at the initial values of the x_i and y_j. Then by successive substitution eliminate $dy_2, \ldots, dy_n$ from the above system (4) of linear equations to obtain an equation of the form

$$dy_1 = f_1^1\, dx_1 + f_2^1\, dx_2 + \cdots + f_m^1\, dx_m \tag{5}$$

where the f_j^1 are functions of the partial derivatives that appear in (4). Now Equation (5) is the total differential of the reduced form for y_1 since dy_1 is a

[1] See, e.g., A. E. Taylor and W. R. Mann, *Advanced Calculus*, 2nd ed., p. 363 (Lexington, Massachusetts: Xerox, 1972).

function of only $dx_1, \ldots, dx_m$. Taking the differential of the first equation of (2) gives

$$dy_1 = \frac{\partial h_1}{\partial x_1} dx_1 + \cdots + \frac{\partial h_1}{\partial x_m} dx_m. \tag{6}$$

From (6) and (5) it therefore follows that

$$f_j^1 = \partial h_1/\partial x_j \qquad \text{for} \quad j = 1, \ldots, n,$$

so that the f_j^1 are the reduced form partial derivatives. Successive substitution in the system (4) will also, of course, yield the differentials of the reduced forms for the other endogenous variables, thereby enabling us to obtain the corresponding reduced form partial derivatives. The reduced form partial derivatives are often called "multipliers" in macroeconomics.

CHAPTER I

THE "CLASSICAL" MODEL

Our model describes the determination of an economy's rate of output and the uses to which it is put. The economy produces a single good, which is produced at a rate per unit time of Y. This rate of output is divided among a real rate of consumption C, a real rate of investment I, a real rate of government purchases G, and a real rate of depreciation of capital δK:

$$Y = C + I + G + \delta K. \tag{1}$$

Equation (1) is the national income identity linking aggregate output and its components.

The economy is organized into three sectors. *Firms* employ capital and labor to produce output. The *government* collects taxes and purchases goods, issues money and bonds, and conducts open-market operations. *Households* own the government's money and bond liabilities and all of the equities of firms. They make both a saving decision and a decision to allocate their portfolios of paper assets among bonds, equities, and money.

1. FIRMS

The economy consists of a large number of n perfectly competitive firms, each of which produces the same single good subject to the same production function. The rate of output of the ith firm at any instant is described by the instantaneous production function

$$Y_i = F(K_i, N_i), \qquad i = 1, \ldots, n, \tag{2}$$

where Y_i is the output of the ith firm per unit time, K_i is the stock of capital employed by the ith firm, and N_i is employment of the ith firm.[1] The variables

[1] Capital is measured in units of the one good in the economy, labor in number of men.

Y_i, K_i, and N_i should each be thought of as functions of time. We have omitted a subscript i from the function F because it is assumed that all firms share the same production function. The production function is assumed to be characterized by positive though diminishing marginal products of capital and labor and a direct dependence of the marginal product of capital (employment) on employment (capital):

$$F_K,\ F_N > 0, \qquad F_{KK},\ F_{NN} < 0, \qquad F_{KN} > 0.$$

The production function F is assumed to be linearly homogeneous in K_i and N_i, so that

$$\lambda F(K_i, N_i) = F(\lambda K_i, \lambda N_i), \qquad \lambda > 0.$$

By virtue of Euler's theorem on homogeneous functions we have

$$Y_i = \frac{\partial F}{\partial K_i}(K_i, N_i)\, K_i + \frac{\partial F}{\partial N_i}(K_i, N_i)\, N_i.$$

Also, by virtue of the linear homogeneity of F we have

$$\frac{\partial}{\partial K_i} F(K_i, N_i) = \frac{\partial F}{\partial \lambda K_i}(\lambda K_i, \lambda N_i);$$

setting $\lambda = 1/N_i$, we have

$$\frac{\partial}{\partial K_i} F(K_i, N_i) = \frac{\partial F}{\partial (K_i/N_i)}\left(\frac{K_i}{N_i}, 1\right),$$

so that the marginal product of capital depends only on the ratio of capital to labor. Similarly, the marginal product of labor depends only on the ratio of capital to labor.

In this one-good economy capital represents the accumulated stock of the one good that is available to assist in production. We assume that at any moment the stock of capital is fixed both to the economy and to each individual firm. Assuming that capital is fixed to the economy amounts to ruling out once-and-for-all gifts of physical capital from abroad or from heaven and once-and-for-all decreases in the capital stock due to natural or human disasters. Assuming that capital is fixed to each firm at each moment in time amounts to ruling out the existence of a perfect market in the existing stock of capital in which individual firms can purchase or sell (or rent) capital, and so effect a discrete change in their stock of capital at a moment in time. The absence of a market in existing capital might be rationalized by positing that once in place capital becomes completely specialized to each firm. Firms simply have no use for the existing capital of another firm, so that there is no opportunity for making a market in existing capital.[2] Regardless of how the assumption is rationalized, ruling out trading of existing stocks of capital is a fundamental feature of the class of "classical" and "Keynesian" models that we shall be describing. It is the feature that makes flow aggregate demand play such an important role. In contradistinction, in Tobin's "dynamic aggregative model," which we study in Chapter III and in which there *is* a perfect market in which firms trade *stocks* of capital, flow aggregate demand plays no role in determining the level of output at a point in time.

While firms cannot trade capital at a point in time, they are assumed to be able to vary employment instantaneously. Firms operate in a competitive labor market in which at any moment they can hire all the labor they want at the going money wage w measured in dollars per man per unit of time. Firms are perfectly competitive in the output market also, and each can sell output at any rate it wishes at the price of the one good in the model, p measured in dollars per good.

The typical firm's profits Π_i are defined by

$$\Pi_i = pF(K_i, N_i) - wN_i - (r + \delta - \pi)pK_i \tag{3}$$

where r is the instantaneous rate of interest on government bonds, δ is the instantaneous rate of physical depreciation of capital, and π is the anticipated rate of increase in the price of (newly produced) capital goods. In a sense to be defined below $r + \delta - \pi$ is the appropriate cost of capital that should be used to define the firm's profits. Were there a rental market in capital, $(r + \delta - \pi)p$ would be the rental rate, expressed in dollars per unit time.

Each firm maximizes its profits per unit time with respect to the employment of labor, taking its capital stock as fixed momentarily. The firm's employment is then described by the first-order condition for maximization of (3),

$$\partial\Pi_i/\partial N_i = pF_{N_i}(K_i, N_i) - w = 0$$

or

$$F_{N_i}(K_i, N_i) = w/p, \tag{4}$$

which states that the firm equates the marginal product of labor to the real wage. Equation (4) is in the nature of a firm's demand function for labor which, given K_i, relates the firm's demand for employment inversely to the real wage. For each firm, Equation (4) determines a capital–labor ratio, which is identical for all firms since all face a common real wage. At any moment the n firms have amounts of capital K_i, $i = 1, \ldots, n$, which might differ across firms. Employment of labor then varies proportionately with K_i across firms.

[2] Alternatively, it is often posited that there are costs of adjusting the capital stock that are internal to the firm and that rise at an increasing rate with increases in the absolute value of the rate of investment. As we shall see in Chapter VI, this can give rise to a Keynesian investment demand schedule of the form assumed in this chapter. In effect, the role of the costs of adjustment is to prevent firms from wanting to make discrete adjustments in their stocks of capital at a point in time.