ALGEBRA *for* COLLEGE STUDENTS

Annotated Instructor's Edition

fifth edition

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Algebra for College Students

PREFACE

Algebra for College Students, Fifth Edition, was written for those college students who need an algebra course to bridge the gap between elementary algebra and the more advanced courses in precalculus mathematics. The first six chapters contain intermediate algebra topics, and the last eight chapters contain a blend of intermediate and college algebra topics. All of the material is at an intermediate level.

The basic concepts of algebra are presented in a simple, straightforward manner. Algebraic ideas are developed in a logical sequence, but in an easy-to-read manner without excessive formalism. Concepts are frequently developed through examples, continuously reinforced through additional examples, and then applied in a variety of problem-solving situations.

The examples demonstrate a large variety of situations; other situations are left for students to think about in the problem sets. In the examples students are guided to organize their work and to decide when a meaningful shortcut might be used.

In the preparation of this edition, a special effort was made to incorporate improvements suggested by reviewers and by users of the earlier editions, while at the same time preserving the book's many successful features.

New in This Edition

- Problems called **Thoughts into Words** are now included in every problem set except the review exercises. These problems are designed to encourage students to express in written form their thoughts about various mathematical ideas. See, for example, Problem Sets 2.1, 3.5, 4.7, 5.5, and 10.2.
- Miscellaneous problem sections, now called **Further Investigations**, have been enhanced by the addition of more problems that lend themselves to small group work. These problems remain as "extras" but add flexibility for the instructor. See, for example, Problem Sets 1.2, 2.7, 5.6, 6.5, and 9.5.
- A **Chapter Test** has been included at the end of each chapter. Along with the **Chapter Review Problem Sets**, these practice tests should provide students with ample opportunity to prepare for the "real" tests. **Cumulative Review Problem Sets** appear at the ends of Chapters 3, 5, 7, and 12.
- The **chapter introductions** have been rewritten in an effort to provide more motivation for students to study algebra. Each introduction begins with at least one application that leads into the material of the chapter.

- **Applications** have been added in several sections, including the following:

  Sections 3.1, 3.2, and 3.3: Examples and problems that connect geometry and the study of polynomials

  Section 5.2: Applications involving radicals

  Section 5.5: Applications involving radical equations

  Section 6.2: Applications of the Pythagorean theorem

  Section 7.1: Applications of slope

- The use of a **graphing utility** is introduced in Section 7.2. Graphics calculator examples (designated by an icon) are then incorporated, as appropriate, throughout Chapters 7 through 14. These examples are written so that students without a graphing utility can read and benefit from them. For example, see Sections 7.2, 7.4, 8.3, 9.4, 9.5, and 10.5.
- A new section of problems called **Graphics Calculator Activities** has been added to many of the problem sets in Chapters 7 through 14. These activities, which are good for either individual or small group work, have been designed to reinforce concepts already presented and lay the groundwork for concepts about to be discussed. They also help students to predict shapes and locations of graphs based on earlier graphing experiences. Through working these problems, students should become more familiar with the capabilities and limitations of a graphics calculator. For example, see Problem Sets 7.2, 7.4, 8.2, 8.3, 9.4, 9.5, and 10.5.
- Parts of **Chapter 8** have been reorganized and a new Section 8.3 has been added. Section 8.2 now discusses linear and quadratic functions and applications of quadratic functions. New Section 8.3 presents transformations of some basic curves. These transformations are then used as appropriate in later sections. Section 8.4 contains the composition of functions in preparation for inverse functions presented in Section 8.5.
- A focal point of every revision is the **problem sets**. Users of the previous editions were very helpful in suggesting problems to be added, deleted, or changed in some way.

Other Special Features

- A common thread runs throughout the book: namely, *learn a skill,* next *use the skill to help solve equations and inequalities,* and then *use equations and inequalities to solve word problems.* This thread influenced some other decisions.

  1. Numerous word problems are scattered throughout the text. These problems deal with a large variety of applications and constantly show the connections between mathematics and the real world.

2. Many problem-solving suggestions are offered throughout, with special discussions in several sections. The problem-solving suggestions are demonstrated in more than 80 worked-out examples.
3. Newly acquired skills are used as soon as possible to solve equations and inequalities, which are, in turn, used to solve word problems. Therefore, the concept of solving equations and inequalities is introduced early and developed throughout the text. The concepts of factoring, solving equations, and solving word problems are tied together in Chapter 3.

- As recommended by the American Mathematical Association of Two-Year Colleges, many basic geometric concepts are integrated in a problem-solving setting. Contained in this text are approximately 20 worked-out examples and 100 problems that connect algebra, geometry, and the real world. Specific discussions of geometric concepts are contained in the following sections:

  Section 2.2: Complementary and supplementary angles; the sum of the angles of a triangle equals 180°

  Section 2.4: Area and volume formulas

  Section 3.4: More on area and volume formulas, perimeter, and circumference formulas

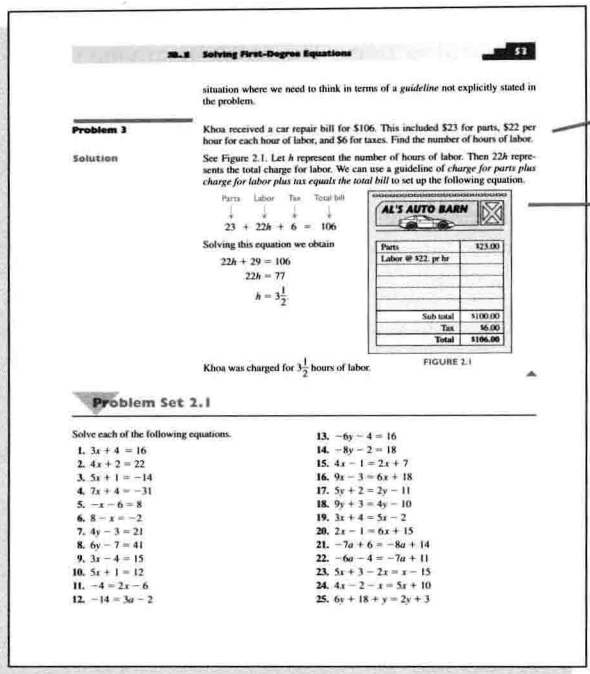
  Section 3.7: Pythagorean theorem

  Section 6.2: More on the Pythagorean theorem, including work with isosceles right triangles and 30°–60° right triangles

- Specific graphing ideas (intercepts, symmetry, restrictions, asymptotes, and transformations) are introduced and used throughout Chapters 7 and 8. In Section 8.3 the work with parabolas from Chapter 7 is used to develop definitions for translations, reflections, stretchings, and shrinkings. These transformations are then applied to the graphs of $f(x) = x^3$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$. Transformations are also used later with polynomial, exponential, and logarithmic curves.
- All answers for Chapter Review Problem Sets, Chapter Tests, and Cumulative Review Problem Sets appear in the back of the text.

Additional Comments About Some of the Chapters

- Chapter 1 is written so that it can be covered quickly, and on an individual basis if so desired, by those needing only a brief review of some basic algebraic concepts.
- Chapter 2 presents an early introduction to the heart of an intermediate algebra course. Problem solving and the solving of equations and inequalities are introduced early so they can be used as unifying themes throughout the text.



2.1 Solving First-Degree Equations 53

situation where we need to think in terms of a *guideline* not explicitly stated in the problem.

Problem 3 Khoa received a car repair bill for $106. This included $23 for parts, $22 per hour for each hour of labor, and $6 for taxes. Find the number of hours of labor.

Solution See Figure 2.1. Let h represent the number of hours of labor. Then $22h$ represents the total charge for labor. We can use a guideline of *charge for parts plus charge for labor plus tax equals the total bill* to set up the following equation.

Parts Labor Tax Total bill

$$23 + 22h + 6 = 106$$

Solving this equation we obtain

$$22h + 29 = 106$$
$$22h = 77$$
$$h = 3\frac{1}{2}$$

AL'S AUTO BARN

Parts	$23.00
Labor @ $22. pr hr	
Sub total	$100.00
Tax	$6.00
Total	$106.00

Khoa was charged for $3\frac{1}{2}$ hours of labor.

FIGURE 2.1

Problem Set 2.1

Solve each of the following equations.

1. $3x + 4 = 16$
2. $4x + 2 = 22$
3. $5x + 1 = -14$
4. $7x + 4 = -31$
5. $-x - 6 = 8$
6. $8 - x = -2$
7. $4y - 3 = 21$
8. $6y - 7 = 41$
9. $3x - 4 = 15$
10. $5x + 1 = 12$
11. $-4 = 2x - 6$
12. $-14 = 3a - 2$
13. $-6y - 4 = 16$
14. $-8y - 2 = 18$
15. $4x - 1 = 2x + 7$
16. $9x - 3 = 6x + 18$
17. $5y + 2 = 2y - 11$
18. $9y + 3 = 4y - 10$
19. $3x + 4 = 5x - 2$
20. $2x - 1 = 6x + 15$
21. $-7a + 6 = -8a + 14$
22. $-6a - 4 = -7a + 11$
23. $5x + 3 - 2x = x - 15$
24. $4x - 2 - x = 5x + 10$
25. $6y + 18 + y = 2y + 3$

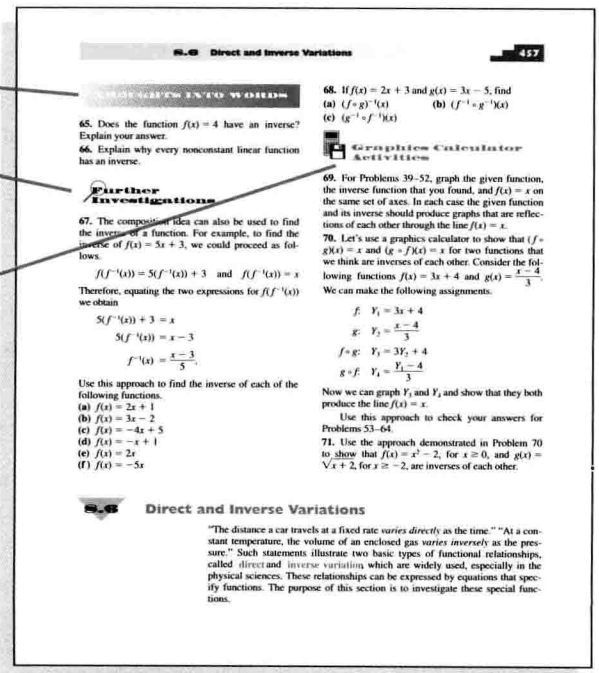
Many sample word problems are fully solved in sections specifically emphasizing problem solving.

Clearly rendered representational art lends interest and helps students visualize the problem.

"Thoughts into Words" problems encourage students to express their mathematical understanding verbally.

"Further Investigations" problems, which require skills learned in the section, are especially appropriate for group work.

"Graphics Calculator Activities," which reinforce concepts and lay the groundwork for new material, ask students to predict the shape and locations of graphs and to draw conclusions from what they see.



5.6 Direct and Inverse Variations 457

Thoughts into Words

65. Does the function $f(x) = 4$ have an inverse? Explain your answer.

66. Explain why every nonconstant linear function has an inverse.

Further Investigations

67. The composition idea can also be used to find the inverse of a function. For example, to find the inverse of $f(x) = 5x + 3$, we could proceed as follows.

$$f(f^{-1}(x)) = 5(f^{-1}(x)) + 3 \quad \text{and} \quad f(f^{-1}(x)) = x$$

Therefore, equating the two expressions for $f(f^{-1}(x))$ we obtain

$$5(f^{-1}(x)) + 3 = x$$
$$5(f^{-1}(x)) = x - 3$$
$$f^{-1}(x) = \frac{x-3}{5}.$$

Use this approach to find the inverse of each of the following functions.

(a) $f(x) = 2x + 1$
(b) $f(x) = 3x - 2$
(c) $f(x) = -4x + 5$
(d) $f(x) = -x + 1$
(e) $f(x) = 2x$
(f) $f(x) = -5x$

68. If $f(x) = 2x + 3$ and $g(x) = 3x - 5$, find
(a) $(f \circ g)^{-1}(x)$ (b) $(f^{-1} \circ g^{-1})(x)$
(c) $(g^{-1} \circ f^{-1})(x)$

Graphics Calculator Activities

69. For Problems 39–52, graph the given function, the inverse function that you found, and $f(x) = x$ on the same set of axes. In each case the given function and its inverse should produce graphs that are reflections of each other through the line $f(x) = x$.

70. Let's use a graphics calculator to show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ for two functions that we think are inverses of each other. Consider the following functions $f(x) = 3x + 4$ and $g(x) = \frac{x-4}{3}$. We can make the following assignments.

f: $Y_1 = 3x + 4$
g: $Y_2 = \frac{x-4}{3}$
$f \circ g$: $Y_3 = 3Y_2 + 4$
$g \circ f$: $Y_4 = \frac{Y_1 - 4}{3}$

Now we can graph Y_3 and Y_4 and show that they both produce the line $f(x) = x$.

Use this approach to check your answers for Problems 53–64.

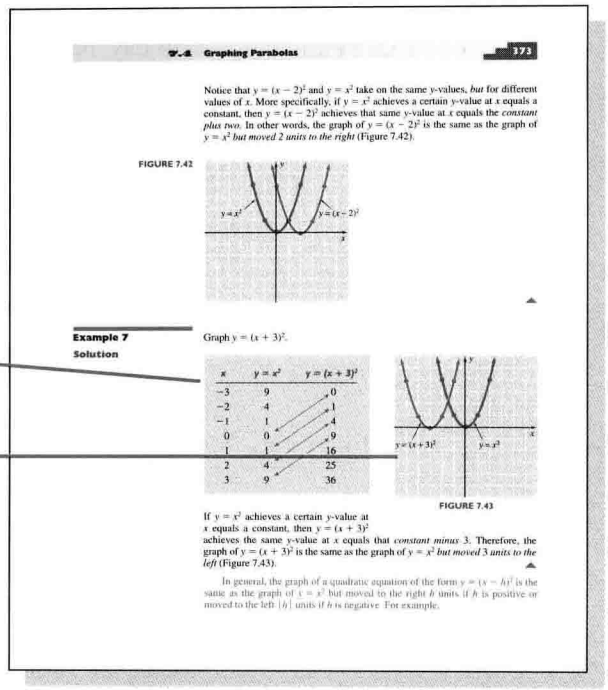
71. Use the approach demonstrated in Problem 70 to show that $f(x) = x^2 - 2$, for $x \geq 0$, and $g(x) = \sqrt{x + 2}$, for $x \geq -2$, are inverses of each other.

5.6 Direct and Inverse Variations

"The distance a car travels at a fixed rate *varies directly* as the time." "At a constant temperature, the volume of an enclosed gas *varies inversely* as the pressure." Such statements illustrate two basic types of functional relationships, called direct and inverse variation, which are widely used, especially in the physical sciences. These relationships can be expressed by equations that specify functions. The purpose of this section is to investigate these special functions.

In sections on graphing, tables of values show numerical approaches to problems.

Large, clear graphs depict curves accurately and restate the equation being graphed.



7.4 Graphing Parabolas 373

Notice that $y = (x - 2)^2$ and $y = x^2$ take on the same y-values, *but* for different values of x. More specifically, if $y = x^2$ achieves a certain y-value at x equals a constant, then $y = (x - 2)^2$ achieves that same y-value at x equals the *constant plus two*. In other words, the graph of $y = (x - 2)^2$ is the same as the graph of $y = x^2$ *but moved 2 units to the right* (Figure 7.42).

FIGURE 7.42

Example 7 Graph $y = (x + 3)^2$.

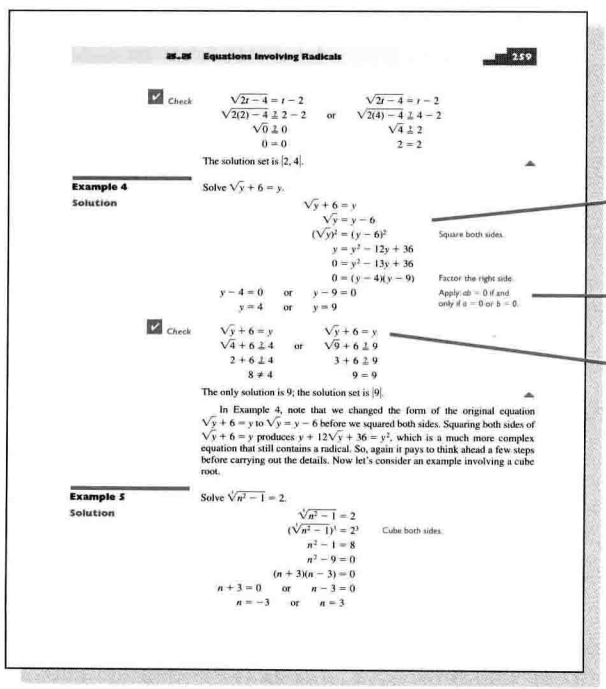
Solution

x	$y = x^2$	$y = (x + 3)^2$
−3	9	0
−2	4	1
−1	1	4
0	0	9
1	1	16
2	4	25
3	9	36

FIGURE 7.43

If $y = x^2$ achieves a certain y-value at x equals a constant, then $y = (x + 3)^2$ achieves the same y-value at x equals that *constant minus* 3. Therefore, the graph of $y = (x + 3)^2$ is the same as the graph of $y = x^2$ *but moved* 3 *units to the left* (Figure 7.43).

In general, the graph of a quadratic equation of the form $y = (x - h)^2$ is the same as the graph of $y = x^2$ but moved to the right h units if h is positive or moved to the left $|h|$ units if h is negative. For example,



2.5 Equations Involving Radicals 259

Check $\sqrt{2t - 4} = t - 2$ or $\sqrt{2t - 4} = t - 2$

$\sqrt{2(2) - 4} \stackrel{?}{=} 2 - 2$ $\sqrt{2(4) - 4} \stackrel{?}{=} 4 - 2$

$\sqrt{0} \stackrel{?}{=} 0$ $\sqrt{4} \stackrel{?}{=} 2$

$0 = 0$ $2 = 2$

The solution set is $\{2, 4\}$.

Example 4 Solve $\sqrt{y} + 6 = y$.

Solution

$\sqrt{y} + 6 = y$

$\sqrt{y} = y - 6$

$(\sqrt{y})^2 = (y - 6)^2$ Square both sides.

$y = y^2 - 12y + 36$

$0 = y^2 - 13y + 36$

$0 = (y - 4)(y - 9)$ Factor the right side.

$y - 4 = 0$ or $y - 9 = 0$ Apply: $ab = 0$ if and only if $a = 0$ or $b = 0$.

$y = 4$ or $y = 9$

Check $\sqrt{y} + 6 = y$ or $\sqrt{y} + 6 = y$

$\sqrt{4} + 6 \stackrel{?}{=} 4$ $\sqrt{9} + 6 \stackrel{?}{=} 9$

$2 + 6 \stackrel{?}{=} 4$ $3 + 6 \stackrel{?}{=} 9$

$8 \neq 4$ $9 = 9$

The only solution is 9; the solution set is $\{9\}$.

In Example 4, note that we changed the form of the original equation $\sqrt{y} + 6 = y$ to $\sqrt{y} = y - 6$ before we squared both sides. Squaring both sides of $\sqrt{y} + 6 = y$ produces $y + 12\sqrt{y} + 36 = y^2$, which is a much more complex equation that still contains a radical. So, again it pays to think ahead a few steps before carrying out the details. Now let's consider an example involving a cube root.

Example 5 Solve $\sqrt[3]{n^2 - 1} = 2$.

Solution

$\sqrt[3]{n^2 - 1} = 2$

$(\sqrt[3]{n^2 - 1})^3 = 2^3$ Cube both sides.

$n^2 - 1 = 8$

$n^2 - 9 = 0$

$(n + 3)(n - 3) = 0$

$n + 3 = 0$ or $n - 3 = 0$

$n = -3$ or $n = 3$

Many worked examples show careful, step-by-step problem solving.

Annotations make clear each step of the problem.

The "Check" feature in worked examples and problems reminds students to complete this important problem-solving step.

- Chapter 6 is organized to give students the opportunity to learn, on a day-to-day basis, different techniques for solving quadratic equations. The process of completing the square is treated as a viable equation-solving process for certain types of quadratic equations. The emphasis on completing the square in this setting pays dividends in Chapter 7 when we graph parabolas and circles. Section 6.5 offers some guidance as to when to use a particular technique for solving quadratic equations. In addition the often overlooked relationships involving the sum and product of roots are discussed and used as an effective checking procedure.
- Chapter 7 is written on the premise that students at this level need more work with coordinate geometry concepts—specifically graphing techniques—*before* functions are defined. My experience indicates a need for students in this course to become proficient at graphing straight lines, parabolas, and circles. In addition at least a little work with graphing ellipses and hyperbolas seems appropriate. Graphing suggestions are offered throughout the chapter.
- Chapter 8 is devoted entirely to functions and the issue is not clouded by the jumping back and forth between functions and relations that are not functions. It includes some work on the composition of functions and the use of quadratic functions in problem-solving situations.
- Chapter 10 presents a modern-day version of the concepts of exponents and logarithms. The emphasis is on making the concepts and their applications understood. The calculator is used as a tool to help with the complicated computational aspects.
- Chapters 11 and 12 contain the various techniques for solving systems of linear equations. This material is organized so that instructors can use as much of the two chapters as needed for a particular course. The work with the elimination-by-addition method in Sections 11.2 and 11.3 emphasizes equivalent systems and sets the stage for the use of matrices in Chapter 12.
- Problem solving is the unifying theme of Chapters 13 and 14. In contrast to most texts of this type, Chapter 14 contains a significant amount of probability. These two chapters lend themselves to individual or small group work.

Ancillaries for Instructors

The following useful ancillaries are available to adopters of this text:

- **Annotated Instructor's Edition** includes answers to all problems in the text—most printed adjacent to the problem.
- **Instructor's Solutions Manual** contains solutions for even-numbered problems and answers for all odd-numbered problems.
- **Test Bank with Chapter Tests** contains all questions and answers from the computerized test bank and three sample tests (two multiple choice,

one open ended) for each chapter. These tests may be duplicated for student testing by instructors using the text.

- **Computerized testing software** is available for the IBM and compatibles and for the Macintosh. The computerized testing programs contain multiple-choice and open-ended questions that allow users to edit, rearrange, and add to the question bank.
- **Videotape Series** follows the organization and style of the textbook. Video lectures include basic instruction and worked examples.

Ancillaries for Students

- **Student's Solutions Manual** contains complete worked-out solutions for all odd-numbered problems.
- **Worksheets and Study Guide** is a text-specific study resource in worktext format. It includes examples and exercises for topics keyed to sections in the text so that students have the opportunity for additional practice and study assistance. The manual is designed to be integrated as an interactive component to lectures or for instructional use outside the classroom.
- **MathQuest Tutorial Software** is an interactive, text-specific intuitive tutorial that runs on both Windows and Macintosh platforms. The program provides fill-in, multiple-choice, and true/false questions. If a student answers a question incorrectly, the program will first respond with hints; if the student answers incorrectly a second time, the program will supply a step-by-step solution. Record-keeping capabilities enable students to monitor their progress.

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Jerome E. Kaufmann
Marble Falls, Texas

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