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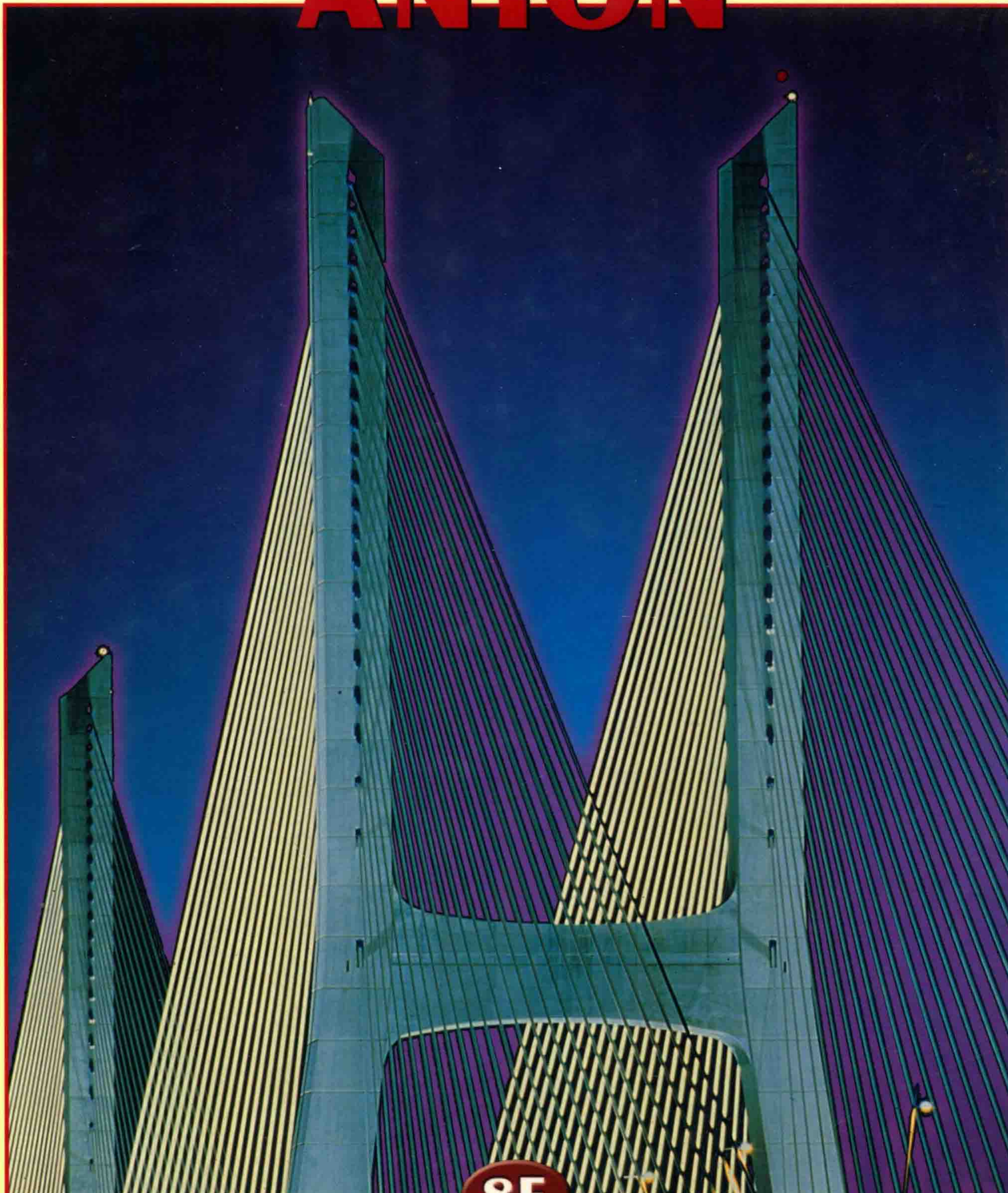

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Elementary Linear Algebra

Elementary Linear Algebra

Eighth Edition

HOWARD ANTON

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Index of Symbols

CHAPTER 1

$[a_{ij}]$, a matrix whose entry in ith row and jth column is a_{ij}
$[A \mid \mathbf{b}]$, augmented matrix of a system of linear equations
$m \times n$, size of a matrix with m rows and n columns
A^T, transpose of a matrix
$\mathrm{tr}(A)$, trace of a matrix A
I_n, $n \times n$ identity matrix
A^{-1}, inverse of a square matrix
$A^T = A$, symmetric matrix
$A^T = -A$, skew-symmetric matrix
$A\mathbf{x} = \mathbf{b}$, system of linear equations
0, zero matrix

CHAPTER 2

$\det(A)$, determinant of a square matrix
E, symbol used to denote an elementary matrix
$\det(\lambda I - A) = 0$, characteristic equation of a square matrix A
M_{ij}, minor of entry a_{ij} in a square matrix A
C_{ij}, cofactor of entry a_{ij} in a square matrix A
$\mathrm{adj}(A)$, adjoint of a square matrix A

CHAPTER 3

$\mathbf{v}$, vector
$\mathbf{0}$, zero vector
$\|\mathbf{u}\|$, norm of a vector $\mathbf{u}$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$, distance formula in 3-space
$\mathbf{u} \cdot \mathbf{v}$, dot product or inner product of two vectors
$\mathrm{proj}_{\mathbf{a}}\mathbf{u}$, vector component of $\mathbf{u}$ along $\mathbf{a}$
$\mathbf{u} - \mathrm{proj}_{\mathbf{a}}\mathbf{u}$, vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$
$\mathbf{u} \times \mathbf{v}$, cross product of two vectors in 3-space
$\mathbf{i}, \mathbf{j}, \mathbf{k}$, standard unit vectors in 3-space
$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$, scalar triple product
$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, point-normal form of the equation of a plane
$ax + by + cz + d = 0$, general form of the equation of a plane
$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$, vector form of the equation of a plane

CHAPTER 4

R^n, set of ordered n-tuples or n-space
$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$, Euclidean inner product
$\mathbf{v}^T\mathbf{u} = \mathbf{u} \cdot \mathbf{v}$, Euclidean inner product using column vectors
$\|\mathbf{u}\| = (\mathbf{u} \cdot \mathbf{u})^{1/2}$, Euclidean norm
$d(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|$, Euclidean distance
T, symbol usually used to denote a linear transformation
$T_A(\mathbf{x}) = A\mathbf{x}$, matrix transformation from R^n to R^m (multiplication by A)
$[T]$, standard matrix for a linear transformation T
$T_B \circ T_A$, composition of a linear transformation T_B with a linear transformation T_A
$[T_2 \circ T_1] = [T_2][T_1]$, standard matrix for the composition $T_2 \circ T_1$
$\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$, standard basis for R^n
$[T] = [T(\mathbf{e}_1) \mid T(\mathbf{e}_2) \mid \cdots \mid T(\mathbf{e}_n)]$, standard matrix for T in terms of the images of the standard basis vectors for R^n

CHAPTER 5

$\mathbf{u}$, a vector
V, usually a general vector space
W, usually a subspace of a vector space V
M_{mn}, vector space of real $m \times n$ matrices
$F(-\infty, \infty)$, space of real-valued functions that are defined on the interval $(-\infty, \infty)$
$F[a, b]$, space of real-valued functions on the closed interval $[a, b]$
$F(a, b)$, space of real-valued functions on the open interval (a, b)
$C(-\infty, \infty)$, space of continuous real-valued functions on the interval $(-\infty, \infty)$
$C^1(-\infty, \infty)$, space of real-valued functions with continuous first derivatives on the interval $(-\infty, \infty)$
$C^m(-\infty, \infty)$, space of real-valued functions with continuous mth derivatives on the interval $(-\infty, \infty)$
$C^\infty(-\infty, \infty)$, space of real-valued functions with continuous derivatives of all orders on the interval $(-\infty, \infty)$
$C[a, b]$, space of real-valued functions that are continuous on the closed interval $[a, b]$
$C(a, b)$, space of real-valued functions that are continuous on the open interval (a, b)
$C^m[a, b]$, space of real-valued functions with continuous mth derivatives on the closed interval $[a, b]$
$C^m(a, b)$, space of real-valued functions with continuous mth derivatives on the open interval (a, b)

$C^\infty[a, b]$, space of real-valued functions with continuous derivatives of all orders on the closed interval $[a, b]$
$C^\infty(a, b)$, space of real-valued functions with continuous derivatives of all orders on the open interval (a, b)
span(S), space of all linear combinations of the vectors in the set S
P_n, space of polynomials of degree $\le n$
$W(x)$, Wronskian
dim(V), dimension of a finite-dimensional vector space V
rank(A), common dimension of the row and column spaces of the matrix A
nullity(A), dimension of the nullspace of an $m \times n$ matrix A
$(\mathbf{v})_S = (c_1, c_2, \ldots, c_n)$, coordinate vector of $\mathbf{v}$ relative to a basis S

CHAPTER 6

$\langle \mathbf{u}, \mathbf{v} \rangle$, inner product of vectors $\mathbf{u}$ and $\mathbf{v}$
$\|\mathbf{u}\|$, norm of a vector $\mathbf{u}$
$d(\mathbf{u}, \mathbf{v})$, distance between vectors $\mathbf{u}$ and $\mathbf{v}$
$W^\perp$, orthogonal complement of a subspace W
$\frac{1}{\|\mathbf{v}\|}\mathbf{v}$, normalized vector
$\text{proj}_W\, \mathbf{u}$, orthogonal projection of the vector $\mathbf{u}$ on the subspace W
$A^{-1} = A^T$, orthogonal matrix
$[\mathbf{v}]_S$, coordinate matrix of $\mathbf{v}$ relative to the basis S

CHAPTER 7

$\det(\lambda I - A) = 0$, characteristic equation of a square matrix A
D, symbol usually used to denote a diagonal matrix

CHAPTER 8

$T\colon V \to W$, linear transformation from a vector space V into a vector space W
ker(T), kernel of the linear transformation T
$R(T)$, range of the linear transformation T
rank(T), rank of the linear transformation T
nullity(T), nullity of a linear transformation
$[T]_{B',B}$, matrix for T with respect to bases B and B'

CHAPTER 9

$Y' = AY$, linear system of first-order differential equations
$\int_a^b [f(x) - g(x)]^2\, dx$, mean square error

CHAPTER 10

i, symbol used to denote $\sqrt{-1}$
$z = a + bi$, a general complex number
(a, b), alternative symbol for a complex number
$\bar{z} = a - bi$, conjugate of a complex number z
Re(z), real part of a complex number z
Im(z), imaginary part of a complex number z
arg z, argument of a complex number z
Arg z, principal argument of a complex number z
$|z| = \sqrt{a^2 + b^2}$, modulus of a complex number z
$z = r(\cos\theta + i \sin\theta)$, polar form of a complex number z
$z = re^{i\theta}$, alternative polar form of a complex number z
C^n, space of n-tuples of complex numbers (complex Euclidean n-space)
complex $C(-\infty, \infty)$, vector space of complex-valued functions of a real variable that are continuous on the interval $(-\infty, \infty)$
complex $C[a, b]$, vector space of complex-valued functions of a real variable that are continuous on the closed interval $[a, b]$
$\mathbf{u} \cdot \mathbf{v} = u_1\bar{v}_1 + u_2\bar{v}_2 + \cdots + u_n\bar{v}_n$, complex Euclidean inner product
$A^* = A$, Hermitian matrix
$A^* = -A$, skew-Hermitian matrix
$A^* = A^{-1}$, unitary matrix
$AA^* = A^*A$, normal matrix

Elementary Linear Algebra

To my wife Pat
and my children
Brian, David, and Lauren

Preface

This edition, in the spirit of its predecessors, gives an elementary treatment of linear algebra that is suitable for students in their freshman or sophomore year. My aim is to present the fundamentals of linear algebra in the clearest possible way—pedagogy is the main consideration. Calculus is not a prerequisite, but there are clearly labeled exercises and examples for students with calculus backgrounds. Those exercises can be omitted without loss of continuity. Technology is also not required, but for those who would like to use MATLAB, Maple, *Mathematica*, or calculators with linear algebra capabilities, I have included exercises at the ends of the chapters for that purpose.

SUMMARY OF CHANGES IN THIS EDITION

This edition is a refinement of the previous edition. I have tried to maintain the clarity and style of the earlier editions, yet accommodate the needs of a new generation of students. Here is a summary of the changes:

- **Addition of Technology Exercises:** A set of technology exercises has been placed at the ends of chapters, but they are categorized according to section, so they can be assigned as part of the regular section exercises. The technology exercises are designed to acquaint the student with most of the basic commands that are required to solve problems in linear algebra using technology. The exercises are written in a generic, syntax-free form with the understanding that the student's own documentation will be the resource for specific commands and procedures. To relieve students of the burden of data entry, data for the technology exercises are provided in MATLAB, Maple, and *Mathematica* formats. These data can be downloaded from **www.wiley.com/college/anton**.
- **Addition of Discussion and Discovery Exercises:** A new category of exercises, identified as *Discussion and Discovery*, has been added to most exercise sets. In keeping with modern pedagogical trends, these exercises are more open-ended than others in the set. They typically include true/false, conjecture, discovery, and informal explanations of how conclusions are reached.
- **Refinement of Exposition:** Parts of the exposition have been refined and improved, but I have not made substantial changes in style, organization, or content, except as previously noted.

Hallmark Features

- **Relationships Between Concepts:** One of the important goals of a course in linear algebra is to establish the intricate thread of relationships between systems of linear equations, matrices, determinants, vectors, linear transformations, and eigenvalues. That thread of relationships is developed through the following crescendo of theorems that link each new idea with ideas that preceded it: 1.5.3, 1.6.4, 2.3.6, 4.3.4, 5.6.9, 6.2.7, 6.4.5, 7.1.5. These theorems bring a coherence to the linear algebra landscape and also serve as a constant source of review.

- **Smooth Transition to Abstraction:** The transition from R^n to general vector spaces is traumatic for most students, so I have tried to smooth out that transition by emphasizing the underlying geometry and developing key ideas in R^n before proceeding to general vector spaces.
- **Early Exposure to Linear Transformations and Eigenvalues:** To ensure that the material on linear transformations and eigenvalues does not get lost at the end of the course, some of the basic concepts relating to those topics are developed early in the text and then reviewed when the topic is treated in more depth later in the text. For example, characteristic equations are discussed briefly in the section on determinants, and linear transformations from R^n to R^m are discussed immediately after R^n is introduced, then reviewed later in the context of general linear transformations.

About the Exercises

Each section exercise set begins with routine drill problems, progresses to problems with more substance, and concludes with theoretical problems. In most sections, the main part of the exercise set is followed by the *Discussion and Discovery* problems described above. Most chapters end with a set of supplementary exercises that tend to be more challenging and force the student to draw on ideas from the entire chapter rather than a specific section. The technology exercises follow the supplementary exercises and are classified according to the section in which we suggest that they be assigned. Data for these exercises can be downloaded from **www.wiley.com/college/anton**.

Supplementary Materials for Students

Student Solutions Manual to Accompany Elementary Linear Algebra, Eighth Edition *Charles A. Grobe, Jr. (Bowdoin College)* and *Elizabeth M. Grobe*. This supplement provides detailed solutions to most theoretical exercises and to at least one nonroutine exercise of every type. (ISBN 0-471-38249-3)

Linear Algebra Applications Software *Howard Anton (Drexel University)*, *Chris Rorres (Drexel University)*, and *Intellipro, Inc.* This is a set of ten modules, each of which focuses on an application of linear algebra. Each module has four parts:

1. A statement of the problem, supported by graphics and animations.
2. A discussion of the ideas relevant to the solution.
3. Interactive practice sessions that allow students to perform "what if" analyses by varying parameters and observing resulting simulations.
4. exercises.

This software, which is provided for Windows 95 and NT, can be downloaded from **www.wiley.com/college/anton**

Data for Technology Exercises is provided in MATLAB, Maple, and *Mathematica* formats. This data can be downloaded from **www.wiley.com/college/anton**

Supplementary Materials for Instructors

Linear Algebra Test Bank: *Randy Schwartz (Schoolcraft College)*. This includes approximately 50 free-form questions, five essay questions for each chapter, and a sample cumulative final examination. Worked out solutions are given for each question. This manual is available in hard copy form or can be downloaded from **www.wiley.com/college/anton**

Wiley Web Tests for Linear Algebra: This is a system for assigning homework or giving examinations over the World Wide Web. Questions are multiple choice, free response, and true/false. The system can be used to administer tests in either practice or proctored modes. Students receive immediate feedback on grades. This powerful testing and homework management system is available to instructors upon adoption of this text.

Web Resources: More information about this text and its resources can be obtained from your Wiley representative or from **www.wiley.com/college/anton**

A GUIDE FOR THE INSTRUCTOR

Linear algebra courses vary widely between institutions in content and philosophy, but most courses fall into two categories: those with about 35–40 lectures (excluding tests and reviews) and those with about 25–30 lectures (excluding tests and reviews). Accordingly, I have created long and short templates as possible starting points for constructing a course outline. In the long template I have assumed that all sections in the indicated chapters are covered, and in the short template I have assumed that instructors will make selections from the chapters to fit the available time. Of course, these are just guides and you may want to customize them to fit your local interests and requirements.

The organization of the text has been carefully designed to make life easier for instructors working under time constraints: A brief introduction to eigenvalues and eigenvectors occurs in Sections 2.3 and 4.3, and linear transformations from R^n to R^m are discussed in Chapter 4. This makes it possible for all instructors to cover these topics at a basic level when the time available for their more extensive coverage in Chapters 7 and 8 is limited. Also, note that Chapter 3 can be omitted without loss of continuity for students who are already familiar with the material.

	Long Template	**Short Template**
Chapter 1	7 lectures	6 lectures
Chapter 2	4 lectures	3 lectures
Chapter 4	3 lectures	3 lectures
Chapter 5	8 lectures	7 lectures
Chapter 6	6 lectures	3 lectures
Chapter 7	4 lectures	3 lectures
Chapter 8	6 lectures	2 lectures
Total	38 lectures	27 lectures

Variations in the Standard Course

Many variations in the long template are possible. For example, one might create an alternative long template by following the time allocations in the short template and devoting the remaining 11 lectures to some of the topics in Chapters 9, 10, and 11.

An Applications-Oriented Course

Chapter 9 contains selected applications of linear algebra that are mostly of a mathematical nature. Instructors who are interested in a wider variety of applications may want to consider the alternative version of this text, *Elementary Linear Algebra, Applications Version*, by Howard Anton and Chris Rorres. That text provides applications to business, biology, engineering, economics, the social sciences, and the physical sciences.

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HOWARD ANTON

Contents

1

Systems of Linear Equations and Matrices

Chapter Contents

INTRODUCTION: Information in science and mathematics is often organized into rows and columns to form rectangular arrays, called "matrices" (plural of "matrix"). Matrices are often tables of numerical data that arise from physical observations, but they also occur in various mathematical contexts. For example, we shall see in this chapter that to solve a system of equations such as

$$\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}$$

all of the information required for the solution is embodied in the matrix

$$\begin{bmatrix} 5 & 1 & 3 \\ 2 & -1 & 4 \end{bmatrix}$$

and that the solution can be obtained by performing appropriate operations on this matrix. This is particularly important in developing computer programs to solve systems of linear equations because computers are well suited for manipulating arrays of numerical information. However, matrices are not simply a notational tool for solving systems of equations; they can be viewed as mathematical objects in their own right, and there is a rich and important theory associated with them that has a wide variety of applications. In this chapter we will begin the study of matrices.