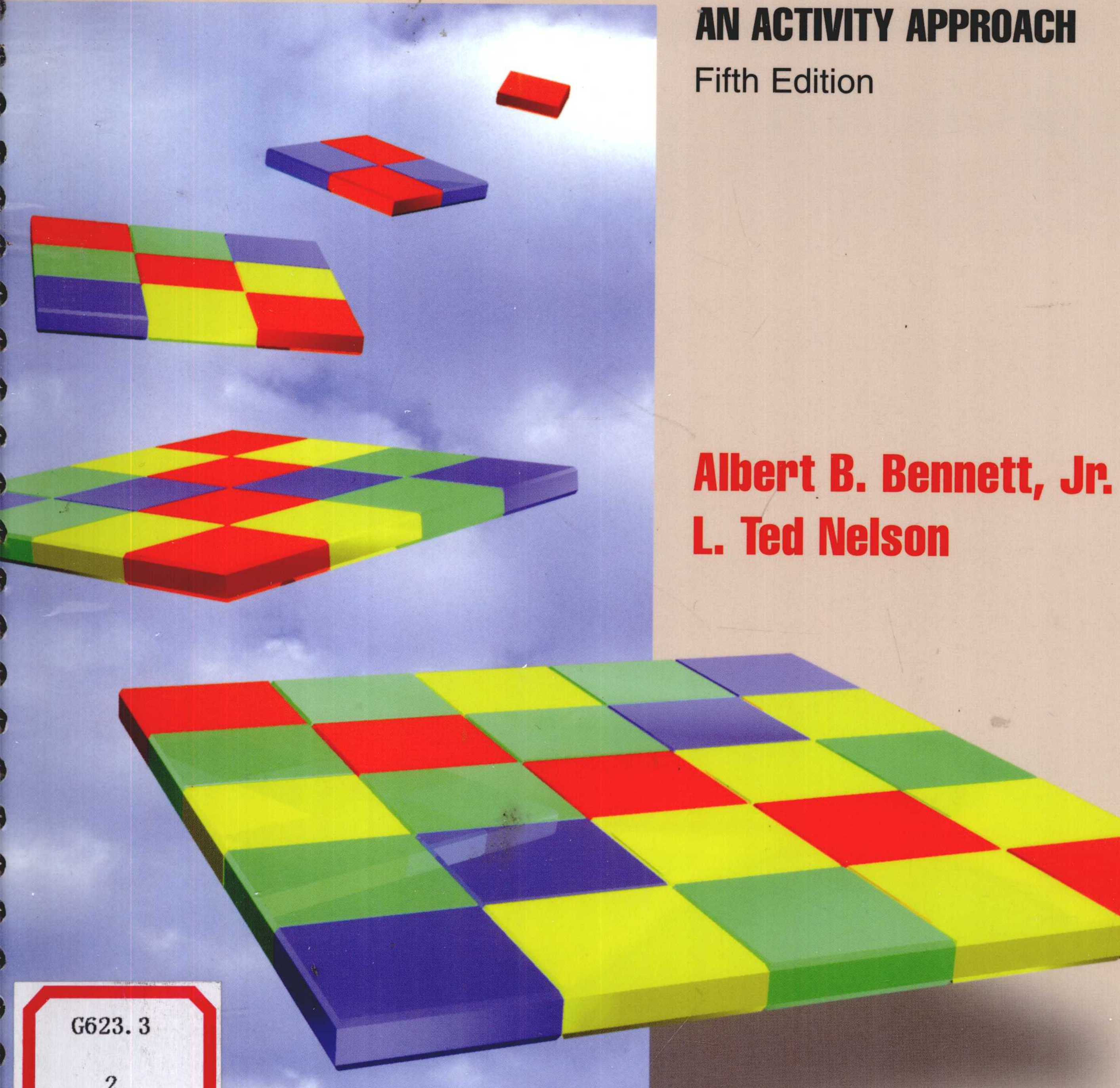
MATHEMATICS

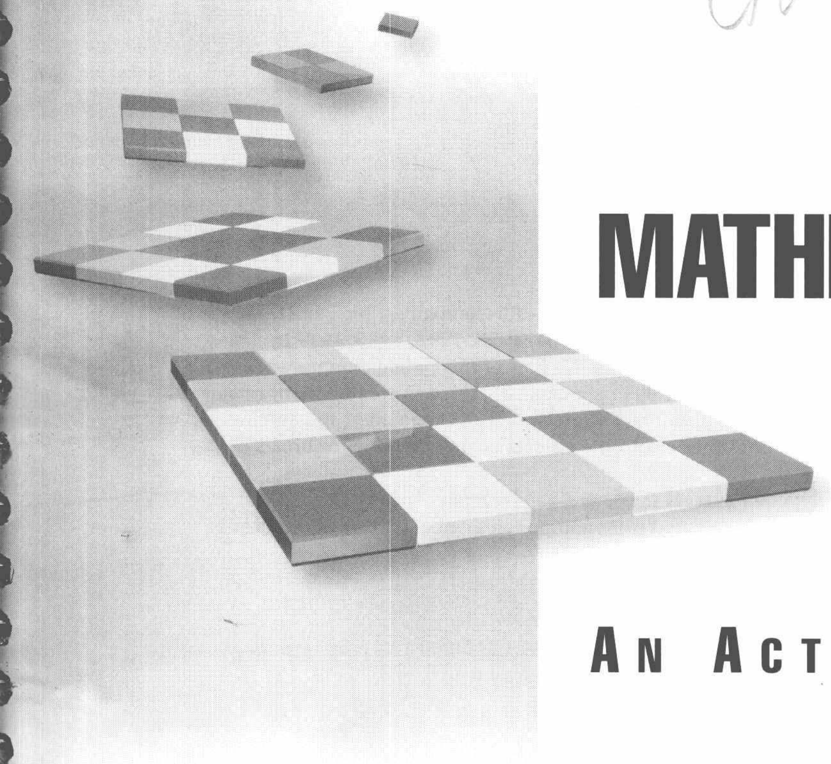
For Elementary Teachers

An Activity Approach

Fifth Edition



Albert B. Bennett, Jr.
L. Ted Nelson



Fifth Edition

MATHEMATICS FOR ELEMENTARY TEACHERS

AN ACTIVITY APPROACH

Albert B. Bennett, Jr.
University of New Hampshire

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McGraw Hill

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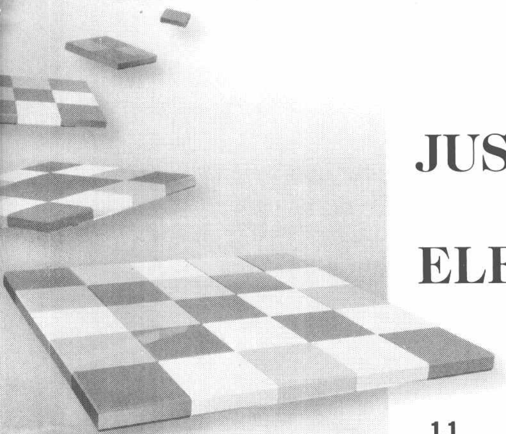
About the Authors

It was at the University of Michigan that Albert Bennett and L. Ted Nelson and their families first met. Bennett and Nelson had been invited to participate in a National Science Foundation sponsored program of graduate studies in mathematics. Ten years later while on sabbaticals at the University of Oregon they collaborated in writing lessons to actively involve prospective teachers in learning the mathematical concepts they would be teaching. These lessons eventually led to the publication of the first editions of *Mathematics for Elementary Teachers: A Conceptual Approach* and *Mathematics for Elementary Teachers: An Activity Approach.*

Albert Bennett completed his undergraduate and masters degrees at the University of Maine in Orono. He taught mathematics at Gorham State College and became active in the summer mathematics institutes that were sponsored by the Association of Teachers of Mathematics in New England. An early bias that was reflected in his teaching of these institutes was the need to encourage intuition in the teaching and learning of mathematics. He received his doctorate in mathematics from the University of Michigan in 1966 and joined the mathematics faculty at the University of New Hampshire to teach mathematics to prospective teachers. There he organized a mathematics lab and started writing laboratory activities for teachers. In the next few years his efforts led to the publication of Fraction Bars, Decimal Squares, and articles and textbooks for elementary and middle school teachers. These publications support methods of using models and concrete materials in the teaching of mathematics.

Ted Nelson is professor of mathematics and education at Portland State University. He taught junior and senior high school mathematics after graduating from St. Cloud State University, and then continued mathematical studies at Bowdoin College and the University of Michigan, where he received his doctorate in 1968. After serving four years as the first mathematics department chair at Southwest Minnesota State University he moved to Oregon to follow his interest in teaching mathematics to teachers. Currently, his main goal is to continue development of three lab-based courses for prospective elementary teachers and eight additional lab-based courses for middle school teachers. His teaching and curriculum efforts led to a faculty achievement award for outstanding university teaching in 1988. Over the past fifteen years he has written curriculum materials and given workshops designed to bring more concrete materials, visual models, and problem-solving investigations into the elementary and middle school mathematics curriculums.



JUST FOR FUN ACTIVITIES AND ELEMENTARY CLASSROOM IDEAS

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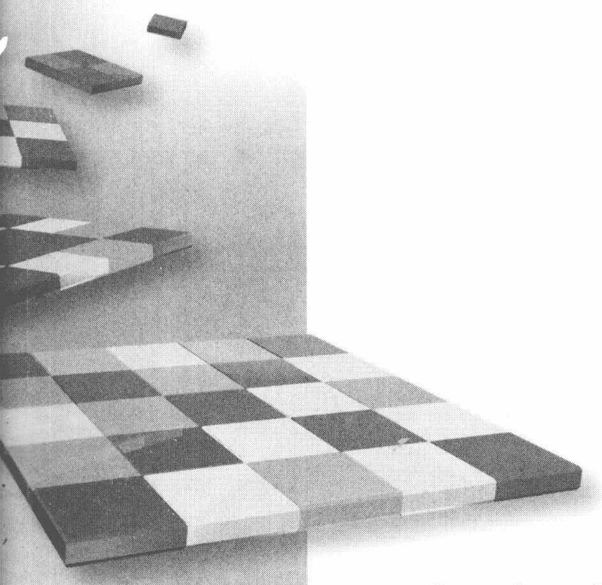
*Decimal Squares® is a registered trademark of Scott Resources.



CHAPTER 1

PROBLEM SOLVING

From the earliest grades, the curriculum should give the students opportunities to focus on regularities in events, shapes, designs, and sets of numbers. Children should begin to see that regularity is the essence of mathematics. The idea of a functional relationship can be intuitively developed through observations of regularity and work with generalizable patterns.[1]

ACTIVITY SET 1.1

SEEING AND EXTENDING PATTERNS WITH PATTERN BLOCKS

Purpose To recognize, describe, construct, and extend geometric patterns

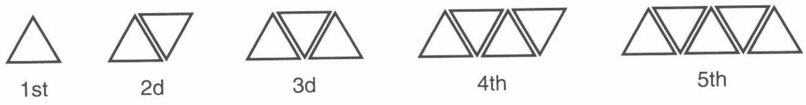
Materials Pattern blocks from the Manipulative Kit

Activity Human beings are pattern-seeking creatures. Babies begin life's journey listening for verbal patterns and looking for visual patterns. Scientists in search of extraterrestrial intelligence send patterned signals into the universe and listen for incoming patterns on radio telescopes. Mathematics is also concerned with patterns. Many mathematicians and educators involved in reforming mathematics teaching and learning at the elementary and middle school levels are suggesting that the notion of mathematics as the study of number and shape needs to be expanded. Some suggest that "mathematics is an exploratory science that seeks to understand every kind of pattern."[2]

In this first activity set colored geometric shapes called pattern blocks will be used to recognize, study, and extend geometric patterns. The set of pattern blocks consists of six different figures: a green triangle, an orange square, a red trapezoid, a blue rhombus, a white rhombus, and a yellow hexagon.

[1]*Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989), 60.
[2]Lynn A. Steen, *On the Shoulders of Giants: New Approaches to Numeracy* (Washington, DC: National Academy Press, 1990), 1–8.

1. The pattern block figures shown here form the first five figures of a sequence. Use your green triangles to construct the sixth and seventh figures that you think extend the given pattern.



★ a. Describe in writing at least three ways that the seventh figure in your sequence differs from the sixth figure.

★ b. Describe in writing what the 15th figure in this sequence would look like so that someone reading your description could build the same figure. Give your written description to another person and ask him or her to build a figure according to your instructions.

2. Use your pattern blocks to construct the sixth and seventh figures of the sequence below.

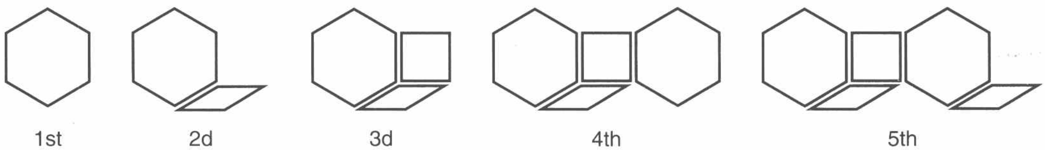
a. Describe in writing how new figures are created as this sequence is extended.

b. Will the 10th figure in an extended sequence have a green triangle or a blue rhombus on the right end? Explain your reasoning.

c. How many triangles and how many rhombuses are in the 25th figure of the extended sequence? Explain how you arrived at your answer.

d. Write a statement that will enable readers to determine the number of triangles and rhombuses in any figure they choose.

3. The pattern block sequence started below uses three different types of pattern blocks. Use your pattern blocks to build the next two figures in the extended sequence.



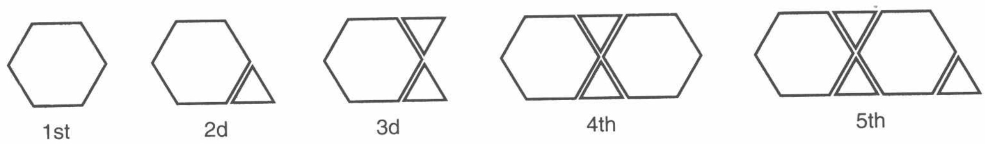
★ **a.** Describe in writing how new figures are created as this sequence is extended.

★ **b.** What pattern block will be on the right end of the 17th figure in this sequence? Explain how you arrived at your answer.

★ **c.** Determine the number of hexagons, squares, and rhombuses in the 20th figure of the sequence without building the figure. Explain how you thought about it.

★ **d.** Repeat part c for the 57th figure in the sequence.

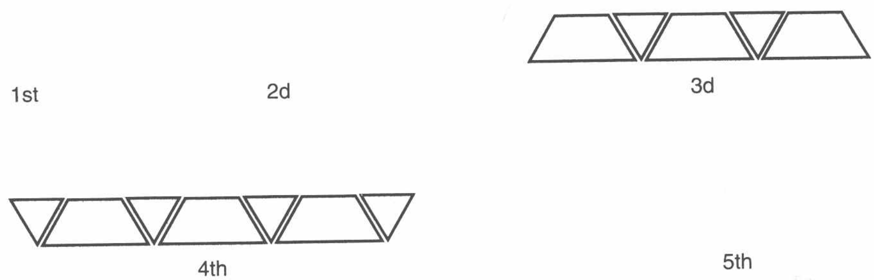
4. Use your pattern blocks to build the sixth and seventh figures of the sequence here.



a. Determine the number of triangles and hexagons in the 10th figure of the extended sequence. Do the same for the 15th figure.

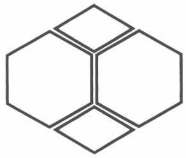
b. Devise a way to determine the number of triangles and hexagons in any given figure in this sequence. Write an explanation of your procedure so that readers can use it to determine the number of triangles and hexagons in any figure they choose.

5. The third and fourth figures of a sequence are given below. Use your pattern blocks to construct the first, second, and fifth figures in this sequence. Sketch diagrams of your figures in the space provided.

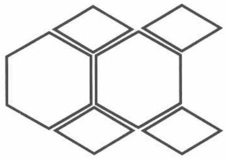


a. Describe how the odd-numbered figures differ from the even-numbered figures.

b. Sketch the missing figures for the sequence on the next page. Explain how you can determine the number of hexagons in any even-numbered figure of the sequence, then explain it for any odd-numbered figure.



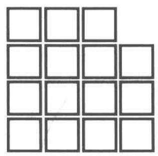
1st 2d 3d



4th 5th

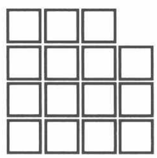
★ 6. The third term of a sequence is shown below. Create more than one sequence for which the given figure is the third term and sketch diagrams of the first, second, and fourth figures in the space provided. Write a rule for extending each pattern you create so that the reader is able to build the next few figures in the sequence. (You may wish to use the colored tiles from your Manipulative Kit for this activity.)

Sequence I



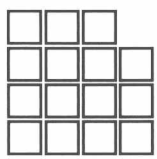
1st 2d 3d 4th

Sequence II



1st 2d 3d 4th

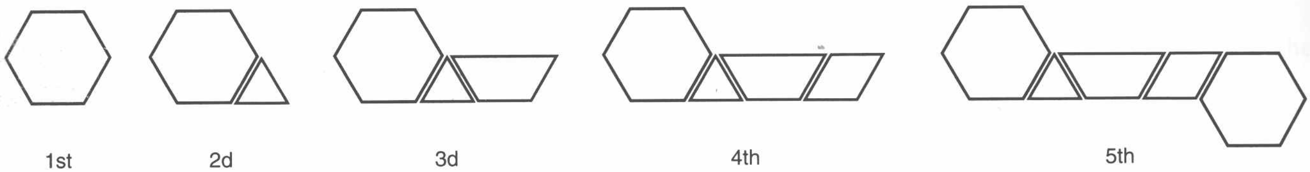
Sequence III



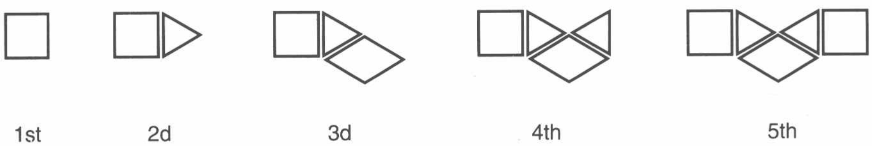
1st 2d 3d 4th

7. Sequences I and II begin repeating in the fifth figure and sequence III begins repeating in the sixth figure. Build the next two figures in each sequence with your pattern blocks. For the 38th figure in each sequence determine which pattern block is at its right end and how many of each type of pattern block it contains. Describe how you reached your conclusion in each case.

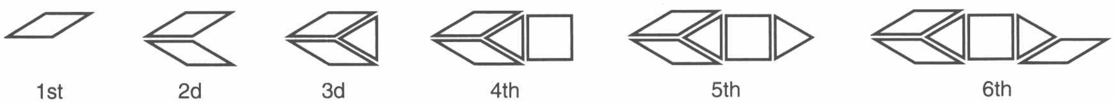
Sequence I



★ *Sequence II*



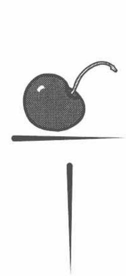
Sequence III



8. Devise your own sequence of figures with pattern blocks. Pose at least three questions about your sequence. Ask another person to build your sequence and answer your questions.

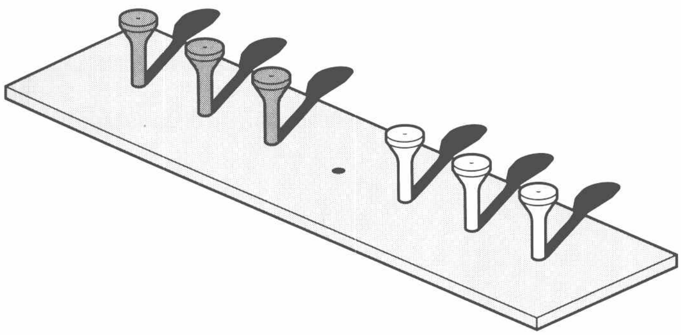
PUZZLER

By moving exactly two toothpicks, get the cherry out of the cup without changing the size and shape of the cup.



JUST FOR FUN

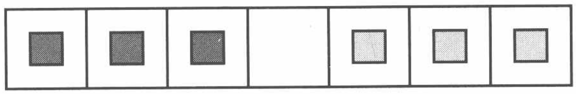
THE PEG-JUMPING PUZZLE



The official peg-jumping puzzle is shown here. A block of wood has seven holes in a row, with three movable black pegs in the holes at one end and three movable red pegs in the holes at the other end. The hole in the center is empty.

The challenge of the peg-jumping puzzle is to interchange the pegs between the right and left sides so that the black pegs move to the positions occupied by the red pegs and vice versa. There are two legal moves: any peg can move to an adjacent empty hole, and a peg of one color can jump a single peg of another color if there is a hole to jump into. (You cannot jump over two or more pegs.)

A simple model for this puzzle uses black and red tiles from the Manipulative Kit on a puzzle grid drawn like the one here.



★ 1. Use three problem-solving strategies as you investigate this puzzle: using a model, simplifying the problem (start with one red and one black peg and three holes), and making a table (to record the numbers of moves).

2. What is the least number of moves required to solve the puzzle?

ACTIVITY SET 1.2

GEOMETRIC NUMBER PATTERNS WITH COLOR TILE

Purpose To use geometric patterns to represent number patterns and provide visual support for extending number sequences

Materials Colored tiles from the Manipulative Kit

Activity How long would it take you to find the sum of the counting numbers from 1 to 100?

$$1 + 2 + 3 + 4 + \cdots + 49 + 50 + 51 + \cdots + 97 + 98 + 99 + 100$$

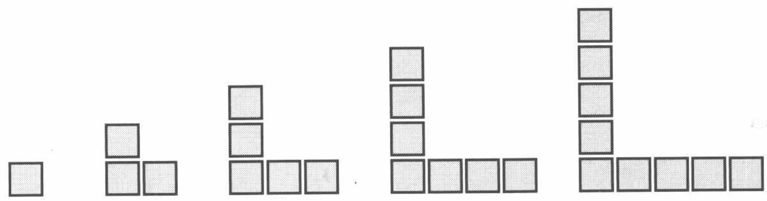
Karl Friedrich Gauss

Karl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time, was asked to compute such a sum when he was 10 years old. As was the custom, the first student to get the answer was to put his or her slate on the teacher's desk. The schoolmaster had barely stated the problem when Gauss placed his slate on the table and said, "There it lies."

No one knows for sure how the young Gauss obtained the sum so quickly. It is possible, however, that he, like many other creative thinkers, made a mental calculation by thinking of this number problem in a pictorial or visual way. Can you think of a picture or diagram that represents Gauss' sum?

Often in mathematics visual information can give valuable insights into numerical questions. Visual images can also help us remember mathematical ideas and concepts. In the following activities, geometric patterns will be used to generate number sequences. The visual information in the patterns will aid you in making numerical generalizations.

1. Find a pattern in the following sequence and use your tiles to construct the sixth and seventh figures of the sequence.



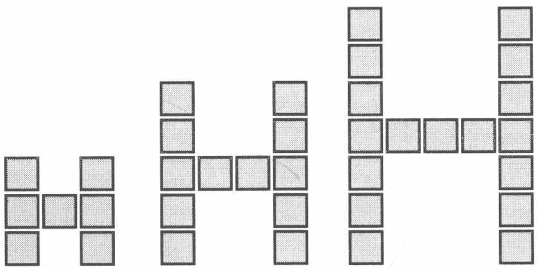
★ **a.** By counting the number of tiles in each figure, we can see that the first seven figures represent the sequence of odd numbers 1, 3, 5, 7, 9, 11, and 13. Use your tiles to build the tenth figure in this sequence. Determine the tenth odd number by counting the tiles in the tenth figure.

★ **b.** Write a sentence or two describing precisely how you would build the 20th figure in this sequence. How many tiles would be needed? What is the 20th odd number?

★ **c.** Write a sentence or two describing what the 50th figure would look like and how many tiles it would contain.

★ **d.** Write a statement that will enable readers to determine the number of tiles in any figure they choose from this sequence.

2. The first three terms of the number sequence represented by the sequence here are 7, 12, and 17. Build the fourth figure. Sketch this figure, and beneath it record the number of tiles needed to build it.



a. Write directions for constructing the eighth figure in the sequence. Ask someone to build the figure by following your directions.

b. How many tiles are in the eighth figure?

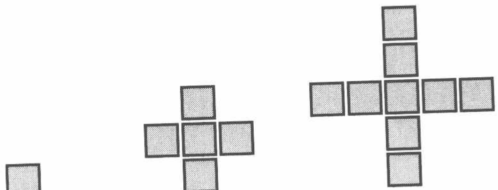
c. Determine the number of tiles in the 15th figure. (Imagine how you would construct that figure.)

d. Describe in words what the 50th figure would look like and how many tiles it would contain.

e. Write a procedure using words or an algebraic expression to determine the number of tiles for any figure, given the figure number.

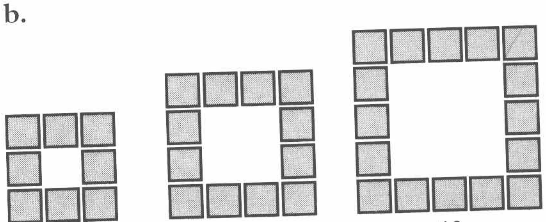
3. Here are three sequences and the number sequences they represent. Build the fourth figure in each sequence, and record the fourth number in each number sequence. Determine the 10th number in each number sequence by imagining how you would construct the 10th figure in each geometric pattern. Write a procedure using words or an algebraic expression that would enable the reader to determine the number of tiles in any figure, given the figure number.

a.



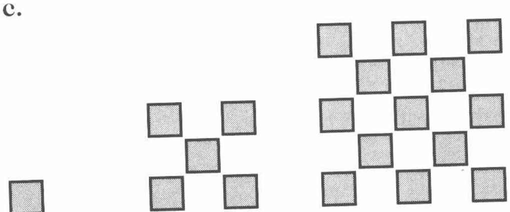
1 5 9

★ b.



8 12 16

c.



1 5 13