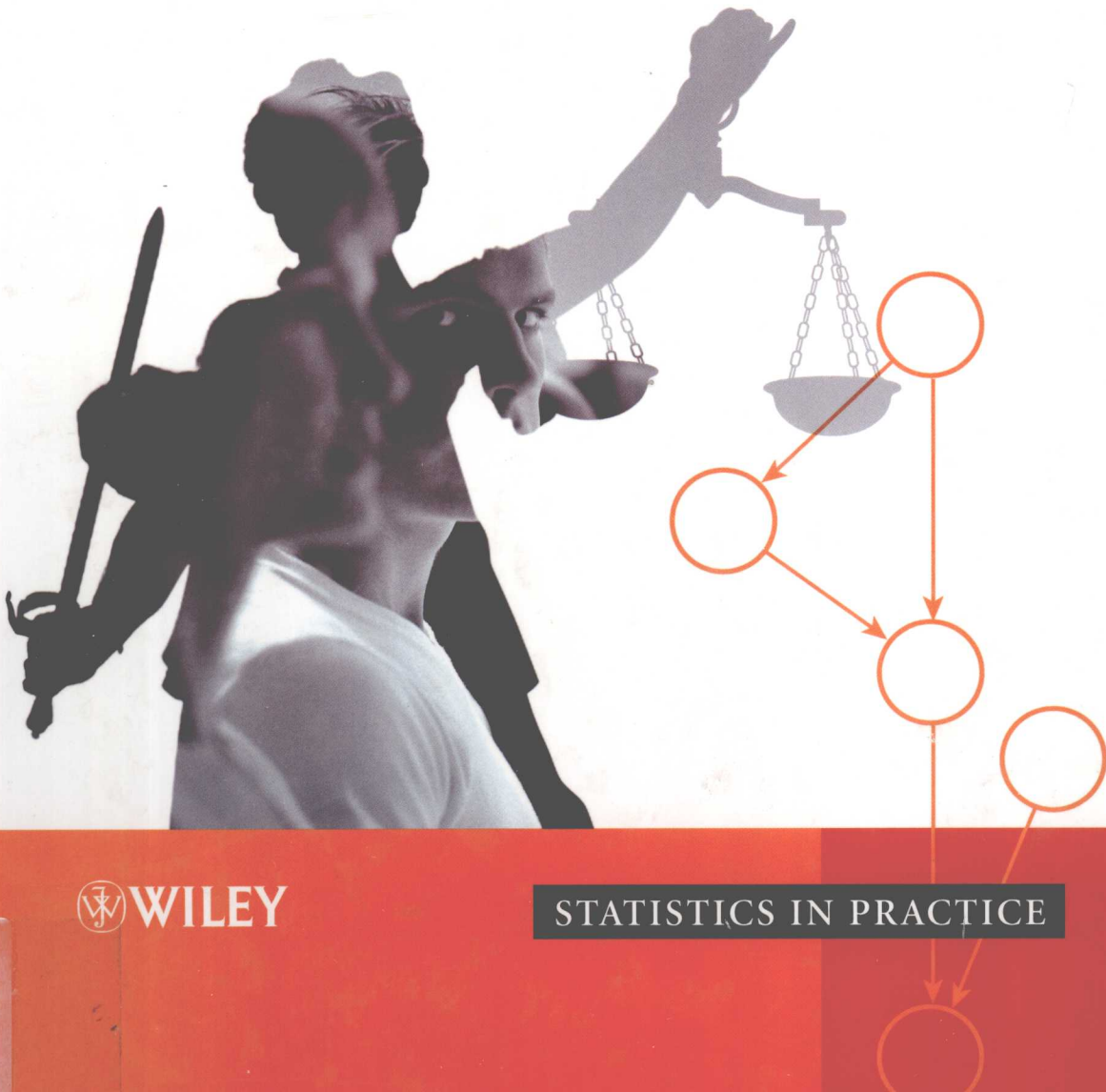


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Bayesian Networks and Probabilistic Inference in Forensic Science



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Preface

Forensic scientists may have hardly ever been able to gather and offer as much information, analytical or otherwise, as is possible today. Owing to advances made in science and technology, today's forensic scientists can choose amongst a broad scope of methods and techniques, applicable to various kinds of evidence even in their remotest quantities. However, despite the potentially abundant amount of available information, there now is an increased awareness amongst a significant part of the forensic community – including legal scholars – that there are risks associated with there is a sense of some sort of overconfidence, for example.

Scientific evidence as encountered in the real world is always incomplete to some degree, thus uncertainty is a prevalent element and one with which forensic scientists have to deal. Evidence does not say anything in itself; its significance needs to be elucidated in the light of competing propositions and background knowledge about the case at hand. There is a great practical necessity for forensic scientists to advise their clients, be they lawyers, prosecutors, jurors or decision makers at large, of the significance of their findings. Forensic scientists are required to qualify and, where possible, quantify their states of knowledge and to be consultants in the assessment of uncertainties associated with the inferences that may be drawn from forensic evidence.

For this task, forensic scientists should consider probability theory as the fundamental concept to govern their reasoning. The aim of this book will be to show that the practical application of probabilistic reasoning in forensic science can be assisted and its rationale substantially clarified if it is conducted in a graphical environment; *i.e.*, conducted through the use of a formalism known as Bayesian networks.

Thus, the idea for a book on *Bayesian networks and probabilistic inference in forensic science* is guided by a series of questions currently asked by forensic scientists and other participants in the criminal justice system. The aim is to offer theoretical and practical elements to help solve the following questions.

- What are the relationships among a set of (usually unobservable) causes and a set of (observable) scientific evidence?
- What are the structural relationships among arguments based on different kinds of evidence in an inference to one or more propositions of interest?
- How can we construct coherent, credible and defensible arguments in reasoning about the evidential value of scientific evidence?

- Given that the known set of evidence is not sufficient to determine the cause(s) of its origins with certain degrees of certainty; *i.e.*, given that inductive inferences are risky, what additional information should be obtained?
- What is the value of each of these additional pieces of information?
- Can we build expert systems to guide forensic scientists and other actors of the criminal justice system in their decision making about forensic evidence?
- How can one collect, organise, store, update and retrieve forensic information (hard data or linked with expert judgement) in expert systems?

The current state of the art in forensic science, notably in scientific evidence evaluation, does not allow scientists to cope adequately with the problems caused by the complexity of evidence (typically the combination of evidence, especially if contradictory) even if such complexity occurs routinely in practice.

Methods of formal reasoning have been proposed to assist the forensic scientist to understand all of the dependencies which may exist between different aspects of evidence and to deal with the formal analysis of decision making. Notably, graphical methods, such as Bayesian networks, have been found to provide valuable assistance for the representation of the relationships amongst characteristics of interest in situations of uncertainty, unpredictability or imprecision. Recently, several researchers (mainly statisticians) have begun to converge on a common set of issues surrounding the representation of problems which are structured with Bayesian networks. Bayesian networks are a widely applicable formalism for a concise representation of uncertain relationships among parameters in a domain (in this case, forensic science). The task of developing and specifying relevant equations can be made invisible to the user and the arithmetic can be almost completely automated. Most importantly, the intellectually difficult task of organising and arraying complex sets of evidence to exhibit their dependencies and independencies can be made visual and intuitive. Bayesian networks are a method for discovering valid, novel and potentially useful patterns in data where uncertainty is handled in a mathematically rigorous, but simple and logical, way. A network can be taken as a concise graphical representation of an evolution of all possible stories related to a scenario. However, the majority of scientists are not familiar with such new tools for handling uncertainty and complexity.

Thus, attention is concentrated here on Bayesian networks essentially because they are relatively easy to develop and – from a practical point of view – they also allow their user to deduce the related formulae (expressed through likelihood ratios) for the assessment of scientific evidence. Examples of the use of Bayesian networks in forensic science have already been presented in several papers. In summary, the use of Bayesian networks has some key advantages that could be described as follows:

- the ability to structure inferential processes, permitting the consideration of problems in a logical and sequential fashion;
- the requirement to evaluate all possible narratives;
- the possibility to calculate the effect of knowing the truth of one proposition or piece of evidence on the plausibility of others;

- the communication of the processes involved in the inferential problems to others in a succinct manner, illustrating the assumptions made;
- the ability to focus the discussion on probability and underlying assumptions.

A complete mastery of these aspects is fundamental to the work of modern forensic scientists.

The level of the book is the same as that of Aitken and Taroni (2004), namely those with a modest mathematical background. Undergraduate lawyers and aspiring forensic scientists attending a course in evidence evaluation should be able to cope with it though it would be better appreciated by professionals in law or forensic science with some experience of evidence evaluation.

The aim of the authors is to present a well-balanced book which introduces new knowledge and challenges for all individuals interested in the evaluation and interpretation of evidence and, more generally, the fundamental principles of the logic of scientific reasoning. These principles are set forth in Chapter 1. Chapter 2 shows how they can be operated within a graphical environment – Bayesian networks – with the reward of being applicable to problems of increased complexity. The discussion of the logic of uncertainty is then continued in the particular context of forensic science (Chapter 3) with studies of Bayesian networks for dealing with general issues affecting the evaluation of scientific evidence (Chapter 4). Later chapters will focus on more specific kinds of forensic evidence, such as DNA (Chapter 5) and transfer evidence (Chapter 6). Bayesian network models studied so far will then be used for the analysis of aspects associated with the joint evaluation of scientific evidence (Chapter 7). In Chapter 8 the discussion will focus on case-preassessment, where the role of the forensic scientists consists of assessing the value of expected results *prior* to laboratory examination. Here, Bayesian networks will be constructed to evaluate the probability of possible outcomes in various cases together with their respective weight. Chapter 9 will emphasise the importance of the structural dependencies among the basic constituents of an argument. It will be shown that qualitative judgements may suffice to agree with the rules of probability calculus and that reasonable ideas can be gained about a model's properties through sensitivity analyses. The book concludes with a discussion of the use of continuous variables (Chapter 10) and further applications including offender profiling and Bayesian decision analysis (Chapter 11).

An important message of the present book is that the Bayesian network formalism should primarily be considered as an aid to structure and guide one's inferences under uncertainty, rather than a way to reach 'precise numerical assessments'. Moreover, none of the proposed models is claimed to be, in some sense, 'right'; a network is a direct translation of one's subjective viewpoint towards an inference problem, which may be structured differently according to one's extent of background information and knowledge about domain properties. It is here that a valuable property of Bayesian networks comes into play: they are flexible enough to accommodate readily structural changes whenever these are felt to be necessary.

The authors believe that their differing backgrounds (*i.e.*, forensic science, statistics and philosophy of science) have common features and interactions amongst them that enable the production of results that none of the disciplines could produce separately. A book on this topic aims to offer insight not only for forensic scientists but also for all persons faced with uncertainty in data analysis.

We are very grateful to Glenn Shafer for his permission to use his own words as the heading of Section 1.1.1, to Michael Dennis for help with Chapter 10 and to Silvia Bozza for commenting on Section 11.2. We thank Hugin Expert A/S who provided the authors with discounted copies of the software to enable them to develop the networks described throughout the book. Other software used throughout the book are R, a statistical package freely available at www.r-project.org, and XFIG, a drawing freeware running under the X Window System and available at www.xfig.org.

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Foreword

In the past we have been far more adept at gathering, transmitting, storing and retrieving information than we have been at making sense out of a mass of information and drawing defensible and persuasive conclusions from it. However, significant progress has been made in recent years in efforts to close this important methodological gap. The authors of this book have a long and distinguished record of success in their attempts to close this gap in the forensic sciences. Forensic scientists, in common with investigators in so many other areas, have the task of establishing the relevance, credibility and inferential or probative force of the various kinds of information they will take as evidence in probabilistic inferences of concern to them. The establishment of these three important credentials of evidence rests on arguments constructed to show how probabilistic variables and their evidential bases are linked in certain ways. These arguments can indeed be complex and are commonly called inference networks.

This book concerns a class of methods for the generation and probabilistic analysis of complex inference networks. The first basis for these methods consists of the generation of graphical representations, in the form of directed acyclic graphs, showing what the persons generating the inference network believe to be patterns of probabilistic linkages amongst the variables or propositions of interest in the situation being represented. As the authors note, the generation of these graphical representations rests upon the knowledge and imagination of the persons generating the inference network. Critical reasoning is also involved as the persons generating the inference network attempt to avoid illogical connections or *non sequiturs* in the network under construction. As the authors acknowledge, and for which they supply numerous examples, an inference network can often be constructed in different ways to emphasise various different distinctions that might be important in the problem at hand. Particularly good examples are provided in their discussion of DNA evidence in Chapter 5. So, one way to view a generated inference network is that it represents an argument showing how the elements of some complex process are linked in their influence upon one or more variables that are of basic interest.

The method for probabilistic analysis of inference networks discussed in this work involves applications of Bayes' rule. Consequently, these networks are commonly called *Bayesian networks*. In most of their discussions of applications of Bayesian networks in the analysis of a wide assortment of different forms of trace evidence of interest in the forensic sciences, the authors wisely focus on likelihoods and their ratios, the crucial ingredients of Bayes' rule that concern the inference or probative force of evidence. In their discussions of the force of evidence they acknowledge that some of the probabilistic ingredients of the force of evidence may have a basis in statistical relative frequencies, but other probabilistic ingredients will rest upon epistemic, judgmental or subjective estimates. Chapters 1 through

3 of this book concern basic tutorial discussions of the graphical and probabilistic elements of Bayes' nets. In these introductory discussions the authors use some examples from Sherlock Holmes stories to begin their explanation of the variety of concepts that are part and parcel of Bayesian inference networks. The inferential issues here are often subtle or complex. Consequently, the terminology that has arisen in the analysis of Bayesian networks is complex, particularly with reference to the graphical structures that can emerge in such analyses. Careful attention to these tutorial comments in Chapters 1 through 3 will be well rewarded as you proceed.

In Chapters 4 through 7 the authors provide an array of examples concerning the construction and analysis of Bayes' nets for a variety of forms of trace evidence including footwear marks, stain evidence, fibre traces, DNA evidence related to various crimes or disputed parentage, handwriting and fingerprints evidence, and ballistics evidence. The authors' discussion of these applications demonstrates their considerable expertise in understanding the complexity of the inferential issues surrounding these forms of trace evidence. But their discussions of these matters bring to my mind one of the most important features of this book. Most prior works on Bayes's nets leave out what I regard as a major virtue of performing the kinds of analyses the authors advocate in this work. What is so often overlooked is the heuristic value of such analyses in the enhancement of investigative or discovery-related processes in the forensic sciences and in so many other contexts of which I am aware. This kind of analysis, including their probabilistic elements, can prompt you to ask questions you might not have thought of asking if you did not perform this detailed analysis. In short, Bayesian analysis can help you open up new lines of inquiry and new potentially valuable evidence. These matters are well documented in Chapter 7, regarding what the authors call Pre-assessment.

Those of us who have employed Bayesian network analyses for various purposes come to recognize their virtues in allowing us to tell different stories about some complex process depending upon what probabilistic ingredients we will include in them. The authors discuss these matters in Chapter 9 concerning sensitivity analyses. The Bayesian probabilistic underpinnings of such analyses allow us to tell how each of these alternative stories will end. In our analysis of the ballistics and other evidence in the case of Nicola Sacco and Bartolomeo Vanzetti, Professor Jay Kadane and I were able to tell different stories, on behalf of the prosecution and on behalf of the defence (Kadane and Schum 1996). We would have experienced considerable difficulties in constructing these stories and saying what their endings should be if we had not used this form of analysis.

I have been pleased and honoured to be asked to provide you with these introductory comments on a truly excellent work of scholarship that will be most helpful to persons in the forensic sciences and to the many other persons to whom I will recommend this book.

David A. Schum

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The logic of uncertainty

1.1 Uncertainty and probability

1.1.1 Probability is not about numbers

The U.S. Federal Rule of Evidence 401 says that

‘Relevant evidence’ means evidence having any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence. (Mueller and Kirkpatrick 1988, p. 33)

The term *probable* here means the degree of belief the fact finder entertains that a certain fact occurred. If it is not known whether the fact occurred, only a degree of belief less than certainty may be assigned to the occurrence of the fact, and there can then be discussion about the strength of this degree. Sometimes we are satisfied with speaking loosely of ‘strong’ or ‘weak’ beliefs; sometimes we would prefer to be more precise because we are dealing with important matters. A way to be more precise is to assign numerical values to our degrees of beliefs, and to use well defined rules for combining them together.

People are usually not very willing to assign numbers to beliefs, especially if they are not actuaries or professional gamblers. In this book we shall ask our readers to assign numbers, but these numbers are not important by themselves: what really matters is the fact that numbers allow us to use powerful rules of reasoning which can be implemented by computer programs. What is really important is not whether numbers are ‘precise’, whatever the meaning of ‘precision’ may be in reference to subjective degrees of belief based upon personal knowledge. What is really important is that we are able to use sound rules of reasoning to check the logical consequences of our propositions, that we are able to answer questions like: ‘What are the consequences with respect to the degree of belief in *A* of assuming that the degree of belief in *B* is high, let us say *x*, or between *x* and *y*?’, ‘how the degree of belief in *A* does change, if we lower the degree of belief in *B* by, let us say, *z*?’. If we are willing to take seriously the task of making up our mind to quantify, as best we can, our degrees of belief, then the reward will be the possibility of using the laws

of *probability calculus* to answer questions like those formulated above. As a distinguished scholar of the logic of uncertainty, Glenn Shafer once said: ‘Probability is not really about numbers; it is about the structure of reasoning’.

1.1.2 The first two laws of probability

In order to be able to ‘measure’ our degree of belief that a certain fact occurred, it is necessary to be precise about what ‘a degree of belief’ is. In this book it will be defined as a *personal degree of belief* that a proposition of a natural language, describing that fact, is true. ‘Evidence’ bearing on that proposition is expressed by means of other propositions. Therefore, it shall be said, on first approximation, that a proposition *B* is *relevant*, according to an opinion, for another proposition *A* if, and only if, knowing the truth (or the falsity) of *B* would change the degree of belief in the truth of *A*.

Having defined what is to be measured, it is next necessary to choose a function that assigns numbers to propositions. There are several alternatives available and, according to the choice, there are different rules for combining degrees of belief. In this book a *probability function* has been chosen, denoted by the symbol $Pr()$ where the $()$ contain the event or proposition, the probability of which is of interest. Numerical degrees of belief must satisfy, for any propositions *A* and *B*, the laws of the mathematical theory of probability.

The first two laws can be formulated as follows:

- Degrees of belief are real numbers between zero and one: $0 \leq Pr(A) \leq 1$.
- If *A* and *B* are mutually exclusive propositions, *i.e.*, they cannot be both true at the same time, then the degree of belief that one of them is true is given by the sum of their degrees of belief, taken separately: $Pr(A \text{ or } B) = Pr(A) + Pr(B)$ (*The addition law*).

The addition law can be extended to any number *n* of exclusive propositions. If A_1, A_2, \dots, A_n cannot be true at the same time, then

$$Pr(A_1 \text{ or } A_2 \text{ or } \dots A_n) = Pr(A_1) + Pr(A_2) + \dots + Pr(A_n).$$

Satisfying the first law means that, if it is known that proposition *A* is true, then the degree of belief should take the maximum numerical value, *i.e.*, $Pr(A) = 1$. The degrees of belief in propositions not known to be true, like, for example, the proposition *A*, ‘this coin lands heads after it is tossed’, are somewhere between the certainty that the proposition is true and the certainty that it is false. A straightforward consequence of the probability laws is that, when the degree of belief for heads is fixed, then it is necessary to assign to the proposition *B*, ‘this coin lands tails after it is tossed’, the degree of belief $Pr(B) = 1 - Pr(A)$, assuming that pathological results such as the coin balancing on its edge do not occur.

This is the simplest example of how probability calculus works as a *logic for reasoning under uncertainty*. The logic places constraints on the ways in which numerical degrees of belief may be combined. Notice that the laws of probability require the degrees of belief in *A* and in *B* to be such that they are non-negative and their sum is equal to one. Within these constraints, there is not an obligation for *A* to take any particular value. Any value between the minimum (0) and the maximum (1) is allowed by the laws.

This result holds, in general, for any two propositions A and B which are said to be mutually exclusive and exhaustive: one and only one of them can be true at any one time and together they include all possible outcomes.

- Given that a proposition and its logical negation are mutually exclusive and exhaustive, the degree of belief in the logical negation of any proposition A is $Pr(not - A) = Pr(\bar{A}) = 1 - Pr(A)$

where \bar{A} is read as A -bar. The logical negation of an event or proposition is also known as the *complement*.

1.1.3 Relevance and independence

A proposition B is said to be *relevant* for another proposition A if and only if the answer to the following question is positive: if it is supposed that B is true, does that supposition change the degree of belief in the truth of A ? A judgement of relevance is an exercise in hypothetical reasoning. There is a search for a certain kind of evidence because it is known in advance that it is relevant; if someone submits certain findings maintaining that they constitute relevant evidence, a hypothetical judgement has to be made as to whether or not to accept the claim. In doing that, a distinction has to be drawn, not only between the *hypothesis* A and *evidence* B for the hypothesis A , but also between that particular evidence B and whatever else is known.

When a proposition's degree of belief is evaluated, there is always exploitation of available *background information*, even though it is not explicit. An assessment of the degree of belief in the proposition 'this coin lands heads after it is tossed' is made on the basis of some background information that has been taken for granted: if the coin looks like a common coin from the mint, and there is no reason for doubting that, then it is usually assumed that it is well balanced. Should it be realised, after inspection, that the coin is not a fair coin, this additional information is 'evidence' that changes the degree of belief about that coin, even though it is still believed that coins from the mint are well balanced. A *relevant proposition* is taken to mean a proposition which is not included in the background information. The distinction between 'evidence' and 'background information' is important because sometimes it has to be decided that certain propositions are to be considered as evidence, while others are to be considered as part of the background information.

For example, suppose a DNA test has been evaluated. Assume that all scientific theories which support the methodology of the analysis are true, that the analysis has been done correctly, and that the chain of custody has not been broken. These assumptions all form part of the background information. Relevant evidence is only those propositions which describe the result of the test, *plus* some other propositions reporting statistical data about the reliability of the evidence. Alternatively, propositions concerning how the analysis has been done, and/or the chain of custody, can also be taken to be part of the evidence while scientific theories are still left in the background. Therefore, it is useful to make a clear distinction between what is considered in a particular context to be 'evidence', and what is considered to be 'background'. For this reason, background information is introduced explicitly from time to time in the notation.

Let $Pr(A | I)$ denote 'the degree of belief that proposition A is true, given background information I ', and let $Pr(A | B, I)$ denote 'the degree of belief that proposition A is true, given that proposition B is assumed to be true, *and* given background information I '.