Ceramic Materials Handbook

Carl Burt

Ceramic Materials Handbook

Edited by **Carl Burt**

NY RESEARCH PRESS
New York

Published by NY Research Press,
23 West, 55th Street, Suite 816,
New York, NY 10019, USA
www.nyresearchpress.com

Ceramic Materials Handbook
Edited by Carl Burt

International Standard Book Number: 978-1-63238-073-9 (Hardback)

Printed in the United States of America.

Ceramic Materials Handbook

Preface

The main aim of this book is to educate learners and enhance their research focus by presenting diverse topics covering this vast field. This is an advanced book which compiles significant studies by distinguished experts in the area of analysis. This book addresses successive solutions to the challenges arising in the area of application, along with it; the book provides scope for future developments.

This book discusses various aspects of ceramic materials, from basics to their industrial applications. Furthermore, the book covers their influence on latest technologies such as ceramic matrix composites, porous ceramics, sintering theory paradigm of modern ceramics, among others.

It was a great honour to edit this book, though there were challenges, as it involved a lot of communication and networking between me and the editorial team. However, the end result was this all-inclusive book covering diverse themes in the field.

Finally, it is important to acknowledge the efforts of the contributors for their excellent chapters, through which a wide variety of issues have been addressed. I would also like to thank my colleagues for their valuable feedback during the making of this book.

Editor

Contents

Part 1

Electronic Ceramics

1

Electrode Size and Dimensional Ratio Effect on the Resonant Characteristics of Piezoelectric Ceramic Disk

Lang Wu[1], Ming-Cheng Chure[1], Yeong-Chin Chen[2], King-Kung Wu[1] and Bing-Huei Chen[3]
[1]*Department of Electronics Engineering, Far-East University,*
[2]*Department of Computer Science & Information Engineering, Asia University,*
[3]*Department of Electrical Engineering, Nan Jeon Institute of Technology,*
Taiwan

1. Introduction

After discovered at 1950, lead zirconate titanate [Pb(Zr,Ti)O_3; PZT] ceramics have intensively been studied because of their excellent piezoelectric properties [Jaffe et al., 1971; Randeraat & Setterington, 1974; Moulson & Herbert, 1997; Newnham & Ruschau, 1991; Hertling, 1999]. The PZT piezoelectric ceramics are widely used as resonator, frequency control devices, filters, transducer, sensor and etc. In practical applications, the piezoelectric ceramics are usually circular, so the vibration characteristics of piezoelectric ceramic disks are important in devices design and application. The vibration characteristics of piezoelectric ceramics disk had been study intensively by many of the researchers [Shaw, 1956; Guo et al., 1992; Ivina, 1990a, 2001b; Kunkel et al., 1990; Masaki et al., 2008]. Shaw [Shaw, 1956] measured vibrational modes in thick barium titanate disks having diameter-to-thickness ratios between 1.0 and 6.6. He used an optical interference technique to map the surface displacement at each resonance frequency, and used a measurement of the resonance and antiresonance frequency to calculate an electromechanical coupling for each mode. Guo et al., [Guo et al., 1992] presented the results for PZT-5A piezoelectric disks with diameter-to-thickness ratios of 20 and 10. There were five types of modes being classified according to the mode shape characteristics, and the physical interpretation was well clarified. Ivina [Ivina, 1990] studied the symmetric modes of vibration for circular piezoelectric plates to determine the resonant and anti-resonant frequencies, radial mode configurations, and the optimum geometrical dimensions to maximize the dynamic electromechanical coupling coefficient. Kunkel et al., [Kunkel et al., 1990] studied the vibration modes of PZT-5H ceramics disks concerning the diameter-to-thickness ratio ranging from 0.2 to 10. Both the resonant frequencies and effective electromechanical coupling coefficients were calculated for the optimal transducer design. Masaki et al., [Masaki et al., 2008] used an iterative automated procedure for determining the complex materials constants from conductance and susceptance spectra of a ceramic disk in the radial vibration mode.

The phenomenon of partial-electroded piezoelectric ceramic disks also study by some researchers. Ivina [Ivina, 2001] analyzed the thickness symmetric vibrations of piezoelectric disks with partial axisymmetric electrodes by using the finite element method. According to the spectrum and value of the dynamic electromechanical coupling coefficient of quasi-thickness vibrations, the piezoelectric ceramics can be divided into two groups. Only for the first group can the DCC be increased by means of the partial electrodes, which depends on the vibration modes. Schmidt [Schmidt, 1972] employed the linear piezoelectric equations to investigate the extensional vibrations of a thin, partly electroded piezoelectric plate. The theoretical calculations were applied to the circular piezoelectric ceramic plate with partial concentric electrodes for the fundamental resonant frequency. Huang [Huang, 2005] using the linear two-dimensional electroelastic theory, the vibration characteristics of partially electrode-covered thin piezoelectric ceramic disks with traction-free boundary conditions are investigated by theoretical analysis, numerical calculation, and experimental measurement.

In this study, the vibration characteristics of a thin piezoelectric ceramic disk with different electrode size and dimensional ratio are study by the impedance analysis method.

2. Vibration analysis of the piezoelectric ceramic disk

Figure 1 shows the geometrical configuration of the piezoelectric ceramic disk with radius R and thickness h. The piezoelectric ceramic disk is assumed to be thin (R>>h) and polarized in the thickness direction. If the cylindrical coordinates (r, θ, z) with the origin in the center of the disk are used. The linear piezoelectric constitutive equations of a piezoelectric ceramic with crystal symmetry C_{6mm}, can be expressed as [IEEE, 1987]:

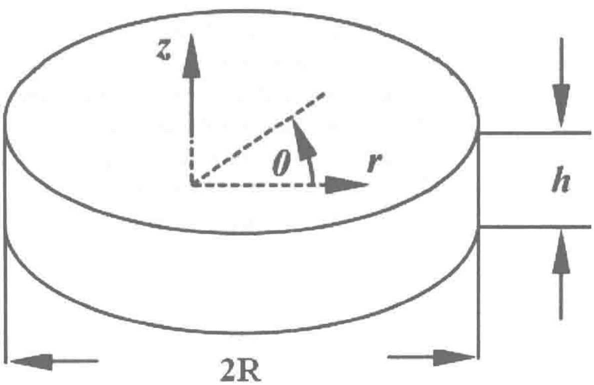

Fig. 1. The geometrical configuration of the piezoelectric ceramic disk.

$$\begin{bmatrix} S_{rr} \\ S_{\theta\theta} \\ S_{zz} \\ S_{\theta z} \\ S_{rz} \\ S_{r\theta} \\ D_r \\ D_\theta \\ D_z \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & s_{44}^E & 0 & d_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15} & 0 & \varepsilon_{11}^T & 0 & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 & 0 & \varepsilon_{11}^T & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33}^E \end{bmatrix} \begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{zz} \\ T_{\theta z} \\ T_{rz} \\ T_{r\theta} \\ E_r \\ E_\theta \\ E_z \end{bmatrix} \tag{1}$$

where T_{rr}, $T_{\theta\theta}$ and T_{zz} are the longitudinal stress components in the r, θ and z directions; $T_{r\theta}$, $T_{\theta z}$ and T_{rz} are the shear stress components. S_{rr}, $S_{\theta\theta}$, S_{zz}, $S_{\theta z}$, S_{rz} and $S_{r\theta}$ are the strain components. D_r, D_θ and D_z are the electrical displacement components, and E_r, E_θ, and E_z are the electrical fields. $s_{11}^E, s_{12}^E, s_{13}^E, s_{33}^E, s_{44}^E$ and s_{66}^E are the compliance constants at constant electrical field, in which $s_{66}^E = 2(s_{11}^E - s_{12}^E)$. d_{15}, d_{31} and d_{33} are the piezoelectric constants; ε_{11}^T and ε_{33}^T are the dielectric constants.

The electric field vector E_i is derivable from a scalar electric potential V_j:

$$E_r = -\frac{\partial V}{\partial r} \tag{2a}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \tag{2b}$$

$$E_z = -\frac{\partial V}{\partial z} \tag{2c}$$

The electric displacement vector D_i satisfies the electrostatic equation for an insulator, and shown as:

$$\frac{\partial D_r}{\partial r} + \frac{1}{r}\frac{\partial D_\theta}{\partial \theta} + \frac{1}{r}D_r + \frac{\partial D_z}{\partial z} = 0 \tag{3}$$

Some basic hypotheses are employed for analysis the vibration of thin disk [Rogacheva, N.N., 1994]:

a. Normal stress T_{zz} is very small, so it can be neglected relative to other stresses, hence $T_{zz} = 0$.
b. The rectilinear element normal to the middle surface before deformation remains perpendicular to the strained surface after deformation, and its elongation can be neglected, i.e., $S_{rz} = S_{\theta z} = 0$.
c. Electrical displacement D_z is a constant with respect to the thickness.

In this study, only the radial axisymmetry vibrations of the disk are considered, so S_{zz}=0. The electrodes are coated on the z axis, so E_r=0 and E_θ=0. The constitutive equations can reduce to:

$$S_{rr} = s_{11}^E T_{rr} + s_{12}^E T_{\theta\theta} + d_{31} E_z \tag{4a}$$

$$S_{\theta\theta} = s_{12}^E T_{rr} + s_{11}^E T_{\theta\theta} + d_{31} E_z \tag{4b}$$

$$S_{r\theta} = s_{66}^E T_{r\theta} = 2(s_{11}^E - s_{12}^E) T_{r\theta} \tag{4c}$$

$$D_z = d_{31} T_{rr} + d_{31} T_{\theta\theta} + \varepsilon_{33}^E E_z \tag{4d}$$

The stresses and the charge density Q on the surface of the disk, can be obtained by inversion of Eq.(4),

$$T_{rr} = \frac{1}{s_{11}^E(1-\sigma^2)}(S_{rr} + \sigma S_{\theta\theta}) - \frac{d_{31}}{s_{11}^E(1-\sigma)}\frac{2V_3}{h} \tag{5a}$$

$$T_{\theta\theta} = \frac{1}{s_{11}^E(1-\sigma^2)}(S_{\theta\theta} + \sigma S_{rr}) - \frac{d_{31}}{s_{11}^E(1-\sigma)}\frac{2V_3}{h} \tag{5b}$$

$$T_{r\theta} = \frac{1}{s_{66}^E} S_{r\theta} = \frac{S_{r\theta}}{2(s_{11}^E - s_{12}^E)} = \frac{S_{r\theta}}{2s_{11}^E(1+\sigma)} \tag{5c}$$

$$D_z = d_{31}(S_{rr} + S_{\theta\theta}) + \varepsilon_{33}^T \frac{2V_3}{h} \tag{5d}$$

where σ is Poisson's ratio and equal to $-(s_{11}^E / s_{12}^E)$, V_3 is the voltage applied in the z-direction.

Assumed the radial extensional displacement of the middle plane as:

$$u_r(r,t)=U(r)e^{i\omega t} \tag{6}$$

where ω is the angular frequency.

The strain-mechanical displacement relations are:

$$S_{rr} = \frac{\partial U}{\partial r} \tag{7a}$$

$$S_{\theta\theta} = \frac{U}{r} \tag{7b}$$

$$S_{r\theta} = \frac{1}{r}\frac{\partial U}{\partial \theta} \tag{7c}$$

Then the stress-displacement relations of the radial vibration are given as:

$$T_{rr} = \frac{1}{s_{11}^E(1-\sigma^2)}\left(\frac{dU}{dr} + \sigma\frac{U}{r}\right) + \frac{d_{31}}{s_{11}^E(1-\sigma)}\frac{2V_3}{h} \tag{8a}$$

$$T_{\theta\theta} = \frac{1}{s_{11}^E(1-\sigma^2)}\left(\frac{U}{r} + \sigma\frac{dU}{dr}\right) + \frac{d_{31}}{s_{11}^E(1-\sigma)}\frac{2V_3}{h} \tag{8b}$$

and the charge density is:

$$Q = -\frac{d_{31}}{s_{11}^E(1-\sigma)}\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) - \frac{2d_{31}^2}{s_{11}^E(1-\sigma)}\frac{2V_3}{h} + \varepsilon_{33}^T\frac{2V_3}{h} \tag{9}$$

The equation of motion in the radial direction is

$$\frac{\partial T_{rr}}{\partial r} - rT_{\theta\theta} + \frac{1}{r}T_{rr} = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{10}$$

where ρ is the density.

Substitution of Eq.(8) into Eq.(10), find

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{1}{r^2}U - \rho\omega^2 s_{11}^E(1-\sigma^2)U = 0 \tag{11}$$

the general solution of Eq.(11) is

$$U(r)=CJ_1(\beta r) \tag{12}$$

where J_1 is the Bessel function of the first kind and first order, C is a constant and

$$\beta^2 = \rho s_{11}^E(1-\sigma^2)\omega^2 \tag{13}$$

For the boundary condition at r = R, it has:

$$\int_{-h/2}^{h/2} T_{rr}dz = 0 \tag{14}$$

So, the constant C is found to be:

$$C = \frac{2Vd_{31}(1+\sigma)}{(1-\sigma)J_1(\beta R) - \beta R J_0(\beta R)}\frac{R}{h} \tag{15}$$

where J_0 is Bessel function of the first kind and zero order.

When the piezoelectric disk in radial vibration, the current can be developed as[Huang et al, 2004]:

$$I=\frac{\partial}{\partial t}\iint_S D_z ds = i\omega\int_0^{2\pi}\int_0^R\left\{\frac{d_{31}(1+\sigma)}{s_{11}^E(1-\sigma^2)}\left[\frac{dU}{dr}+\frac{U}{r}\right]+\frac{2\varepsilon_{33}^T V}{h}(k_p^2-1)\right\}rdrd\theta$$
$$=i\omega\frac{2\pi R^2 V\varepsilon_{33}^T}{h}\frac{\left[1-\sigma+(1+\sigma^2)\dfrac{k_p^2}{1-k_p^2}\right]J_1(\beta R)-\beta RJ_0(\beta R)}{(1-\sigma)J_1(\beta R)-\beta RJ_0(\beta R)} \tag{16}$$

where S is the area of the electrode and

$$k_p=\sqrt{\frac{2d_{31}^2}{\varepsilon_{33}^T s_{11}^E(1-\sigma)}} \tag{17}$$

is the planar effective electromechanical coupling factor.

The frequencies corresponding to the current I approaches infinity is the resonant frequencies. The characteristic equation of resonant frequencies for radial vibrations is given by:

$$\eta J_0(\eta)=(1-\sigma)J_1(\eta) \tag{18}$$

where $\eta=\beta R$.

The antiresonant frequencies for which the current through the piezoelectric ceramic disk equal to zero are determined from the roots of the following equation:

$$\left[1-\sigma+(1+\sigma)\frac{k_p^2}{k_p^2-1}\right]J_1(\eta)=\eta J_0(\eta) \tag{19}$$

From Eqs(12), (18) and (19), under free boundary conditions, the resonant frequency of the piezoelectric ceramic disk with fully coated electrode can be expressed as:

$$f_r=\frac{\eta}{2\pi R}\sqrt{\frac{1}{\rho s_{11}^E(1-\sigma^2)}} \tag{20}$$

3. Experimental process

The piezoceramic disks used in this study were prepared by conventional powder processing technique, starting from high purity raw materials, TiO_2 (Merck, 99%), ZrO_2 (Aldrich, 99%) and PbO (Merck, 99%). The compositions of the ceramics were in the vicinity of the MPB of PZT in the tetragonal range, and doped with minor MnO_2 and Sb_2O_3.

After 2h ball milling with ZrO_2 balls, the mixed powders were calcined in air for 2h at 850°C. The calcined powders were then cold pressed into disk type pellets. The pellets were sintered at 1250°C for 2h with the double crucible arrangement, with $PbZrO_3$ atmosphere powder for PbO compensation.

Two groups of samples were used in this study, one group used for electrode size study, the other group used for dimensional ratio effect. The diameter and thickness of samples used