MATHEMATICS

ITS POWER AND UTILITY

KARL J. SMITH

4TH EDITION

To my son, Shannon

Contemporary Undergraduate Mathematics Series,
Robert J. Wisner, Editor

Brooks/Cole Publishing Company
A Division of Wadsworth, Inc.

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging-in-Publication Data
Smith, Karl J.
Mathematics : its power and utility / Karl J. Smith. — 4th ed.
p. cm.
Includes index.
ISBN 0-534-20484-8
1. Mathematics. I. Title.
QA39.2.S598 1994
510—dc20 93-28515
CIP

Sponsoring Editor: Gary W. Ostedt
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Cover Design: Vernon T. Boes
Cover Photo: Garry Gay, The Image Bank
Interior Illustration: Precision Graphics
Cartoons: Ryan Cooper
Photo Researcher: Chris Pullo
Typesetting: Bookman Productions; Graphic World
Cover Printing: Lehigh Graphics
Printing and Binding: R.R. Donnelley & Sons, Crawfordsville

Preface

If you plan for a year, plant a seed.
If for ten years, plant a tree.
If for a hundred years, teach the people.
CONFUCIUS

SITUATION

"What are you taking this semester?" "Well, I'm taking English and history; I made the swimming team; and I finally get to take band. Oh, I forgot, I also **have** to take math."
In this book we look at the power *of mathematics, not the drudgery of number crunching. We also look at the* utility *of mathematics, and you are NOT told that you should do this just because it might be useful* someday. *The first part of the book, "The Power of Mathematics," develops some ideas in arithmetic, algebra, and geometry; and the second, "The Utility of Mathematics," develops the ideas around* ***SITUATIONS*** *with which I hope you, the student, can identify. Each section begins with a* ***SITUATION****, develops some related concepts, and then concludes (in the problem set) by asking you to answer a question introduced by the* ***SITUATION*** *as well as offering suggestions for writing a related paper.*

As a teacher and author, I am faced with transferring some knowledge of the **power**, the beauty, and the **utility** of mathematics to my students. But I can't instruct until my students have overcome their initial fears and attitudes about mathematics and are *ready* to listen. I have often wondered why, when I tell people that I'm a math teacher, more often than not I hear about their unpleasant experiences in mathematics. Why is it that we *get* to participate in sports or music but *have* to take mathematics? For most people, mathematics is presented as the

ultimate lesson in delayed gratification. Year after year, the students are told, "You must learn this thoroughly so you can do your math tomorrow, next week, next month, next year." The implication is that, in twenty or so years, we'll let you see the benefit of what you have been learning.

As I began to write this book, I asked myself, "What do my students *need*?" Why should there be a new mathematics book in this world? What can I hope to present that is not already available? I believe too much lip service is paid to "getting past the hurdles," and to asking "How little can I do and still finish this course?" This book was written to give the student who has previously not been successful with mathematics a fresh and innovative approach to arithmetic, beginning algebra, and geometry in order to satisfy basic competencies in mathematics in today's world. I have long believed that, if students have avoided or have been unsuccessful with a particular aspect of mathematics in the past, simply presenting it to them again in the same way will generally not meet with a great deal more success. Nevertheless, because of the tremendous importance of mathematics in the world today, a student must have some degree of mathematical competency to be successful in almost any discipline. Therefore, even though I have presented the essential ideas of arithmetic, beginning algebra, and geometry, I have presented them in settings different from the usual or traditional ones.

My students need to be able to relate to the real world. You will find this book filled with practical information rather than abstract (and meaningless) made-up word problems. For example, when we take up graphs (page 455), we look at graphs showing the blood alcohol level and its effects on driving or at political graphs found in a newspaper.

My students need to be able to solve problems outside the classroom. Take a look at the word problems we consider in Section 3.6. You will *not* find the usual mixture, distance, and age problems. Instead, we develop a *procedure* and work problems dealing with miles per gallon, price comparisons, and money exchange. We seek to develop higher-order reasoning in which the student must use, explain, and exploit newly learned knowledge. For example (page 403), we motivate probability by considering a Monopoly game.

My students need to be able to estimate. I believe this is one of the most avoided topics in mathematics textbooks. I have spent a great deal of effort discussing this topic, and you will notice that many of the problem sets have multiple-choice questions that ask students to estimate, and not calculate. My students need to develop a *number sense* which is different from "getting right answers." Answers in the back of the book are, of course, desirable and necessary, but in the book of life and problem-solving there are no "answers in the back."

My students need money sense. Why do we learn to solve equations but not learn how to handle credit cards or purchase a car or home? (See Chapter 7.) I believe both skills are important in today's world.

My students need to develop communication skills and need to be able to think critically. In many of the sections of this book I have assigned problems that solicit a written paper or a book report. Students need to be able to *communicate*

in mathematics, and should not stop after they can "find a few right answers." You will find problems throughout the book that encourage higher-order reasoning. For example, we analyze the lyrics of "By the Time I Get to Phoenix," and we look for the errors in Anacin and Saab advertisements.

I believe we (teachers and authors) have done a disservice to most of our past students. We have taught mathematics as if we were preparing them for a career in mathematics, when instead we should have been teaching them an appreciation of mathematics. It is not necessary to learn all the technical details in order to use and appreciate math. For example, most people like music of some kind; almost anyone can find a radio station playing something they like to hear. But suppose we were to learn about music in the same way we learn about mathematics. Suppose we said that, before you could listen to the radio, you had to take three years of sand blocks and recorder, followed by five years of practice piano (graded by your proficiency at scales and rhythm), and finally, after some high school courses on theory of music, you would be allowed to hear your first composed piece. On the contrary, composers, musicians, conductors, and music teachers see themselves as engaged in a common enterprise of bringing music to the world; and they believe that, whatever techniques they are trying to teach, the lessons should always occur in the context of the wonderful sound of music. We should expect no less of mathematics.

Organization of This Book

The main theme throughout the book is *problem solving*. In "The Power of Mathematics," we begin by discussing math anxiety and how to formulate the problem. The most difficult first step for many students is to determine the exact nature of the problem. All too often we try to solve a problem before we are even sure what we are trying to solve. Techniques from arithmetic, algebra, and geometry are all applied to problem solving.

These techniques of problem solving are then used in the second part of the book, "The Utility of Mathematics." Each topic in this part of the book was selected because of its usefulness to the student. The topics include managing money, using the ideas of interest, installment buying, using a credit card, and inflation; buying a car or home; sets; probability; contests; statistics; surveys; and the influence of these topics on our lives.

The material of this book can be adapted to almost any course arrangement. Chapter 1 on calculators and arithmetic is required for the rest of the book, but it may be treated lightly or skipped by those familiar with its contents. Topics from beginning algebra are presented in Chapters 2, 3, and 4 and are then used in developing much of the material that follows. For example, percents in Chapter 5 are described with proportions and simple equations. Chapter 6 introduces the

ideas of geometry and measurement in a practical and down-to-earth manner. Chapters 7–11 give students a chance to use mathematics in a variety of ways, including interest, consumer applications, sets, logic, probability, statistics, and graphs. I have written this book with the idea that different classes will pick different topics as interest and competency requirements dictate. The chapters are independent and can be covered in any order.

Acknowledgments

I am most grateful to those who have assisted me in the development of this material: to my students who made many valuable suggestions about the material, and especially to those who were bold enough to share their fears and anxieties with me; to my colleagues who shared ideas and teaching suggestions; and to the reviewers who offered many valuable suggestions. In particular, I would like to thank

Eugene Brown, Northern Virginia Community College
Margaret Finster, Erie Community College
Harvey Lambert, University of Nevada-Reno
Ann Loving, J. Sargeant Reynolds Community College
Maryanne C. Petruska, Pensacola Jr. College
Jane Rood, Eastern Illinois University

I would also like to thank the reviewers of previous editions:

Carol Achs, Mesa Community College
Elton Beougher, Fort Hays State University
James W. Brown, Northern Essex Community College
Mary Clarke, Cerritos College
Gary Donica, Florence-Darlington Technical College
William Durand, Henderson State College
Carolyn Ehr, Fort Hays State University
Ben V. Flora, Jr., Moorehead State University
Elton Fors, Northern State College, South Dakota
Roy E. Garland, Minnersville State College
Armando Gingras, Metropolitan State College, Denver
Donald K. Hostetler, Mesa Community College
Glenn E. Johnston, Moorehead State University
Keith S. Joseph, Metropolitan State College, Denver
Virginia Keen, Western Michigan University
Roxanne King, Prince George's Community College
Genevieve M. Knight, Hampton Institute
Robert Levine, Community College of Allegheny County

Chicha Lynch, Capuchino High School
Tucker Maney, Northern Virginia Community College
Glen E. Mills, Pensacola Junior College
Carol B. Olmstead, University of Nevada, Reno
Diana Pors, Eastside Union High School
George C. Sethares, Bridgewater State College
Clifford Tremblay, Pembroke State College
Erma Williams, Hampton University
Susan Wood, J. Sargent Reynolds Community College

I would like to thank Diana Gerardi for working all the problems and for providing helpful suggestions. Carol Dondrea and Robin Lockwood of Bookman Productions were pleasant colleagues and made the hard work easier.

The production staff at Brooks/Cole also deserves special credit. In particular, Craig Barth was most helpful not only in doing extensive market research for this book, but also in suggesting material and for taking an active part in the development of this book. He went far beyond the job requirements of a good editor.

Karl J. Smith
Sebastopol, California

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*Optional sections

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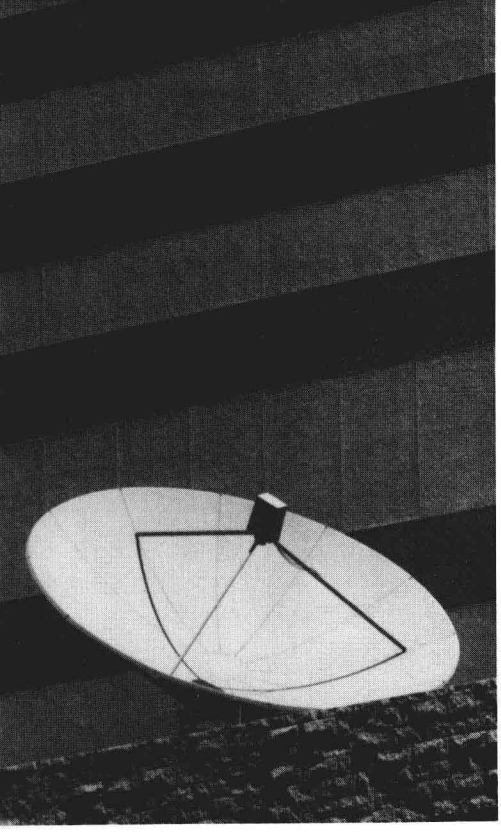


1 FOUNDATIONS: THE POWER OF MATHEMATICS

The first part of this book, "The Power of Mathematics," attempts to develop an appreciation for mathematics by displaying the intrinsic **power** of the subject. We will begin by looking at some of the causes and effects of *math anxiety*. We take natural steps—small at first, and then a little larger as you gain confidence—to review and learn about calculators, fractions, percents, algebra, equations, metrics, and geometry.

Most people view mathematics as a series of techniques of use only to the scientist, the engineer, or the specialist. In fact, the majority of our population could be classified as math-avoiders, who consider the assertion that mathematics can be creative, beautiful, and significant not only as an "impossible dream," but also something they don't even want to discuss.

At each turn of the page, I hope you will find something new and interesting to you. I want you to participate and become involved with the material. I want you to experience what I mean when we speak of the *beauty* of mathematics. I hope you are now ready to begin your study of a new course; I wish you success.

Mathematics, rightly viewed, possesses . . . supreme beauty—a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The pure spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

Bertrand Russell

CHAPTER

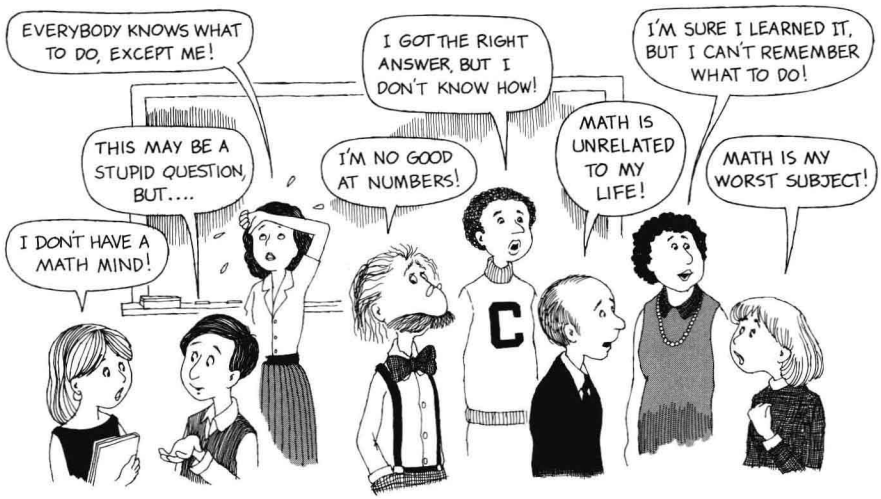
1

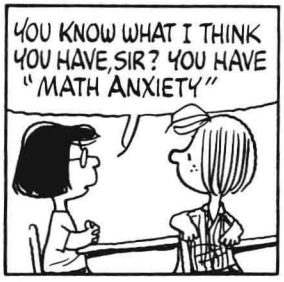
Arithmetic, Calculators, and Problem Solving

1.1 Math Anxiety

There are many reasons for reading a book but the best reason is because you want to read it. Although you are probably reading this first page because you were requested to do so by your instructor, it is my hope that in a short while you will be reading this book because you want to read it.

This book was written for people who are math-avoiders, people who think they can't work math problems, and people who think they are never going to use math. Do you see yourself making any of these statements?





Peanuts, *reprinted by permission of UFS, Inc.*

Do you feel that you can be reasonably successful in other subjects but are unable to do math? Do you make career choices based on avoidance of mathematics courses? If so, you have *math anxiety.* If you reexamine your negative feelings toward mathematics, you can overcome them. In this book, I'll constantly try to help you overcome these feelings.

Sheila Tobias, an educator, feminist, and founder of an organization called Overcoming Math Anxiety, has become one of our nation's leading spokespersons on math anxiety. She is not a mathematician, and in fact describes herself as a math-avoider. She has written a book titled *Overcoming Math Anxiety* (New York: W. W. Norton & Company, 1978; available in paperback). I recommend this book to anyone who has ever said "I'm no good at numbers." In this book, she describes a situation that characterizes anxiety (p. 45):

> Paranoia comes quickly on the heels of the anxiety attack. "Everyone knows," the victim believes, "that I don't understand this. The teacher knows. Friends know. I'd better not make it worse by asking questions. Then everyone will find out how dumb I really am." This paranoid reaction is particularly disabling because fear of exposure keeps us from constructive action. We feel guilty and ashamed, not only because our minds seem to have deserted us but because we believe that our failure to comprehend this one new idea is proof that we have been "faking math" for years.

The reaction described in this paragraph sets up a vicious cycle. The more we avoid math, the less able we feel; and the less able we feel, the more we avoid it. The cycle can also work in the other direction. What do you like to do? Chances are, if you like it, you do it. The more you do something, the better you become at it. In fact, you've probably thought, "I like to do it, but I don't get to do it as often as I'd like to." This is the normal reaction toward something you like to do. In this book, I attempt to break the negative cycle concerning math and replace it with a positive cycle. However, I will need your help and willingness to try.

The central theme in this book is problem solving. Through problem solving I'll try to dispel your feelings of panic. Once you find that you are capable of doing mathematics, we'll look at some of the foundations and uses. There are no prerequisites for this book; and as we progress through the book, I'll include a review of the math you never quite learned in school—from fractions, decimals, percents, and metrics to algebra and geometry. I hope to answer the questions that, perhaps, you were embarrassed to ask.

I hope you will enjoy reading this book; but if you feel an anxiety attack coming, STOP and put it aside for a while. Talk to your instructor, or call me. My telephone number is

(707) 829-0606

I care about your progress with the course, and I'd like to hear your reactions to this book. You can write to me at the following address:

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At the end of each section in this book is a problem set. This first problem set is built around 12 math myths. These myths come from another book on math anxiety, *Mind Over Math,* by Stanley Kogelman and Joseph Warren (New York: Dial Press, 1978), which I highly recommend. These commonly believed myths have resulted in false impressions about how math is done, and they need to be dispelled.

Math Anxiety Bill of Rights*

by Sandra L. Davis

1. I have the right to learn at my own pace and not feel put down or stupid if I'm slower than someone else.
2. I have the right to ask whatever questions I have.
3. I have the right to need extra help.
4. I have the right to ask a teacher or TA for help.
5. I have the right to say I don't understand.
6. I have the right not to understand.
7. I have the right to feel good about myself regardless of my abilities in math.
8. I have the right not to base my self-worth on my math skills.
9. I have the right to view myself as capable of learning math.
10. I have the right to evaluate my math instructors and how they teach.
11. I have the right to relax.
12. I have the right to be treated as a competent adult.
13. I have the right to dislike math.
14. I have the right to define success in my own terms.

PROBLEM SET 1.1

In Problems 1–12, comment on each math myth. There are no right or wrong answers, but you will gain insight into your own attitudes as well as begin to dispel some false notions you might have about the subject.

1. Myth 1: Men are better in math than women.
2. Myth 2: Math requires logic, not intuition.
3. Myth 3: You must always know how you got the answer.
4. Myth 4: Math is not creative.
5. Myth 5: There is a best way to do a math problem.
6. Myth 6: It's always important to get the answer exactly right.

*From *Overcoming Math Anxiety,* by Sheila Tobias, pp. 236–237.

7. Myth 7: It's bad to count on your fingers.
8. Myth 8: Mathematicians do problems quickly, in their heads.
9. Myth 9: Math requires a good memory.
10. Myth 10: Math is done by working intensely until the problem is solved.
11. Myth 11: Some people have a "math mind" and some don't.
12. Myth 12: There is a magic key to doing math.
13. Summarize your math experiences in elementary school.
14. Summarize your math experiences in high school.

1.2 Formulating the Problem

Read the story given below. No questions are asked, but try to imagine yourself sitting in a living room with several others who share your feelings about math. Your job is to read the story and make up a problem you know how to solve from any part of the story. You should have a pencil and paper and you can have as much time as you want; nobody will look at what you are doing, but I want you to keep track of your feelings as you read the story and follow the directions.

> On the way to the market, which is 12 miles from home, I stopped at the drugstore to pick up a get-well card. I selected a series of cards with puzzles on them. The first one said "A bottle and a cork cost $1.10 and the bottle is a dollar more than the cork. How much is the bottle and how much is the cork?" I thought that would be a good card for Joe, so I purchased it for $1.75, along with a six-pack of cola for $2.79. The total bill was $4.81, which included 6% sales tax. My next stop was the market, which was exactly 3.4 miles from the drugstore. I bought $15.65 worth of groceries and paid for it with a $20 bill. I deposited the change in a charity bank on the counter and left the store. On the way home I bought 8.5 gallons of gas for $13.60. Since I had gone 238 miles since my last fill-up, I was happy with the mileage on my new car. I returned home and made myself a ham and cheese sandwich.

Have you spent enough time on the story? Take time to reread it (spend at least 10 minutes with this exercise). Now, write a math question based on this story that you can answer without difficulty. Can you summarize your feelings? If my experiences in doing this exercise with my students apply to you, I would guess that you encountered some difficulty, some discomfort, perhaps despair or anger, or even indifference. Most students tend to focus on the more difficult questions (perhaps a miles per gallon problem) instead of following the directions to formulate a problem that will give you no difficulty.