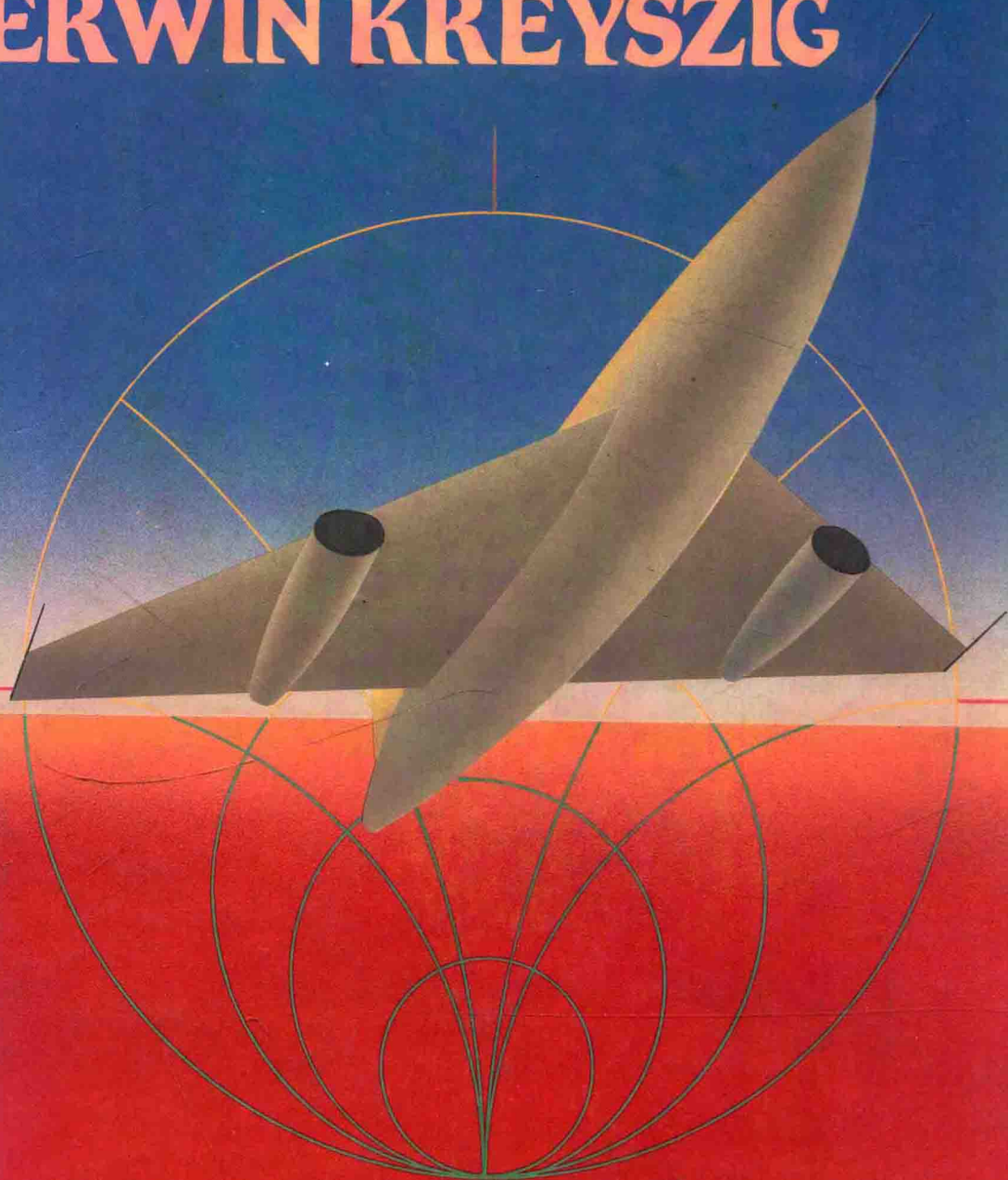


SIXTH EDITION

ADVANCED ENGINEERING MATHEMATICS

ERWIN KREYSZIG



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Systems of Units. Some Important Conversion Factors

The most important systems of units are shown in the table below. The Mks System is also known as the *International System of Units* (abbreviated *SI System*), and the abbreviations s (instead of sec) and N (instead of nt) are also used.

System of units	Length	Mass	Time	Force
Cgs system	centimeter (cm)	gram (gm)	second (sec)	dyne
Mks system	meter (m)	kilogram (kg)	second (sec)	newton (nt)
Engineering system	foot (ft)	slug	second (sec)	pound (lb)

$$1 \text{ inch (in.)} = 2.54000 \text{ cm}$$

$$1 \text{ foot (ft)} = 12 \text{ in.} = 30.48006 \text{ cm}$$

$$1 \text{ yard (yd)} = 3 \text{ ft} = 91.44018 \text{ cm}$$

$$1 \text{ statute mile (mi)} = 5280 \text{ ft} = 1.60935 \text{ km}$$

$$1 \text{ nautical mile} = 6080.2 \text{ ft} = 1.8532 \text{ km}$$

$$1 \text{ acre} = 4840 \text{ yd}^2 = 4046.773 \text{ m}^2$$

$$1 \text{ mi}^2 = 640 \text{ acres} = 2.58999 \text{ km}^2$$

$$1 \text{ fluid ounce} = 29.5737 \text{ cm}^3$$

$$1 \text{ U.S. gallon} = 4 \text{ quarts (liq)} = 8 \text{ pints (liq)} = 128 \text{ fl oz} = 3785.432 \text{ cm}^3$$

$$1 \text{ British Imperial and Canadian gallon} = 1.20094 \text{ U.S. gallons} = 4546.1 \text{ cm}^3$$

$$1 \text{ slug} = 14.59390 \text{ kg}$$

$$1 \text{ pound (lb)} = 4.448444 \text{ nt}$$

$$1 \text{ newton (nt)} = 10^5 \text{ dynes}$$

$$1 \text{ British thermal unit (Btu)} = 1054.8 \text{ joules} \quad 1 \text{ joule} = 10^7 \text{ ergs}$$

$$1 \text{ calorie (cal)} = 4.1840 \text{ joules}$$

$$1 \text{ kilowatt-hour (kWh)} = 3413 \text{ Btu} = 3.6 \cdot 10^6 \text{ joules}$$

$$1 \text{ horsepower (hp)} = 2545 \text{ Btu/h} = 178.2 \text{ cal/sec} = 0.74570 \text{ kW}$$

$$1 \text{ kilowatt (kW)} = 1000 \text{ watts} = 3413 \text{ Btu/h} = 238.9 \text{ cal/sec}$$

$$^{\circ}\text{F} = ^{\circ}\text{C} \cdot 1.8 + 32$$

$$1^{\circ} = 60' = 3600'' = 0.01745 \text{ radian}$$

Differentiation

$$(cu)' = cu' \quad (c \text{ constant})$$

$$(u + v)' = u' + v'$$

$$(uv)' = u'v + v'u$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} \quad (\text{Chain rule})$$

$$(x^n)' = nx^{n-1}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{\log_a e}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\text{arc cot } x)' = -\frac{1}{1+x^2}$$

Integration

$$\int uv' dx = uv - \int u'v dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln |x| + c$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln |\cos x| + c$$

$$\int \cot x dx = \ln |\sin x| + c$$

$$\int \sec x dx = \ln |\sec x + \tan x| + c$$

$$\int \csc x dx = \ln |\csc x - \cot x| + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1} \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + c$$

$$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + c$$

$$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + c$$

$$\int \tan^2 x dx = \tan x - x + c$$

$$\int \cot^2 x dx = -\cot x - x + c$$

$$\int \ln x dx = x \ln x - x + c$$

$$\begin{aligned} \int e^{ax} \sin bx dx \\ = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c \end{aligned}$$

$$\begin{aligned} \int e^{ax} \cos bx dx \\ = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c \end{aligned}$$

**ADVANCED
ENGINEERING
MATHEMATICS**

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JOHNS WILEY & SONS
New York, New York

Preface

Purpose of the Book

This book introduces students of engineering, physics, mathematics and computer science to those areas of mathematics which, from a modern point of view, are most important in connection with practical problems.

The content and character of mathematics needed in applications are changing rapidly. Linear algebra—especially matrices—and numerical methods for computers are of increasing importance. Statistics and graph theory play more prominent roles. Real analysis (ordinary and partial differential equations) and complex analysis remain indispensable. The material in this book is arranged accordingly, in seven independent parts (see also the diagram on the next page):

- A** Ordinary Differential Equations (Chaps. 1–5)
- B** Linear Algebra, Vector Calculus (Chaps. 6–9)
- C** Fourier Analysis and Partial Differential Equations (Chaps. 10, 11)
- D** Complex Analysis (Chaps. 12–17)
- E** Numerical Methods (Chaps. 18–20)
- F** Optimization, Graphs (Chaps. 21, 22)
- G** Probability and Statistics (Chaps. 23, 24)

This is followed by

- References (App. 1)
- Answers to Problems (App. 2)
- Auxiliary Material (App. 3 and inside of covers)
- Tables of Functions (App. 4).

This book has helped to pave the way for the present development and will prepare students for the present situation and the future by a modern approach to the areas listed above and the ideas—some of them computer-related—that are presently causing basic changes: Many methods have become obsolete. New ideas are emphasized, for instance stability, error estimation and structural problems of algorithms, to mention just a few. Trends are driven by supply and demand: supply of powerful new mathematical and computational methods and of enormous computer capacities, demand to solve problems of growing complexity and size, arising from more and more sophisticated systems or production processes, from extreme physical conditions (e.g., those in space travel), from materials with unusual properties (plastics, alloys, superconductors, etc.), or from entirely new tasks in computer vision, robotics and other new fields.

PART A
Chaps. 1–5
Ordinary differential equations
Chaps. 1–3 Basic material
↓ Chap. 4 Series solutions, Orthogonality
Chap. 5 Laplace transformation

PART B
Chaps. 6–9
Linear algebra. Vector calculus
Chap. 6 Vectors
Chap. 7 Matrices
↓ Chap. 8 Vector differential calculus
↓ Chap. 9 Integral theorems

PART C
Chaps. 10, 11
Fourier analysis. Partial differential equations
Chap. 10 Fourier analysis
↓ Chap. 11 Partial differential equations

PART D
Chaps. 12–17
Complex analysis
Chaps. 12–15 Basic material
↓ Chap. 16 Conformal mapping
↓ Chap. 17 Potential theory

PART E
Chaps. 18–20
Numerical methods
Chap. 18 General numerical methods
Chap. 19 Methods for linear algebra
Chap. 20 Methods for differential equations

PART F
Chaps. 21, 22
Optimization. Graphs
Chap. 21 Linear programming
Chap. 22 Graphs. Combinatorial optimization

PART G
Chaps. 23, 24
Probability. Statistics
Chap. 23 Probability theory
↓ Chap. 24 Mathematical statistics

Parts of the Book and Corresponding Chapters

The general trend seems clear. Details are more difficult to predict. Accordingly, students need solid knowledge of basic principles, methods and results, and a clear perception of what engineering mathematics is all about, in all three phases of solving problems:

- **Modeling:** Translating given physical or other information and data into mathematical form, into a mathematical *model* (a differential equation, a system of equations or some other expression).
- **Solving:** Obtaining the solution by selecting and applying suitable mathematical methods, and in most cases doing numerical work on a computer.
- **Interpreting:** Understanding the meaning and the implications of the mathematical solution for the original problem in terms of physics—or wherever the problem comes from.

It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead, it is important that students become familiar with ways to think mathematically, recognize the need for applying mathematical methods to engineering problems, realize that mathematics is a systematic science built on relatively few basic concepts and involving powerful unifying principles, and get a firm grasp for the interrelation between theory, computing and experiment.

The rapid ongoing developments just sketched have led to many changes and new features in the present edition of this book, causing it to differ very substantially from previous editions.

Changes and New Features Throughout the Book

The book has been simplified by rewriting various sections in a more detailed and leisurely fashion and by placing more emphasis on applications, algorithms and examples.

We first list some major changes and additions pertaining to the book as a whole and then some of the many changes and additions in individual chapters.

- **Problem sets** changed and expanded to contain *over 6000 carefully selected problems*, including more applied problems and more routine problems
- **Chapter review problems** added, to give students practice in choosing a method from the great variety of methods in a whole chapter
- **Worked-out examples** increased to *over 600*, for help in problem solving and better understanding of the text
- **Key formulas boxed**
- **Chapter summaries** added, for quick orientation and survey of the most important facts in each chapter

Changes in chapters are listed on the next page.

Changes and New Features in Chapters

- **Ordinary differential equations** (Chaps. 1–4): More systematic treatment of *integrating factors* (Sec. 1.6). *Linear differential equations* (Chap. 2) cast in simpler and more logical form. *Frobenius method* (Sec. 4.4) greatly simplified.
- **Laplace transformation** (Chap. 5). New: shifted data problems, impulsive forcés, *Dirac's delta*, list of general formulas, (in addition to the list of transforms)
- **Matrices** (Chap. 7): More *applications* (Markov processes, Leslie matrices, etc.). More on *eigenvalues* and *diagonalization*. Additional modern numerical methods (see below)
- **Vector differential and integral calculus** (Chaps. 8, 9) streamlined by omitting some material of minor interest or making it optional. Grad, div, curl now close together; their forms in curvilinear coordinates (new). Greater emphasis on the types of integrals needed in the integral theorems in Chap. 9.
- **Fourier transformation, Fourier sine and cosine transformations** (Secs. 10.10–10.12, new) with applications to partial differential equations (Sec. 11.14)
- **Complex analysis** (Chaps. 12–17) reorganized to make it more teachable:
 1. *Mappings* by elementary functions added to Chap. 12 (Sec. 12.9).
 2. *Conformal mapping* moved to Chap. 16, to have it close to its applications in Chap. 17 on potential theory, which has been extended by stationary heat problems, etc.
 3. The lengthy introductory chapter on series now reduced to two sections that precede the discussion of power, Taylor and Laurent series.
 4. More on evaluating real integrals by complex integration.
- **Numerical methods** (Chaps. 18–20) modernized throughout, by adding new and more detailed *algorithms* and discussing more worked-out examples, by including *computer-related aspects*, on operations count, pivoting, numerical stability, rounding errors etc.; by giving more extensive treatments of *Newton interpolation*, *splines*, *LU-factorization* (Doolittle, Crout, Cholesky), and adding new material, such as *matrix norms*, *condition numbers*, *matrix deflation* and *tridiagonalization*, *QR*, *spectral shift*, etc.
- **Graph theory**: A new self-contained chapter (Chap. 22) on *graphs* and *digraphs* and their application in *combinatorial optimization* (traveling salesman and other *shortest path problems*, *shortest spanning trees*, *network flows*, *matching*, etc.).
- **Probability and statistics** (Chaps. 23, 24) reorganized by moving sections on sampling to Chap. 24.
- **References** (App. 1) updated and extended, notably those on numerical methods and optimization
- **Auxiliary material added**: Review of partial derivatives (App. 3.2), real series (App. 3.3), first-aid kits of differentiation formulas and integrals, conversion table, Greek alphabet (all on the inside covers).

Suggestions for Courses: A Four-Semester Sequence

The material may be taken in sequence and is suitable for four consecutive semester courses, meeting 3–5 hours a week:

- First semester.* Ordinary differential equations (Chaps. 1–5)
- Second semester.* Linear algebra and vector analysis (Chaps. 6–9)
- Third semester.* Complex analysis (Chaps. 12–17)
- Fourth semester.* Numerical methods (Chaps. 18–20)

For the remaining chapters, see below. Possible interchanges are obvious; for instance, numerical methods could precede complex analysis, etc.

Suggestions for Courses: Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week; for example:

- Introduction to ordinary differential equations (Chaps. 1, 2)
- Laplace transformation (Chap. 5)
- Vector algebra and calculus (Chaps. 6, 8)
- Matrices and systems of linear equations (Chap. 7)
- Fourier series and partial differential equations (Chaps. 10, 11, Secs. 20.4–20.7)
- Introduction to complex analysis (Chaps. 12–15)
- Numerical analysis (Chaps. 18, 20)
- Numerical linear algebra (Chap. 7 for review, Chap. 19)
- Optimization (Chaps. 21, 22)
- Graphs and combinatorial optimization (Chap. 22)
- Probability and statistics (Chaps. 23, 24)

General Features of This Edition

The selection, arrangement and presentation of the material has been made with greatest care, based on past and present teaching, research and consulting experience. Some major features of the book are these:

The book is **self-contained**, except for a few clearly marked places where a proof would be beyond the level of a book of the present type and a reference is given instead. Hiding difficulties or oversimplifying would be of no real help to students.

The presentation is **detailed**, to avoid irritating readers by frequent references to details in other books.

The examples are **simple**, to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?

The notations are **modern and standard**, to help students read articles in journals or other *modern* books and understand other mathematically oriented courses.

The chapters are largely **independent**, providing flexibility in teaching special courses (see above).

The end of a proof is marked by ■. This sign is also used at the end of some of the definitions and at the end of examples followed by further text.

Acknowledgment

I am indebted to many of my former teachers, colleagues and students who directly or indirectly helped me in preparing this book, in particular the present edition of it. Various parts of the manuscript were distributed to my classes in mimeographed form and returned to me with suggestions for improvement. Discussions with engineers and mathematicians (as well as written comments) were of great help to me; I want to mention particularly Professors S. L. Campbell, J. T. Cargo, P. L. Chambré, V. F. Connolly, A. Cronheim, J. Delany, J. W. Dettman, D. Dicker, D. Ellis, W. Fox, R. G. Helsel, W. N. Huff, J. Keener, E. C. Klipple, V. Komkow, H. Kuhn, G. Lamb, H. B. Mann, I. Marx, K. Millet, J. D. Moore, W. D. Munroe, J. N. Ong, Jr., P. J. Pritchard, H.-W. Pu, W. O. Ray, P. V. Reichelderfer (who helped me very much with the new Chap. 22) J. T. Scheick, H. A. Smith, J. P. Spencer, J. Todd, H. Unz, A. L. Villone, H. J. Weiss, A. Wilansky, C. H. Wilcox, L. Zia, A. D. Ziebur, all from this country, Professors H. S. M. Coxeter and R. Vaillancourt and Mr. H. Kreyszig (whose computer expertise was of great help in Chaps. 18–20) from Canada, and Professors H. Florian, F. Hohenberg, M. Kracht, F. Reutter, C. Schmieden, H. Unger, H. Wielandt, all from Europe. I can offer here only an inadequate acknowledgment of my appreciation.

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Suggestions of many readers were evaluated in preparing the present edition. Any further comment and suggestion for improvement of the book will be gratefully received.

ERWIN KREYSZIG

CONTENTS

Part A. ORDINARY DIFFERENTIAL EQUATIONS	1
CHAPTER 1 Differential Equations of the First Order	2
1.1 Basic Concepts and Ideas, 2	
1.2 Separable Differential Equations, 12	
1.3 Modeling: Separable Equations, 15	
1.4 Reduction to Separable Form, 24	
1.5 Exact Differential Equations, 27	
1.6 Integrating Factors, 31	
1.7 Linear Differential Equations, 34	
1.8 Modeling: Electric Circuits, 41	
1.9 Families of Curves. Orthogonal Trajectories, 47	
1.10 Approximate Solutions: Direction Fields, Iteration, 53	
1.11 Existence and Uniqueness of Solutions, 59	
<i>Chapter Review Problems, 65</i>	
<i>Chapter Summary, 67</i>	
CHAPTER 2 Linear Differential Equations	69
2.1 Homogeneous Linear Equations of the Second Order, 70	
2.2 Homogeneous Equations with Constant Coefficients, 74	
2.3 General Solution. Basis. Initial Value Problem, 77	
2.4 Real Roots, Complex Roots, Double Root of the Characteristic Equation, 83	
2.5 Differential Operators, 89	
2.6 Modeling: Free Oscillations, 92	
2.7 Euler-Cauchy Equation, 103	
2.8 Existence of Solutions, Uniqueness, 107	
2.9 Homogeneous Linear Equations of Arbitrary Order n , 112	
2.10 Equations of Order n with Constant Coefficients, 117	
2.11 Nonhomogeneous Equations, 121	
2.12 Nonhomogeneous Equations: Solving by the Method of Undetermined Coefficients, 125	
2.13 Modeling: Forced Oscillations. Resonance, 129	
2.14 Modeling of Electric Circuits, 136	
2.15 Complex Method for Particular Solutions, 142	
2.16 Nonhomogeneous Equations: Solving by the Method of Variation of Parameters, 145	
<i>Further Proof, 148</i>	
<i>Chapter Review Problems, 150</i>	
<i>Chapter Summary, 152</i>	

CHAPTER 3	Systems of Differential Equations, Phase Plane, Stability	154
3.1	Systems of Differential Equations, 154	
3.2	Phase Plane, 162	
3.3	Critical Points. Stability, 168	
	<i>Chapter Review Problems, 176</i>	
	<i>Chapter Summary, 179</i>	
CHAPTER 4	Series Solutions of Differential Equations, Orthogonal Functions	180
4.1	Power Series Method, 181	
4.2	Theory of the Power Series Method, 184	
4.3	Legendre's Equation. Legendre Polynomials $P_n(x)$, 190	
4.4	Extended Power Series Method. Indicial Equation, 196	
4.5	Bessel's Equation. Bessel Functions of the First Kind, 205	
4.6	Bessel Functions of the Second Kind, 213	
4.7	Orthogonal Sets of Functions, 218	
4.8	Sturm-Liouville Problem, 225	
4.9	Orthogonality of Bessel Functions and Legendre Polynomials, 231	
	<i>Further Proof, 236</i>	
	<i>Chapter Review Problems, 239</i>	
	<i>Chapter Summary, 240</i>	
CHAPTER 5	Laplace Transformation	242
5.1	Laplace Transform. Inverse Transform. Linearity, 243	
5.2	Laplace Transforms of Derivatives and Integrals, 249	
5.3	Shifting on the s -Axis, Shifting on the t -Axis, Unit Step Function, 256	
5.4	Further Applications. Dirac's Delta Function, 262	
5.5	Differentiation and Integration of Transforms, 268	
5.6	Convolution. Integral Equations, 271	
5.7	Partial Fractions. Systems of Differential Equations, 278	
5.8	Periodic Functions. Further Applications, 288	
5.9	Basic General Formulas for the Laplace Transformation, 298	
5.10	Table of Laplace Transforms, 299	
	<i>Chapter Review Problems, 301</i>	
	<i>Chapter Summary, 303</i>	
Part B. LINEAR ALGEBRA, VECTOR CALCULUS		305
CHAPTER 6	Vectors	306
6.1	Scalars and Vectors, 306	
6.2	Components of a Vector, 308	
6.3	Addition of Vectors, Multiplication by Scalars, 312	
6.4	Vector Spaces, 316	

- 6.5 Inner Product (Dot Product), 322
- 6.6 Inner Product Spaces, 329
- 6.7 Vector Product (Cross Product), 332
- 6.8 Components of Vector Products, 335
- 6.9 Scalar Triple Product. Other Repeated Products, 339
- Further Proof, 344*
- Chapter Review Problems, 345*
- Chapter Summary, 347*

CHAPTER 7 Matrices and Determinants**349**

- 7.1 Basic Concepts, 350
- 7.2 Addition of Matrices, Multiplication by Scalars, 352
- 7.3 Matrix Multiplication, 357
- 7.4 Transpose of a Matrix, 368
- 7.5 Systems of Linear Equations. Gauss Elimination, 372
- 7.6 Rank of a Matrix, 381
- 7.7 Systems of Linear Equations: General Properties of Solutions, 386
- 7.8 Inverse of a Matrix, 389
- 7.9 Determinants of Second and Third Order, 395
- 7.10 Determinants of Arbitrary Order, 402
- 7.11 Rank in Terms of Determinants, Cramer's Rule, 409
- 7.12 Eigenvalues, Eigenvectors, 415
- 7.13 Hermitian, Skew-Hermitian and Unitary Matrices, 424
- 7.14 Eigenvalues of Hermitian, Skew-Hermitian, and Unitary Matrices, 429
- 7.15 Properties of Eigenvectors. Diagonalization, 434
- 7.16 Systems of Differential Equations, 441
- Further Proof, 448*
- Chapter Review Problems, 450*
- Chapter Summary, 454*

CHAPTER 8 Vector Differential Calculus**457**

- 8.1 Scalar Fields and Vector Fields, 457
- 8.2 Vector Calculus, 460
- 8.3 Curves, 464
- 8.4 Tangent, Arc Length of a Curve, 467
- 8.5 Velocity and Acceleration, 472
- 8.6 Curvature and Torsion of a Curve (Optional), 476
- 8.7 Functions of Several Variables: Chain Rule, Mean Value Theorem, 480
- 8.8 Directional Derivative. Gradient of a Scalar Field, 485
- 8.9 Divergence of a Vector Field, 492
- 8.10 Curl of a Vector Field, 496
- 8.11 Grad, Div, Curl in Curvilinear Coordinates (Optional), 498
- Further Proofs, 504*
- Chapter Review Problems, 508*
- Chapter Summary, 509*

CHAPTER 9 Line and Surface Integrals. Integral Theorems 512

- 9.1 Line Integrals, 512
- 9.2 Double Integrals, 519
- 9.3 Transformation of Double Integrals into Line Integrals (Green's Theorem in the Plane), 527
- 9.4 Surfaces for Surface Integrals, 534
- 9.5 Surface Integrals, 540
- 9.6 Triple Integrals. Divergence Theorem of Gauss, 550
- 9.7 Further Applications of the Divergence Theorem, 556
- 9.8 Stokes's Theorem, 562
- 9.9 Line Integrals Independent of Path, 568
- Chapter Review Problems, 577*
- Chapter Summary, 579*

Part C. FOURIER ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS 581**CHAPTER 10 Fourier Series, Fourier Integrals, Fourier Transforms 582**

- 10.1 Periodic Functions. Trigonometric Series, 583
- 10.2 Fourier Series, 586
- 10.3 Functions of Any Period $p = 2L$, 593
- 10.4 Even and Odd Functions, 596
- 10.5 Half-Range Expansions, 601
- 10.6 Calculating Fourier Coefficients Without Integration (Method of Jumps), 605
- 10.7 Forced Oscillations, 611
- 10.8 Approximation by Trigonometric Polynomials. Square Error, 614
- 10.9 Fourier Integral, 617
- 10.10 Fourier Cosine Transformation, Fourier Sine Transformation, 626
- 10.11 Fourier Transformation, 630
- 10.12 Tables of Fourier Cosine Transforms, Fourier Sine Transforms and Fourier Transforms, 637
- Chapter Review Problems, 640*
- Chapter Summary, 642*

CHAPTER 11 Partial Differential Equations 644

- 11.1 Basic Concepts, 645
- 11.2 Modeling: Vibrating String. One-Dimensional Wave Equation, 647
- 11.3 Method of Separating Variables (Product Method), 649
- 11.4 D'Alembert's Solution of the Wave Equation, 657
- 11.5 Heat Flow, 661
- 11.6 Heat Flow in an Infinite Bar, 671
- 11.7 Modeling: Vibrating Membrane. Two-Dimensional Wave Equation, 676

11.8	Rectangular Membrane, 678	
11.9	Laplacian in Polar Coordinates, 686	
11.10	Circular Membrane. Bessel's Equation, 689	
11.11	Laplace's Equation. Potential, 696	
11.12	Laplace's Equation in Spherical Coordinates. Legendre's Equation, 699	
11.13	Laplace Transformation Applied to Partial Differential Equations, 705	
11.14	Fourier Transformations Applied to Partial Differential Equations, 709	
	<i>Chapter Review Problems, 713</i>	
	<i>Chapter Summary, 716</i>	

Part D. COMPLEX ANALYSIS

719

CHAPTER 12	Complex Numbers. Complex Analytic Functions	720
12.1	Complex Numbers, 720	
12.2	Polar Form of Complex Numbers. Powers and Roots, 726	
12.3	Curves and Regions in the Complex Plane, 733	
12.4	Limit. Derivative. Analytic Function, 735	
12.5	Cauchy-Riemann Equations, 740	
12.6	Exponential Function, 746	
12.7	Trigonometric Functions, Hyperbolic Functions, 750	
12.8	Logarithm. General Power, 753	
12.9	Mapping by Special Functions, 757	
	<i>Further Proof, 762</i>	
	<i>Chapter Review Problems, 763</i>	
	<i>Chapter Summary, 765</i>	
CHAPTER 13	Complex Integration	767
13.1	Line Integral in the Complex Plane, 767	
13.2	Two Integration Methods. Examples, 771	
13.3	Cauchy's Integral Theorem, 777	
13.4	Existence of Indefinite Integral, 785	
13.5	Cauchy's Integral Formula, 788	
13.6	Derivatives of Analytic Functions, 791	
	<i>Further Proof, 796</i>	
	<i>Chapter Review Problems, 798</i>	
	<i>Chapter Summary, 800</i>	
CHAPTER 14	Power Series, Taylor Series, Laurent Series	801
14.1	Sequences and Series, 801	
14.2	Convergence Tests for Series, 805	
14.3	Power Series, 812	
14.4	Functions Given by Power Series, 817	
14.5	Taylor Series, 823	
14.6	Taylor Series of Elementary Functions, 827	