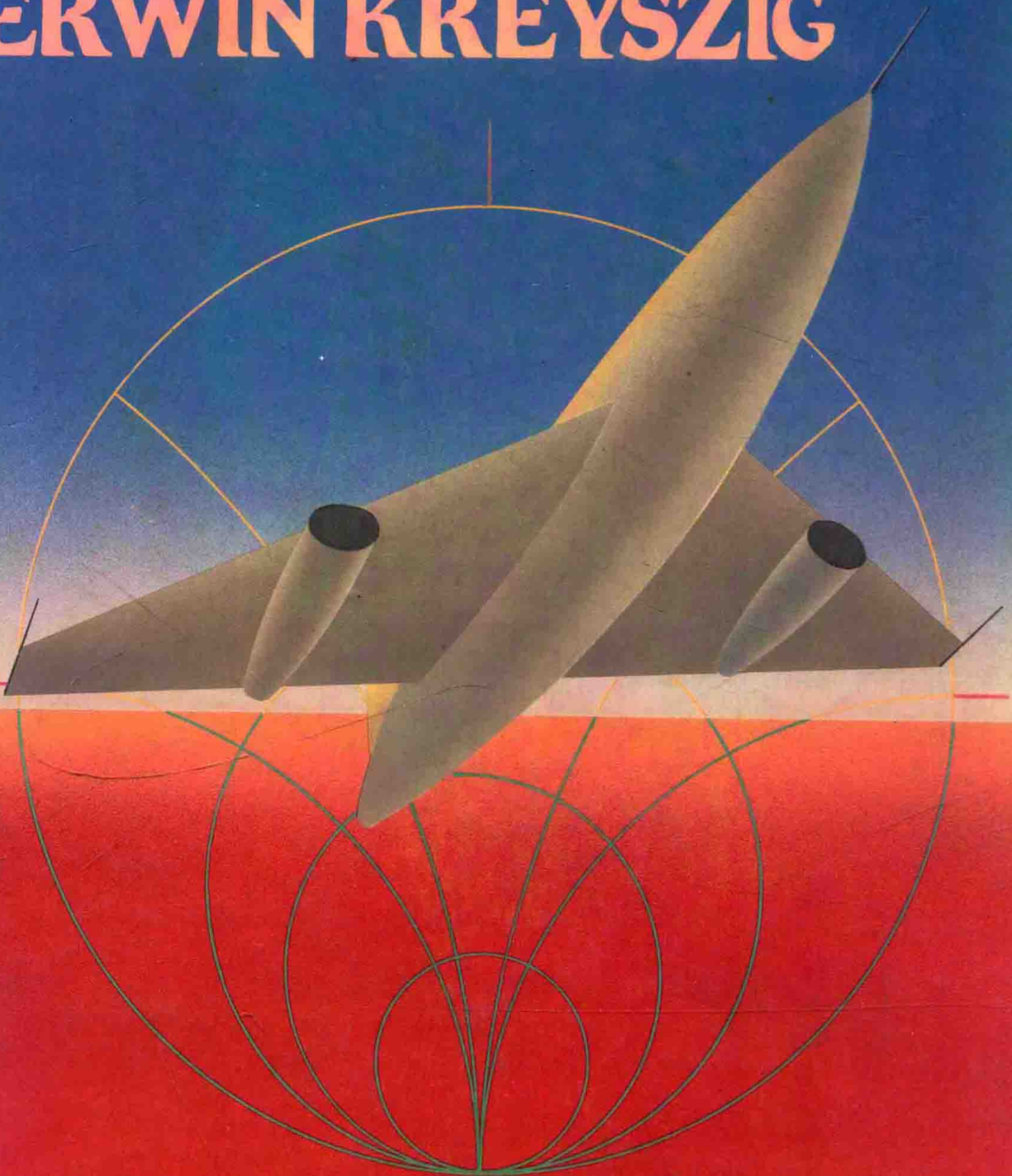
SIXTH EDITION

ADVANCED ENGINEERING MATHEMATICS

ERWIN KREYSZIG



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ADVANCED ENGINEERING MATHEMATICS

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Systems of Units. Some Important Conversion Factors

The most important systems of units are shown in the table below. The Mks System is also known as the *International System of Units* (abbreviated *SI System*), and the abbreviations s (instead of sec) and N (instead of nt) are also used.

System of units	Length	Mass	Time	Force
Cgs system	centimeter (cm)	gram (gm)	second (sec)	dyne
Mks system	meter (m)	kilogram (kg)	second (sec)	newton (nt)
Engineering system	foot (ft)	slug	second (sec)	pound (lb)

1 inch (in.) = 2.54000 51 cm

1 foot (ft) = 12 in. = 30.48006 12 cm

1 yard (yd) = 3 ft = 91.44018 36 cm

1 statute mile (mi) = 5280 ft = 1.60935 km

1 nautical mile = 6080.2 ft = 1.8532 km

1 acre = 4840 yd^2 = 4046.773 m^2

1 mi^2 = 640 acres = 2.58999 87 km^2

1 fluid ounce = 29.5737 cm^3

1 U.S. gallon = 4 quarts (liq) = 8 pints (liq) = 128 fl oz = 3785.432 cm^3

1 British Imperial and Canadian gallon = 1.20094 U.S. gallons = 4546.1 cm^3

1 slug = 14.59390 kg

1 pound (lb) = 4.448444 nt

1 newton (nt) = 10^5 dynes

1 British thermal unit (Btu) = 1054.8 joules

1 joule = 10^7 ergs

1 calorie (cal) = 4.1840 joules

1 kilowatt-hour (kWh) = 3413 Btu = $3.6 \cdot 10^6$ joules

1 horsepower (hp) = 2545 Btu/h = 178.2 cal/sec = 0.74570 kW

1 kilowatt (kW) = 1000 watts = 3413 Btu/h = 238.9 cal/sec

$°F = °C \cdot 1.8 + 32$

$1° = 60' = 3600' = 0.01745$ radian

For further details see, e.g., D. Halliday and R. Resnick, *Physics*. 3rd ed., New York: Wiley, 1978. See also AN American National Standard, ASTM/IEEE Standard Metric Practice, Institute of Electrical and Electronics Engineers, Inc., 345 East 47th Street, New York, N.Y. 10017.

Differentiation

$(cu)' = cu' \quad (c \text{ constant})$

$(u + v)' = u' + v'$

$(uv)' = u'v + v'u$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

$\dfrac{du}{dx} = \dfrac{du}{dy} \cdot \dfrac{dy}{dx} \quad$ (Chain rule)

$(x^n)' = nx^{n-1}$

$(e^x)' = e^x$

$(a^x)' = a^x \ln a$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\cot x)' = -\csc^2 x$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$(\ln x)' = \dfrac{1}{x}$

$(\log_a x)' = \dfrac{\log_a e}{x}$

$(\text{arc}\sin x)' = \dfrac{1}{\sqrt{1 - x^2}}$

$(\text{arc}\cos x)' = -\dfrac{1}{\sqrt{1 - x^2}}$

$(\text{arc}\tan x)' = \dfrac{1}{1 + x^2}$

$(\text{arc}\cot x)' = -\dfrac{1}{1 + x^2}$

Integration

$\int uv'\, dx = uv - \int u'v\, dx$

$\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

$\int \dfrac{1}{x}\, dx = \ln |x| + c$

$\int e^{ax}\, dx = \dfrac{1}{a} e^{ax} + c$

$\int \sin x\, dx = -\cos x + c$

$\int \cos x\, dx = \sin x + c$

$\int \tan x\, dx = -\ln |\cos x| + c$

$\int \cot x\, dx = \ln |\sin x| + c$

$\int \sec x\, dx = \ln |\sec x + \tan x| + c$

$\int \csc x\, dx = \ln |\csc x - \cot x| + c$

$\int \dfrac{dx}{x^2 + a^2}\, dx = \dfrac{1}{a} \text{arc}\tan \dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \text{arc}\sin \dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1} \dfrac{x}{a} + c$

$\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \dfrac{x}{a} + c$

$\int \sin^2 x\, dx = \tfrac{1}{2}x - \tfrac{1}{4} \sin 2x + c$

$\int \cos^2 x\, dx = \tfrac{1}{2}x + \tfrac{1}{4} \sin 2x + c$

$\int \tan^2 x\, dx = \tan x - x + c$

$\int \cot^2 x\, dx = -\cot x - x + c$

$\int \ln x\, dx = x \ln x - x + c$

$\int e^{ax} \sin bx\, dx$

$\quad = \dfrac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$

$\int e^{ax} \cos bx\, dx$

$\quad = \dfrac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$

ADVANCED ENGINEERING MATHEMATICS

Preface

Purpose of the Book

This book introduces students of engineering, physics, mathematics and computer science to those areas of mathematics which, from a modern point of view, are most important in connection with practical problems.

The content and character of mathematics needed in applications are changing rapidly. Linear algebra—especially matrices—and numerical methods for computers are of increasing importance. Statistics and graph theory play more prominent roles. Real analysis (ordinary and partial differential equations) and complex analysis remain indispensable. The material in this book is arranged accordingly, in seven independent parts (see also the diagram on the next page):

A Ordinary Differential Equations (Chaps. 1–5)

B Linear Algebra, Vector Calculus (Chaps. 6–9)

C Fourier Analysis and Partial Differential Equations (Chaps. 10, 11)

D Complex Analysis (Chaps. 12–17)

E Numerical Methods (Chaps. 18–20)

F Optimization, Graphs (Chaps. 21, 22)

G Probability and Statistics (Chaps. 23, 24)

This is followed by

References (App. 1)

Answers to Problems (App. 2)

Auxiliary Material (App. 3 and inside of covers)

Tables of Functions (App. 4).

This book has helped to pave the way for the present development and will prepare students for the present situation and the future by a modern approach to the areas listed above and the ideas—some of them computer-related—that are presently causing basic changes: Many methods have become obsolete. New ideas are emphasized, for instance stability, error estimation and structural problems of algorithms, to mention just a few. Trends are driven by supply and demand: supply of powerful new mathematical and computational methods and of enormous computer capacities, demand to solve problems of growing complexity and size, arising from more and more sophisticated systems or production processes, from extreme physical conditions (e.g., those in space travel), from materials with unusual properties (plastics, alloys, superconductors, etc.), or from entirely new tasks in computer vision, robotics and other new fields.

PART A
Chaps. 1–5
Ordinary differential equations

- Chaps. 1–3 Basic material
- Chap. 4 Series solutions, Orthogonality
- Chap. 5 Laplace transformation

PART B
Chaps. 6–9
Linear algebra. Vector calculus

- Chap. 6 Vectors
- Chap. 7 Matrices
- Chap. 8 Vector differential calculus
- Chap. 9 Integral theorems

PART C
Chaps. 10, 11
Fourier analysis. Partial differential equations

- Chap. 10 Fourier analysis
- Chap. 11 Partial differential equations

PART D
Chaps. 12–17
Complex analysis

- Chaps. 12–15 Basic material
- Chap. 16 Conformal mapping
- Chap. 17 Potential theory

PART E
Chaps. 18–20
Numerical methods

- Chap. 18 General numerical methods
- Chap. 19 Methods for linear algebra
- Chap. 20 Methods for differential equations

PART F
Chaps. 21, 22
Optimization. Graphs

- Chap. 21 Linear programming
- Chap. 22 Graphs. Combinatorial optimization

PART G
Chaps. 23, 24
Probability. Statistics

- Chap. 23 Probability theory
- Chap. 24 Mathematical statistics

Parts of the Book and Corresponding Chapters

The general trend seems clear. Details are more difficult to predict. Accordingly, students need solid knowledge of basic principles, methods and results, and a clear perception of what engineering mathematics is all about, in all three phases of solving problems:

- **Modeling:** Translating given physical or other information and data into mathematical form, into a mathematical *model* (a differential equation, a system of equations or some other expression).
- **Solving:** Obtaining the solution by selecting and applying suitable mathematical methods, and in most cases doing numerical work on a computer.
- **Interpreting:** Understanding the meaning and the implications of the mathematical solution for the original problem in terms of physics—or whereever the problem comes from.

It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead, it is important that students become familiar with ways to think mathematically, recognize the need for applying mathematical methods to engineering problems, realize that mathematics is a systematic science built on relatively few basic concepts and involving powerful unifying principles, and get a firm grasp for the interrelation between theory, computing and experiment.

The rapid ongoing developments just sketched have led to many changes and new features in the present edition of this book, causing it to differ very substantially from previous editions.

Changes and New Features Throughout the Book

The book has been simplified by rewriting various sections in a more detailed and leisurely fashion and by placing more emphasis on applications, algorithms and examples.

We first list some major changes and additions pertaining to the book as a whole and then some of the many changes and additions in individual chapters.

- **Problem sets** changed and expanded to contain *over 6000 carefully selected problems,* including more applied problems and more routine problems
- **Chapter review problems** added, to give students practice in choosing a method from the great variety of methods in a whole chapter
- **Worked-out examples** increased to ***over 600***, for help in problem solving and better understanding of the text
- **Key formulas *boxed***
- **Chapter summaries** added, for quick orientation and survey of the most important facts in each chapter

Changes in chapters are listed on the next page.

Changes and New Features in Chapters

- **Ordinary differential equations** (Chaps. 1–4): More systematical treatment of *integrating factors* (Sec. 1.6). *Linear differential equations* (Chap. 2) cast in simpler and more logical form. *Frobenius method* (Sec. 4.4) greatly simplified.
- **Laplace transformation** (Chap. 5). New: shifted data problems, impulsive forces, ***Dirac's delta,*** list of general formulas, (in addition to the list of transforms)
- **Matrices** (Chap. 7): More *applications* (Markov processes, Leslie matrices, etc.). More on *eigenvalues* and *diagonalization*. Additional modern numerical methods (see below)
- **Vector differential and integral calculus** (Chaps. 8, 9) streamlined by omitting some material of minor interest or making it optional. Grad, div, curl now close together; their forms in curvilinear coordinates (new). Greater emphasis on the types of integrals needed in the integral theorems in Chap. 9.
- **Fourier transformation, Fourier sine and cosine transformations** (Secs. 10.10–10.12, new) with applications to partial differential equations (Sec. 11.14)
- **Complex analysis** (Chaps. 12–17) reorganized to make it more teachable: **1.** *Mappings* by elementary functions added to Chap. 12 (Sec. 12.9). **2.** *Conformal mapping* moved to Chap. 16, to have it close to its applications in Chap. 17 on potential theory, which has been extended by stationary heat problems, etc.
  3. The lengthy introductory chapter on series now reduced to two sections that precede the discussion of power, Taylor and Laurent series.
  4. More on evaluating real integrals by complex integration.
- **Numerical methods** (Chaps. 18–20) modernized throughout, by adding new and more detailed ***algorithms*** and discussing more worked-out examples, by including ***computer-related aspects,*** on operations count, pivoting, numerical stability, rounding errors etc.; by giving more extensive treatments of *Newton interpolation, splines, LU-factorization* (Doolittle, Crout, Cholesky), and adding new material, such as *matrix norms, condition numbers, matrix deflation* and *tridiagonalization, QR, spectral shift,* etc.
- **Graph theory:** A new self-contained chapter (Chap. 22) on *graphs* and *digraphs* and their application in ***combinatorial optimization*** (traveling salesman and other *shortest path problems, shortest spanning trees, network flows, matching,* etc.).
- **Probability and statistics** (Chaps. 23, 24) reorganized by moving sections on sampling to Chap. 24.
- **References** (App. 1) updated and extended, notably those on numerical methods and optimization
- **Auxiliary material added:** Review of partial derivatives (App. 3.2), real series (App. 3.3), first-aid kits of differentiation formulas and integrals, conversion table, Greek alphabet (all on the inside covers).

Suggestions for Courses: A Four-Semester Sequence

The material may be taken in sequence and is suitable for four consecutive semester courses, meeting 3–5 hours a week:

First semester. Ordinary differential equations (Chaps. 1–5)
Second semester. Linear algebra and vector analysis (Chaps. 6–9)
Third semester. Complex analysis (Chaps. 12–17)
Fourth semester. Numerical methods (Chaps. 18–20)

For the remaining chapters, see below. Possible interchanges are obvious; for instance, numerical methods could precede complex analysis, etc.

Suggestions for Courses: Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week; for example:

Introduction to ordinary differential equations (Chaps. 1, 2)
Laplace transformation (Chap. 5)
Vector algebra and calculus (Chaps. 6, 8)
Matrices and systems of linear equations (Chap. 7)
Fourier series and partial differential equations (Chaps. 10, 11, Secs. 20.4–20.7)
Introduction to complex analysis (Chaps. 12–15)
Numerical analysis (Chaps. 18, 20)
Numerical linear algebra (Chap. 7 for review, Chap. 19)
Optimization (Chaps. 21, 22)
Graphs and combinatorial optimization (Chap. 22)
Probability and statistics (Chaps. 23, 24)

General Features of This Edition

The selection, arrangement and presentation of the material has been made with greatest care, based on past and present teaching, research and consulting experience. Some major features of the book are these:

The book is **self-contained,** except for a few clearly marked places where a proof would be beyond the level of a book of the present type and a reference is given instead. Hiding difficulties or oversimplifying would be of no real help to students.

The presentation is **detailed,** to avoid irritating readers by frequent references to details in other books.

The examples are **simple,** to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?

The notations are **modern and standard,** to help students read articles in journals or other *modern* books and understand other mathematically oriented courses.

The chapters are largely **independent,** providing flexibility in teaching special courses (see above).

The end of a proof is marked by ▌. This sign is also used at the end of some of the definitions and at the end of examples followed by further text.

Acknowledgment

I am indebted to many of my former teachers, colleagues and students who directly or indirectly helped me in preparing this book, in particular the present edition of it. Various parts of the manuscript were distributed to my classes in mimeographed form and returned to me with suggestions for improvement. Discussions with engineers and mathematicians (as well as written comments) were of great help to me; I want to mention particularly Professors S. L. Campbell, J. T. Cargo, P. L. Chambré, V. F. Connolly, A. Cronheim, J. Delany, J. W. Dettman, D. Dicker, D. Ellis, W. Fox, R. G. Helsel, W. N. Huff, J. Keener, E. C. Klipple, V. Komkow, H. Kuhn, G. Lamb, H. B. Mann, I. Marx, K. Millet, J. D. Moore, W. D. Munroe, J. N. Ong, Jr., P. J. Pritchard, H.-W. Pu, W. O. Ray, P. V. Reichelderfer (who helped me very much with the new Chap. 22) J. T. Scheick, H. A. Smith, J. P. Spencer, J. Todd, H. Unz, A. L. Villone, H. J. Weiss, A. Wilansky, C. H. Wilcox, L. Zia, A. D. Ziebur, all from this country, Professors H. S. M. Coxeter and R. Vaillancourt and Mr. H. Kreyszig (whose computer expertise was of great help in Chaps. 18–20) from Canada, and Professors H. Florian, F. Hohenberg, M. Kracht, F. Reutter, C. Schmieden, H. Unger, H. Wielandt, all from Europe. I can offer here only an inadequate acknowledgment of my appreciation.

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Suggestions of many readers were evaluated in preparing the present edition. Any further comment and suggestion for improvement of the book will be gratefully received.

ERWIN KREYSZIG

CONTENTS