

R. J. Glauber

Quantum Theory of Optical Coherence

Selected Papers and Lectures

It is convenient to allow $P(\alpha)$ to have delta function singularities so that we may think of a pure coherent state as represented by a special case of Eq. (7.3). A real-valued two-dimensional delta function which is suited to this purpose may be defined as

$$\delta^{(2)}(\alpha) = \delta(\text{Re } \alpha) \delta(\text{Im } \alpha).$$

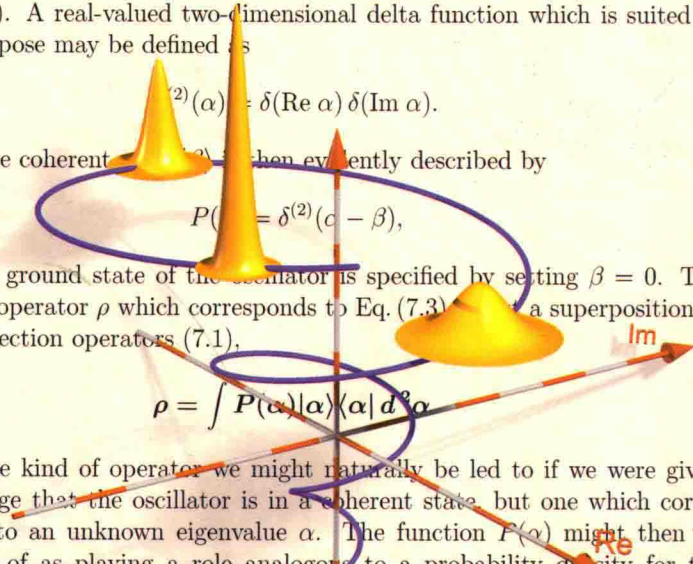
The pure coherent state is then evidently described by

$$P(\alpha) = \delta^{(2)}(\alpha - \beta),$$

and the ground state of the oscillator is specified by setting $\beta = 0$. The density operator ρ which corresponds to Eq. (7.3) is a superposition of the projection operators (7.1),

$$\rho = \int P(\alpha) |\alpha\rangle \langle \alpha| d^2\alpha$$

It is the kind of operator we might naturally be led to if we were given knowledge that the oscillator is in a coherent state, but one which corresponds to an unknown eigenvalue α . The function $P(\alpha)$ might then be thought of as playing a role analogous to a probability density for the distribution of values of α over the complex plane. Such an interpretation may, as we shall see, be justified at times. In general, however, it is not possible to interpret the function $P(\alpha)$ as a probability distribution in any precise way since the projection operators $|\alpha\rangle \langle \alpha|$ with which it is associated are not orthogonal to one another for different values of α .



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1807–2007 Knowledge for Generations

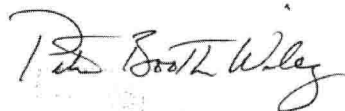
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Foreword

It was in the first days of January 1944 that a young man of nineteen years climbed out of the train at the lonely train station in Lamy, New Mexico, where the Santa Fe Railroad comes closest to the city of Santa Fe. The young man was not the only one getting out of the train; he was joined by a pudgy gentleman. However, it was only when both approached 109 East Palace Avenue in Santa Fe that they realized they were heading in the same direction. The older man registered with Dorothy McKibbin, who was running the place, using the name of Jonny Newman. The younger one wrote the name Roy Glauber into the book. The first was the famous mathematician, physicist and computer scientist John von Neumann, the second one had used his real name. Their common destination was Los Alamos.

Why, you may well ask, is a nineteen year old being recruited for the Manhattan project? Simple – Roy Glauber was a child prodigy. He constructed his first telescope as a pre-teenager, won the Westinghouse Prize and entered Harvard at the age of sixteen. There he studied physics under several notables and by the age of nineteen had learned most of what was known in physics at the time.

Nuclear Physics and First Experience

During his one and a half years in Los Alamos Roy worked in the group of Robert Serber on neutron diffusion. He was also close to Richard Feynman who was directing a small theory department. In this way Roy had first-hand experience of many of Feynman's pranks which are nowadays common knowledge due to the book *Surely you are joking Mr. Feynman*.

On the occasion of the Trinity test shot near Alamogordo, NM, there was only room for the senior staff (such as Fermi). So our resourceful hero Roy teamed up with a group to view the event on Sandia peak some 100 miles from the test site. The appointed hour came and they saw – nothing. After having waited a while longer, they decided that the bomb hadn't worked after all. So they started packing up for the trip back to Los Alamos. Just then the southern sky lit up with an eerie purple glow; you know the rest of the story.

After his time in Los Alamos, Roy started to work on his PhD thesis on meson theory under the guidance of Julian Schwinger (Nobel Prize in physics 1965) at Harvard. After graduating in the fall of 1949 he received a grant to go to the Institute of Advanced Studies in Princeton. A couple of months earlier he had realized, in a hotel in Berkeley, an interesting relation concerning the multiplication of two exponential operators which contain creation and annihilation operators. This identity impressed the director of the institute, Robert Oppenheimer, enormously. Also the book Messiah's on quantum mechanics refers to Glauber in this context. Later, this relation turned out to be very important in the context of coherent states.

During his first year in Princeton, Roy met Wolfgang Pauli who had come to visit the Institute. Pauli offered Roy a position for the summer of 1950. He accepted and traveled to Europe with the luxury liner *Île de France*. Even today Roy keeps talking about the impressive life that was taking place on this ship: champagne, cocktails and an unusual wealth of dishes at any time of the day or night and a plentitude of young women on their pilgrimage to Europe in the Holy Year of 1950. His first stop in the Old World was Paris, where a large European physics conference was held at the Institute Poincaré. On this occasion he had his first experience of Pauli's direct way of dealing with people. Roy had caught a cold during his first days in Paris and had lost his voice. When he saw Pauli in Paris he could barely say "I have lost my voice". Pauli with a huge smile answered "Yeh, you have lost your voice and you don't look so good either".

After the meeting, Roy took a few days off and enjoyed the life in Paris. Since he had never been to this town before, "The postcards came to life", as he used to call it later. For this reason he arrived in Zürich a few weeks after Pauli. His mother had been worried about her son since she hadn't heard from him since he had left America. For this reason she had written a letter to Prof. Pauli asking him about Roy. Pauli immediately forced Roy to write a letter to his mother. Later Pauli used every opportunity to tease Roy about his mother's concern. He often interrupted his lectures when he saw Roy entering the class room to ask "and how is your mother doing?".

Another example of the impish nature of Pauli showed up during an excursion of his department to Stansstad. In the late afternoon the group played soccer and some people were swimming in the lake. From time to time Pauli kicked the ball purposefully into the lake and the students and assistants had to swim out to get it back. Finally he took aim at Roy, who had carried around a large camera all afternoon and was now taking pictures of Pauli during the soccer game. There exists a remarkable photograph of Pauli as he kicks the ball towards the camera. Moments later Roy was knocked down to the floor because of the transfer of momentum.

A New Scattering Approximation

In the summer of 1951, Roy got his first permanent position. Caltech had just hired Feynman from Cornell. However, he wanted to spend his first year on a Sabbatical in

Brazil. It is well known that Feynman learned how to play the bongo drums during this visit. Roy substituted for Feynman and took over his class in quantum mechanics. On this occasion he also collaborated, in the spring of 1952, with the quantum chemistry group of Linus Pauling. Here he became interested in the scattering of electrons from molecules. The experiments of V. Schomaker showed some rather remarkable results which could not be explained by first-order perturbation theory, i.e. by the Born approximation. For this reason Roy developed a new approximation which is also valid at short wavelengths. This theory has found applications in many branches of physics and is known in scattering theory as “Glauber approximation”.

In the meantime, Schwinger had negotiated with the administration at Harvard that an assistant should be attached to him. Roy was offered this position and returned to Harvard. After the first year he became Assistant Professor and stayed at Harvard until today. Since 1976 he is Mallinckrodt Professor. In the following years Roy dedicated much of his time to complete his scattering theory. Of central importance in this context is his discovery of diffractive dissociation.

The Birth of Quantum Optics

In the early sixties Roy was drawn into a completely different topic. He developed the quantum theory of optical coherence and photon detection, for which he was finally awarded the Nobel Prize in 2005. In this way he laid the foundations for modern quantum optics.

The quantum theory of radiation had already been developed in the early days of quantum mechanics by Max Born, Pascual Jordan, Werner Heisenberg, and Paul A. M. Dirac. Nevertheless, this theory could not provide a quantitative description of physical processes since it contained singularities. It was not until 1947 and the experiments of Willis E. Lamb and Polykarp Kusch on the shift of the energy levels of the hydrogen atom and the anomalous magnetic moment of the electron that it was generally accepted that light needs to be treated quantum-mechanically. These revolutionary experiments led to the development of quantum electrodynamics. However, conventional wisdom still believed that quantization of the radiation field had little relevance for optical processes.

In the mid-fifties, interferometric measurements of the size of distant stars accomplished by Robert Hanbury Brown and Richard Q. Twiss brought a dramatic turn of events. They used intensity correlations of photo currents from two spatially separated detectors and observed an enhancement when the difference of the optical wavelengths of the two signals disappeared. Our hero Roy interpreted this enhancement as a quantum effect. Thermal light comes in clumps and is therefore bunched. The probability to detect a photon immediately after another one has been found is higher than at a later moment. This photon bunching experiment with a thermal light source can be considered the birth of quantum optics.

Quantum Theory of Optical Coherence

After the discovery of the maser and the laser in the early sixties new ideas for quantum effects of the radiation field were in the air. However, there was no theory for their observation. It was only in 1963 that Roy had developed the quantum theory of optical coherence. Here the concept of a coherence state plays a central role. Coherent states had been proposed for the first time by Erwin Schrödinger in 1927 in order to show that a wavepacket needs not always be bound to spread. The coherent state became the crucial tool for Roy's theory of optical coherence. In particular, he could show that for coherent fields all correlation functions factorize. As a consequence, the intensity correlations are independent of the delay time. This effect is also observed using laser light.

However, besides photon bunching there is also the opposite effect, i.e. anti-bunching. For short delay times the intensity correlations vanish. The probability to detect two photons right after each other vanishes. Anti-bunched light appears for example in the resonance fluorescence of a single atom and was detected independently by the groups of Leonhard Mandel (Rochester) and Herbert Walther (Garching). In these cases, the light is in a non-classical state. The characterization of non-classical light fields relies on coherent states and the introduction of distributions in the quantum mechanical phase space. They provide a bridge between the classical and the quantum mechanical description of the radiation field.

The Glauber representation of the density operator in terms of coherent states was also important for understanding the laser. The development of the quantum theory of the laser pursued by the groups of Willis Lamb and Marlan O. Scully (Yale), Melvin Lax (Bell Labs), and Hermann Haken with Hannes Risken (Stuttgart) and even the theory of the atom laser benefits from the early work of Roy. Indeed, the quantum theory of coherence is not restricted to light but can be applied to any bosonic field and, in particular, to a Bose–Einstein-Condensate (BEC). Recently, intensity correlations of a BEC have been measured and are in full agreement with theory. Moreover, with the help of Grassmann algebras Roy could also extend his theory to fermionic fields which can be tested using fermionic atoms. First experimental results have been reported.

The Old Question: What is a Photon?

During the Les Houches summer school of 1964, Lamb, who had received the Nobel Prize in 1955 for discovering the shift of the energy levels in a hydrogen atom, lectured on his semi-classical theory of the laser. In this formalism light is treated by using classical electrodynamics and matter by using quantum mechanics. In Lamb's theory there is no need for the concept of the photon. Despite this fact, many scientists in Les Houches were using the word "photon" even when they referred to an effect whose

explanation did not rely on the quantum theory of radiation. This misuse of the word “photon” annoyed Lamb and he introduced a licence which entitled its owner to use the word “photon”. Scientists without licence were not allowed to even mention photons. Roy was one of the very few colleagues who received such a licence from Lamb.

The problem of the photon is best summarized by a 1951 quote from Albert Einstein: “In 50 years of pondering, I still have not come closer to an answer for the question what light quanta are. Today every Tom, Dick and Harry thinks he knows, but he is wrong”. Roy’s theory of a photodetector has, however, furnished considerable progress in this direction. He could connect a light quantum in the field with a click in the detector. The relevant theory had to take into account that the absorption of a photon in the detector changes the state of the radiation field. This condition leads to the normal ordering of creation and annihilation operators of the field. Roy once jokingly summarized his theory of photo detection by the sentence: “I don’t know anything about photons, but I know one when I see one”. This sentence is reminiscent of the American Supreme Court Justice Potter Stewart who in 1964 was asked to define pornography and said: “I know it when I see it”.

The Teacher and the Human Being

Roy can be proud to have an impressive list of PhD students and Postdocs who themselves have started famous schools in theoretical physics. His PhD student Dan Walls who unfortunately died very early had established a highly successful school of quantum optics in New Zealand and Australia. In Germany it was Fritz Haake in Essen and Maciej Lewenstein in Hannover who propagated the Glauber fame. Roy has had always strong ties to Germany. He was a Humboldt awardee at the Max Planck Institute for Quantum Optics and has spent many summers at the University of Ulm.

We are also proud of Roy’s dedication to his children, Val and Jeff. As a single parent, Roy did all the hard work of running the home, being a soccer Dad and bringing up children. Doing all this while carrying his duties as Harvard Professor and father figure to us all, is indeed reason enough for a “Noble Prize”.

Roy has also a great sense of humor. For this reason he is often asked to give the after-dinner speech at conference banquets. An excellent illustration is his after-dinner speech at the *Nato Summer School on Gravitation and Squeezed States* in 1981. This meeting took place in the city of Bad Windsheim close to Nuremberg. Roy had noticed that the town had been promoted to the status of a spa after World War II and now carried the name “Bad Windsheim”. In his speech he remarked, “I have no idea when this city went bad, but we have had such a great time here that from now on we will call it Good Windsheim”.

September 2006
Marlan Scully, College Station, TX
Wolfgang P. Schleich, Ulm

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1

The Quantum Theory of Optical Coherence¹

1.1

Introduction

Correlation, it has long been recognized, plays a fundamental role in the concept of optical coherence. Techniques for both the generation and detection of various types of correlations in optical fields have advanced rapidly in recent years. The development of the optical maser, in particular, has led to the generation of fields with a range of correlation unprecedented at optical frequencies. The use of techniques of coincidence detection of photons^[1,2] has, in the same period, shown the existence of unanticipated correlations in the arrival times of light quanta. The new approaches to optics, which such developments will allow us to explore, suggest the need for a fundamental discussion of the meaning of coherence.

The present paper, which is the first of a series on fundamental problems of optics, is devoted largely to defining the concept of coherence. We do this by constructing a sequence of correlation functions for the field vectors, and by discussing the consequences of certain assumptions about their properties. The definition of coherence which we reach differs from earlier ones in several significant ways. The most important difference, perhaps, is that complete coherence, as we define it, requires that the field correlation functions satisfy an infinite succession of coherence conditions. We are led then to distinguish among various orders of incomplete coherence, according to the number of conditions satisfied. The fields traditionally described as coherent in optics are shown to have only first-order coherence. The fields, generated by the optical maser, on the other hand, may have a considerably higher order of coherence. A further difference between our approach and previous ones is that it is constructed to apply to fields of arbitrary time dependence, rather than just to those which are, on the average, stationary in time. We have also attempted to develop the discussion in a fully quantum theoretical way.

It would hardly seem that any justification is necessary for discussing the theory of light quanta in quantum theoretical terms. Yet, as we all know, the successes of classical theory in dealing with optical experiments have been so great that we feel no hesitation in introducing optics as a sophomore course. The quantum theory, in other

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