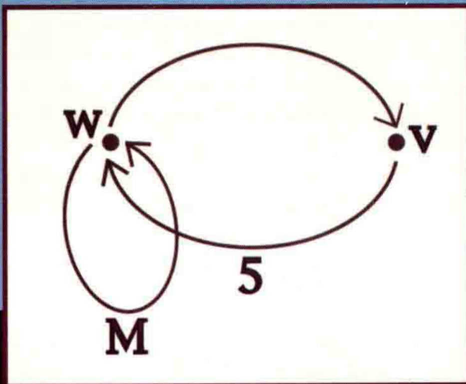
James W. Garson



MODAL LOGIC for PHILOSOPHERS

Second Edition

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Modal Logic for Philosophers

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This book on modal logic is especially designed for philosophy students. It provides an accessible yet technically sound treatment of modal logic and its philosophical applications. Every effort is made to simplify the presentation by using diagrams instead of more complex mathematical apparatus. These and other innovations provide philosophers with easy access to a rich variety of topics in modal logic, including a full coverage of quantified modal logic, non-rigid designators, definite descriptions, and the *de-re de-dicto* distinction. Discussion of philosophical issues concerning the development of modal logic is woven into the text. The book uses natural deduction systems, which are widely regarded as the easiest to teach and use. It also includes a diagram technique that extends the method of truth trees to modal logic. This provides a foundation for a novel method for showing completeness that is easy to extend to quantifiers.

This second edition contains a new chapter on logics of conditionals and an expanded bibliography, and is updated throughout. A number of technical results have also been clarified and streamlined.

James W. Garson is Professor of Philosophy at the University of Houston. His research interests include logic, especially modal logic, the philosophy of mind, neural networks, formal semantics, natural language processing, and philosophical issues concerning the impact of information technology. He has held grants from the National Endowment for the Humanities, the National Science Foundation, and the Apple Education Foundation to study the use of computers in education and to develop software for training students in logic and computer science. He is the author of numerous articles in logic, semantics, linguistics, the philosophy of cognitive science, and computerized education. His review article on quantified modal logic in the *Handbook of Philosophical Logic* is a standard reference in the area. His new book, *What Logics Mean: From Proof Theory to Model-Theoretic Semantics*, is forthcoming from Cambridge University Press.

for Nuel Belnap, who is responsible for anything he likes about this book

Preface to the Second Edition

In the years since the first publication of *Modal Logic for Philosophers*, I have received many suggestions for its improvement. The most substantial change in the new edition is a response to requests for a chapter on logics for conditionals. This topic is widely mentioned in the philosophical literature, so any book titled "Modal Logic for Philosophers" should do it justice. Unfortunately, the few pages on the topic provided in the first edition did no more than whet the reader's appetite for a more adequate treatment. In this edition, an entire chapter (Chapter 20) is devoted to conditionals. It includes a discussion of material implication and its failings, strict implication, relevance logic, and (so-called) conditional logic. Although this chapter still qualifies as no more than an introduction, I hope it will be useful for philosophers who wish to get their bearings in the area.

While the structure of the rest of the book has not changed, there have been improvements everywhere. Thanks to several classes in modal logic taught using the first edition, and suggestions from attentive students, a number of revisions have been made that clarify and simplify the technical results. The first edition also contained many errors. While most of these were of the minor kind from which a reader could easily recover, there were still too many where it was difficult to gather what was intended. A list of errata for the first edition has been widely distributed on the World Wide Web, and this has been of some help. However, it is time to gather these corrections together to produce a new edition where (I can hope) the remaining errors are rare.

I am grateful to the authors of the many messages I have received concerning the first edition, which are far too numerous to list here. I am also

indebted to my student Alireza Fatollahi and especially to my colleague Gregory Brown, who joined with me in a semester-long collaboration covering just about every part of the first edition. Their sharp eyes and helpful suggestions made invaluable contributions to the new edition.

Preface

The main purpose of this book is to help bridge a gap in the landscape of modal logic. A great deal is known about modal systems based on propositional logic. However, these logics do not have the expressive resources to handle the structure of most philosophical argumentation. If modal logics are to be useful to philosophy, it is crucial that they include quantifiers and identity. The problem is that quantified modal logic is not as well developed, and it is difficult for the student of philosophy who may lack mathematical training to develop mastery of what is known. Philosophical worries about whether quantification is coherent or advisable in certain modal settings partly explain this lack of attention. If one takes such objections seriously, they exert pressure on the logician to either eliminate modality altogether or eliminate the allegedly undesirable forms of quantification.

Even if one lays those philosophical worries aside, serious technical problems must still be faced. There is a rich menu of choices for formulating the semantics of quantified modal languages, and the completeness problem for some of these systems is difficult or unresolved. The philosophy of this book is that this variety is to be explored rather than shunned. We hope to demonstrate that modal logic with quantifiers can be simplified so that it is manageable, even teachable. Some of the simplifications depend on the foundations – in the way the systems for propositional modal logic are developed. Some ideas that were designed to make life easier when quantifiers are introduced are also genuinely helpful even for those who will study only the propositional systems. So this book can serve a dual purpose. It is, I hope, a simple and accessible introduction to propositional modal logic for students who have had a first course in formal logic (preferably one that covers natural deduction rules and truth trees). I hope, however, that students who had planned to use this book to learn only propositional modal logic will be inspired to move on to study quantification as well.

A principle that guided the creation of this book is the conviction that visualization is one of the most powerful tools for organizing one's thoughts. So the book depends heavily on diagrams of various kinds. One of the central innovations is to combine the method of Haus diagrams (to represent Kripke's accessibility relation) with the truth tree method. This provides an easy and revealing method for checking validity in a wide variety of modal logics. My students have found the diagrams both easy to learn and fun to use. I urge readers of this book to take advantage of them.

The tree diagrams are also the centerpiece for a novel technique for proving completeness – one that is more concrete and easier to learn than the method of maximally consistent sets, and one that is extremely easy to extend to the quantifiers. On the other hand, the standard method of maximally consistent sets has its own advantages. It applies to more systems, and many will consider it an indispensable part of anyone's education in modal logic. So this book covers both methods, and it is organized so that one may easily choose to study one, the other, or both.

Three different ways of providing semantics for the quantifiers are introduced in this book: the substitution interpretation, the intensional interpretation, and the objectual interpretation. Though some have faulted the substitution interpretation on philosophical grounds, its simplicity prompts its use as a centerpiece for technical results. Those who would like a quick and painless entry to the completeness problem may read the sections on the substitution interpretation alone. The intensional interpretation, where one quantifies over individual concepts, is included because it is the most general approach for dealing with the quantifiers. Furthermore, its strong kinships with the substitution interpretation provide a relatively easy transition to its formal results. The objectual interpretation is treated here as a special case of the intensional interpretation. This helps provide new insights into how best to formalize systems for the objectual interpretation.

The student should treat this book more as a collection of things to do than as something to read. Exercises in this book are found embedded throughout the text rather than at the end of each chapter, as is the more common practice. This signals the importance of doing exercises as soon as possible after the relevant material has been introduced. Think of the text between the exercises as a preparation for activities that are the foundation for true understanding. Answers to exercises marked with a star (*) are found at the end of the book. Many of the exercises also include hints. The best way to master this material is to struggle through the exercises on your own as far as humanly possible. Turn to the hints or answers only when you are desperate.

Many people should be acknowledged for their contributions to this book. First of all, I would like to thank my wife, Connie Garson, who has unfailingly and lovingly supported all of my odd enthusiasms. Second, I would like to thank my students, who have struggled through the many drafts of this book over the years. I have learned a great deal more from them than any of them has learned from me. Unfortunately, I have lost track of the names of many who helped me make numerous important improvements, so I apologize to them. But I do remember by name the contributions of Brandy Burfield, Carl Feierabend, Curtis Haaga, James Hulgan, Alistair Isaac, JoBeth Jordon, Raymond Kim, Kris Rhodes, Jay Schroeder, Steve Todd, Andy Tristan, Mako Voelkel, and especially Julian Zinn. Third, I am grateful to Johnathan Raymon, who helped me with the diagrams. Finally, I would like to thank Cambridge University Press for taking an interest in this project and for the excellent comments of the anonymous readers, some of which headed off embarrassing errors.

Contents

Introduction: What Is Modal Logic?

Strictly speaking, modal logic studies reasoning that involves the use of the expressions 'necessarily' and 'possibly'. The main idea is to introduce the symbols □ (necessarily) and ◊ (possibly) to a system of logic so that it is able to distinguish three different *modes* of assertion: □ A (A is necessary), A (A is true), and ◊ A (A is possible). Introducing these symbols (or operators) would seem to be essential if logic is to be applied to judging the accuracy of philosophical reasoning, for the concepts of necessity and possibility are ubiquitous in philosophical discourse.

However, at the very dawn of the invention of modal logics, it was recognized that necessity and possibility have kinships with many other philosophically important expressions. So the term 'modal logic' is also used more broadly to cover a whole family of logics with similar rules and a rich variety of different operators. To distinguish the narrow sense, some people use the term 'alethic logic' for logics of necessity and possibility. A list describing some of the better known of these logics follows.

System	Symbols	Expression Symbolized
Modal logic	□	It is necessary that
(or Alethic logic)	◊	It is possible that
Tense logic	G	It will always be the case that
	F	It will be the case that
	H	It has always been the case that
	P	It was the case that
Deontic logic	O	It is obligatory that
	P	It is permitted that
	F	It is forbidden that

Locative logic	Tx	It is the case at x that
Doxastic logic	Bx	x believes that
Epistemic logic	Kx	x knows that

This book will provide you with an introduction to all these logics, and it will help sketch out the relationships among the different systems. The variety found here might be somewhat bewildering, especially for the student who expects uniformity in logic. Even within the above subdivisions of modal logic, there may be many different systems. I hope to convince you that this variety is a source of strength and flexibility and makes for an interesting world well worth exploring.