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Mathematics*

**Gerry Stahl**

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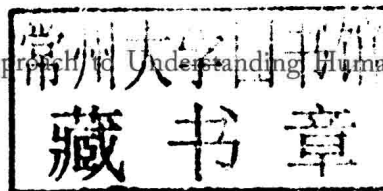
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Gerry Stahl

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# Translating Euclid

## *Designing a Human-Centered Mathematics*

Gerry Stahl

The iSchool & the Math Forum, Drexel University

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## ABSTRACT

*Translating Euclid* reports on an effort to transform geometry for students from a stylus-and-clay-tablet corpus of historical theorems to a stimulating computer-supported collaborative-learning inquiry experience.

The origin of geometry was a turning point in the pre-history of informatics, literacy, and rational thought. Yet, this triumph of human intellect became ossified through historic layers of systematization, beginning with Euclid's organization of the *Elements* of geometry. Often taught by memorization of procedures, theorems, and proofs, geometry in schooling rarely conveys its underlying intellectual excitement. The recent development of dynamic-geometry software offers an opportunity to translate the study of geometry into a contemporary vernacular. However, this involves transformations along multiple dimensions of the conceptual and practical context of learning.

*Translating Euclid* steps through the multiple challenges involved in redesigning geometry education to take advantage of computer support. Networked computers portend an interactive approach to exploring dynamic geometry as well as broadened prospects for collaboration. The proposed conception of geometry emphasizes the central role of the construction of dependencies as a design activity, integrating human creation and mathematical discovery to form a human-centered approach to mathematics.

This book chronicles an iterative effort to adapt technology, theory, pedagogy and practice to support this vision of collaborative dynamic geometry and to evolve the approach through on-going cycles of trial with students and refinement of resources. It thereby provides a case study of a design-based research effort in computer-supported collaborative learning from a human-centered informatics perspective.

## KEYWORDS

Computer-Supported Collaborative Learning (CSCL), Design-Based Research (DBR), Virtual Math Teams (VMT), group cognition, dynamic geometry, post-cognitive philosophy, interaction analysis, creative discovery, multi-user software, interactional resources.

# Acknowledgments

The effort reported here to develop a contemporary approach to the learning of collaborative dynamic geometry has itself been a collaborative learning experience. The interdisciplinary, international VMT research team from 2003–2009 is documented in (Stahl, 2009, esp. pp. xix–xxi). In its current phase from 2011–2016, the project is a collaboration among the iSchool at Drexel, the Math Forum at Drexel, the School of Education at Drexel, and the Department of Urban Education at Rutgers-Newark.

Since the conception of the Virtual Math Teams (VMT) Project in 2002, the project has been a major research effort at the Math Forum, a pioneer online resource for math education. Stephen Weimar, the Director of the Math Forum, has contributed generously of his deep understanding of math education and collaboration practices, participating in the weekly project meetings for a decade.

The recent project team includes HCI faculty from the iSchool: Sean Goggins, Michael Khoo, Andrea Forte, and Jennifer Rode. Professional development of mathematics teachers is conducted by Arthur Powell (Urban Education at Rutgers-Newark), Loretta Dicker (Rutgers), Jason Silverman (Director, Graduate Programs in Mathematics Learning and Teaching at Drexel's School of Education), and Annie Fetter (Math Forum). In addition, Dragana Martinovic (Windsor University, Canada) and Diler Öner (Bogazici Universitesi, Turkey) are visiting researchers on the project.

Carolyn Rosé (HCI Institute, Carnegie-Mellon University) and her students have collaborated in experiments with software discourse agents in VMT. Math Forum programmers Baba Weusijana and Jimmy Xiantong Ou developed the VMT software in 2009 and 2010. Since then, Math Forum programmer Anthony Mantoan has been the VMT software developer. Drexel iSchool students contributing to the project include Nora McDonald, Rachel Magee, Christopher Mascaro, and Robert Hedges-Goettl; Alan Black is developing learning analytics and visualization tools.

During the past year, the Project offered a ten-week teacher professional-development course on Collaborative Dynamic Mathematics to 24 math teachers to prepare them to use the VMT software in their schools. Twelve of the teachers then formed small online groups of students, who each engaged in eight hour-long sessions of dynamic mathematics. The teachers and students provided the Project with invaluable experiences, thoughtful reflections and rich data. They confirmed in many ways that the translation of Euclid is promising, inspiring and productive.

The VMT Project has been supported by the following grants from the US National Science Foundation and the Office of Naval Research:

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Although the VMT Project has been a collaborative effort, the author is solely responsible for the views expressed in this book.

*Gerry Stahl, VMT Project Director and PI  
Philadelphia, March 16, 2013*

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