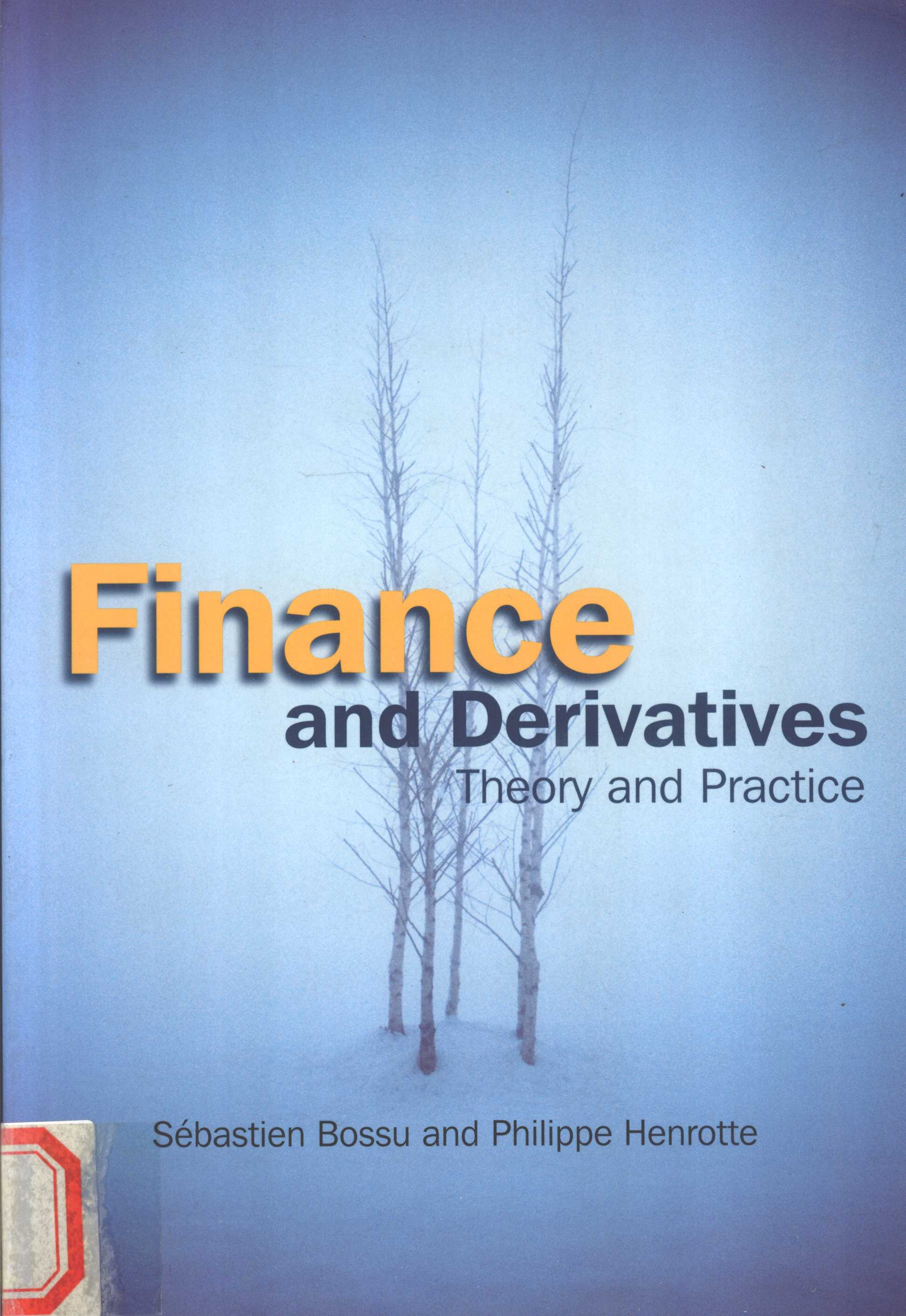

Finance and Derivatives

Theory and Practice

Sébastien Bossu and Philippe Henrotte

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Preface

This is the revised edition in English of *Exercices de Finance des Marchés*, published in Paris by Dunod in 2002. The first textbook of its kind to compile course notes and exercises with solutions in the field of quantitative finance, it is designed as a complement to an MBA sequence in finance or an MS class in financial mathematics. It may also serve as a self-teaching guide for beginning practitioners and confirmed financiers who wish to catch up with financial theory and its applications.

From basic notions on interest rates to option pricing theory through portfolio optimization, we wanted this book to cover the essential theoretical and practical knowledge in the field, and yet be accessible to neophytes. As such, the only required background is an undergraduate level in mathematics – no prior knowledge in finance is assumed, except for a general understanding of basic economic concepts found in the press. A probability and calculus review is included in an appendix for those readers who need to refresh their memory in these particular areas.

We had to make some choices on the extent of the concepts we could cover, and the level of mathematical machinery we could employ. Our approach was to focus on first principles such as arbitrage or the risk-return trade-off; key concepts such as yield or Greek letters; and the major numerical methods such as binomial trees or Monte-Carlo simulations. We strived to keep the course notes as concise and straightforward as possible, while covering a fair amount of fundamental results illustrated with 'real-life' examples. We left non-essential proofs or advanced concepts in the exercises of higher difficulty, which we identified with an asterisk (*).

The result is ten chapters in ascending order of difficulty, which we recommend you to read in sequence:

- Chapters 1 and 2 deal with the time value of money and correspond to a core finance course;
- Chapters 3 and 5 cover bonds, arbitrage and risk-return, which are usually taught in an advanced core finance course;

- Chapters 4 and 6 introduce derivative securities and their valuation, which are taught in an advanced core finance course or in an elective;
- Chapters 7 and 8 cover the advanced valuation and management of options, corresponding to an intermediate MBA elective or a core MS course;
- Chapters 9 to 10 introduce continuous-time finance which would be taught in an advanced MBA elective or a core MS course.

We hope that our book will prove insightful and useful to students and practitioners. We are committed to continually improve successive editions and we will appreciate all feedback from our readers.

Foreword

As revolutions go, the one in finance over the last thirty years may not seem to be that dramatic. It's been quite a lengthy revolution, led by the least likely revolutionaries, mathematicians. But nevertheless this revolution has had an enormous impact on the financial markets, and on the entire world. Not so long ago the financial markets were governed by middle-aged men with undergraduate degrees in History, a wardrobe of Savile Row suits and dandruff. In their place are now the freshly minted Physics PhDs, in chinos and Ferraris.

The change in emphasis from finance being a social science to a more hardcore science began with Paul Samuelson, and later Fischer Black, Myron Scholes and Robert Merton. With this change, and because of the large financial rewards associated with the City and Wall Street, the number of people wanting to join in the fun has exploded. And all of these people are scrambling to learn the subject ASAP. This is where the book by Bossu and Henrotte comes in. *Finance and Derivatives* teaches all of the fundamentals of quantitative finance clearly and concisely without going into unnecessary technicalities. You'll pick up the most important theoretical concepts, tools and vocabulary without getting bogged down in arcane derivations or enigmatic theoretical considerations.

But Bossu and Henrotte have an important responsibility with so much money being controlled by spotty scientists just out of school. When educating people in this field you must always remember to emphasise that this is not really a hardcore science – that a model that worked for the past decade could stop working, tomorrow, with an impact which could cause financial earthquakes on a global scale. So, pick up the tools of the trade from this book, but also appreciate the experience of its authors, it's the practice that makes perfect, not the theory.

Paul Wilmott

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Contents

1 Interest rate

1.1 Measuring time

In finance the most common unit of time is the year, perhaps because it is one that everyone presumes to know well. Although, as we will see, the year can actually create confusion and give an edge to the better-informed investor. How many days are there in 1 year? 365. But what about the 366 days of a leap year? What fraction of a year does the first 6 months represent? Is it 0.5, or 181/365 (except, again, for leap years)?

Financial markets have regulations and conventions to answer these questions. The problem is that these conventions vary by country. Worse still, within a given country different conventions may be used for different financial products.

We leave it to readers to become familiar with these day count conventions while in this book we will use the following rule, which professionals call **30/360**. Note that the first day starts at noon and the last day ends at noon. Thus, there is only 1 whole day between 2 February 2007 and 3 February 2007.

Rule	*Result*	*Example*: from 15 January 2006 to 13 March 2009
1. Count the number of whole years	Y	3 (from 15 January 2006 to 15 January 2009)
2. Count the number of remaining months and divide by 12	M/12	1/12 (from 15 January 2009 to 15 February 2009)
3. Count the number of remaining days (the last day of the month counting as the 30th unless it is the final date) and divide by 360	D/360	28/360 (under the 30/360 convention there are 16 days from 15 February 2009 at noon to 1 March 2009 at noon and 12 days from 1 March 2009 at noon to 13 March 2009 at noon)
TOTAL	Y + M/12 + D/360	3 + 1/12 + 28/360 = 3.161111 . . .

From this rule, we can arrive at the following simplified measures:

Semester (half year)	0.5 year
Quarter	0.25 year
Month	1/12 year
Week	7/360 year
Day	1/360 year

In practice...

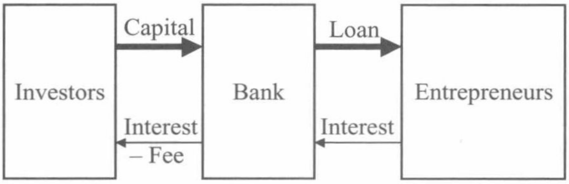
The Excel function DAYS360(*Start_date*, *End_date*) counts the number of days on a 30/360 basis.

1.2 Interest rate

In the economic sphere there are two types of agents whose *interests* are by definition opposed to each other:

- **Investors**, who have money and want that money to make them richer while they remain idle.
- **Entrepreneurs**, who don't have money but want to get rich actively using the money of others.

Banks help to reconcile these two interests by serving as an intermediary, placing the money of the investor at the entrepreneur's disposal and assuming the risk of bankruptcy. In exchange, the bank demands that the entrepreneur pay *interest* at regular intervals, which serves to pay for the bank's service and the investor's capital.



1.2.1 Gross interest rate

If I is the *total* interest paid on a capital K, the **gross interest rate** *over the considered period* is defined as:

$$r = \frac{I}{K}$$

Examples

- €10 of interest paid over 1 year on a capital of €200 corresponds to an annual gross interest rate of 5%.
- \$10 of interest paid *every year* for 5 years on a capital of \$200 corresponds to a 25% gross interest rate over 5 years, which is five times the annual rate in the preceding example.

We must emphasize that **an interest rate is meaningless if no time period is specified**; a 5% gross interest rate every 6 months is far more lucrative than every year.

This rate is called 'gross' because it does not take into consideration the **compounding of interest**, which is explained in the next section.

1.2.2 Compounding: compound interest rate

Hearing the question 'How much interest does one receive over 2 years if the annual interest rate is 10%?', a distressing proportion of individuals reply in a single cry: '20%!' However, the correct answer is 21%, because **interest produces more interest.** In fact, a good little capitalist, rather than foolishly spend the 10% interest paid by the bank after the first year, would immediately reinvest it the second year. Therefore, his total capital after 1 year is 110% of his initial investment on which he will receive 10% interest the second year. His gross interest over the 2-year period is thus: 10% + 10% × 110% = 21%.

More generally, starting with initial capital K one can build a **compounding table** of the capital at the end of each interest period:

Compounding table of capital *K* at interest rate *r* over *n* periods

Period	*Capital*	*Example*: $r = 10\%$
0	K	\$2000
1	$K(1+r)$	$2000 \times (1 + 10\%) = \$2200$
2	$K(1+r)^2$	$2200 \times (1 + 10\%) = \$2420$
...	...	...
n	$K(1+r)^n$	$2000 \times (1 + 10\%)^n$

From this table we obtain a formula for the amount of accumulated interest after n periods:

$$I_n = K(1+r)^n - K$$

We may now define the **compound interest rate** over n periods, corresponding to the total accumulated interest:

$$r^{[n]} = \frac{I_n}{K} = (1+r)^n - 1$$

(To avoid confusion we prefer the notation $r^{[n]}$ over r_n to indicate compounding over n periods, as r_n typically denotes a series of time-dependent variables.)

Example. The total accumulated interest over 3 years on an initial investment of \$2000 at a semi-annual compound rate of 5% is: $I_6 = 2000 \times (1 + 0.05)^6 - 2000 =$ \$680. The compound interest rate over 3 years (six semesters) is $r^{[6]} = 34\%$. Note that this result would be different with a 10% *annual* compound rate.

1.2.3 Conversion formula

Two compound interest rates over periods τ_1 and τ_2 are said to be **equivalent** if they satisfy:

$$\left[1 + r^{[\tau_1]}\right]^{\frac{1}{\tau_1}} = \left[1 + r^{[\tau_2]}\right]^{\frac{1}{\tau_2}}$$

Here τ_1 and τ_2 are two positive real numbers (for instance, $\tau_1 = 1.5$ represents a year and a half) and $r^{[\tau_1]}$ and $r^{[\tau_2]}$ are the equivalent interest rates over τ_1 and τ_2 years respectively.

This formula is very useful to convert a compound rate into a different period than the 'physical' interest payment period. A good way to remember it is to think that for a given investment all expressions of type $[1 + r^{[\text{period}]}]^{\text{frequency}}$ are equal, where frequency is the number of periods per year.

Example. An investment at a semi-annual compound rate of 5% is equivalent to an investment at a 2-year compound rate of:

$$r^{[2]} = (1 + 5\%)^{\frac{2}{0.5}} - 1 \approx 21.55\%$$

1.2.4 Annualization

Annualization is the process of converting a given compound interest rate into its annual equivalent. This allows one to rapidly compare the profitability of investments whose interests are paid out over different periods.

In this book, unless mentioned otherwise, all interest rates are understood to be on an annual basis or annualized. With this convention, the compound interest rate over T years can always be written as:

$$r^{[T]} = (1 + r^{[annual]})^T - 1$$

Example. The annualized rate equivalent to a semi-annual rate of 5% is:

$$r^{[1]} = (1 + 5\%)^{\frac{1}{0.5}} - 1 = 1.05^2 - 1 = 10.25\%$$

From which we obtain the 2-year compound rate found in the previous example:

$$r^{[2]} = (1 + 10.25\%)^2 - 1 \approx 21.55\%$$

1.3 Discounting

'*Time is money.*' In finance, this principle of the businessman has a very precise meaning: **a dollar today is worth more than a dollar tomorrow**. Two principal reasons can be put forward:

- **Inflation**: the increase in consumer prices implies that one dollar will buy less tomorrow than today.
- **Interest**: one dollar today produces interest between today and tomorrow.

With this principle in mind the next step is to determine **the value today of a dollar tomorrow** – or generally the **present value** of an amount received or paid in the future.

1.3.1 Present value

The **present value** of an amount C paid or received in T years is the equivalent amount that, invested today at the compound rate r, will grow to C over T years: $PV \times (1+r)^T = C$. Equivalently:

$$PV = \frac{C}{(1+r)^T}$$

Example. A supermarket chain customarily pays its suppliers with a 3-month delay. With a 5% interest rate the present value of a delivery today of €1 000 000 worth of goods paid in 3 months is:

$$\frac{1\,000\,000}{(1+5\%)^{0.25}} \approx €987\,877$$

The 3-month payment delay is thus implicitly equivalent to a €12 123 discount, or 1.21%.

Discounting is the process of computing the present value of various future cash flows. Similar to annualization, it is a key concept in finance as it makes amounts received or paid at different points in time comparable to what they are worth today. Thus, an investment which pays one million dollars in 10 years is 'only' worth approximately $614 000 assuming a 5% annual interest rate.

1.3.2 Discount rate and expected return

In practice, the choice of the **discount rate** r is crucial when calculating a present value and depends on the **expected return** of each investor. The minimum expected return for all investors is the interest rate offered by such 'infallible' institutions as central banks or government treasury departments. In the USA, the generally accepted benchmark rate is the yield[1] of the 10-year Treasury Note. In Europe, the 10-year Gilt (UK), OAT (France) or Bund (Germany) are used, and in Japan the 10-year JGB. However, an investor who is willing to take more risk should expect a higher return and use a higher discount rate r in her calculations. In investment banking it is not uncommon to use a 10–15% discount rate when assessing the profitability of such risky investments as financing a film production or providing seed capital to a start-up company.

[1] See Chapter 3 for the definition of this term.

Exercises

Exercise 1

Calculate, in years, the time that passes between 30 November 2006 and 1 March 2008 on a 30/360 basis. What is the annualized interest rate of an investment at a gross rate of 10% over this period?

Exercise 2: savings account

On 1 January 2005 you invested €1000 in a savings account. On 1 January 2006 the bank sent a summary statement indicating that you received a total of €40 in interest in 2005.

1. What is the gross annual interest rate of this savings account?
2. How much interest will you receive in 2006?
3. How much interest would you have received in 2005 if you had closed your account on 1 July 2005?

Your bank calculates and pays your interest every month based on your balance.

Exercise 3

Ten years ago you invested £500 in a savings account. The last bank statement shows a balance of £1030.52. What will your savings amount to in 10 years if the interest rate stays the same?

Exercise 4: from Russia with interest

You are a reputed financier and your personal credit allows you to borrow up to $100 000 at a rate of 6.5% (with a little bit of imagination). The annual interest rate offered on deposits by the Russian Central Bank is 150%. The exchange rate of the Russian ruble against the US dollar is 25 RUB/USD and your analysts believe that this exchange rate will remain stable during the coming year. Can you find a way to make money? Analyse the risks that you have taken.

Exercise 5

Sort the interest rates below from the most lucrative to the least lucrative:

(a) 6% per year;
(b) 0.5% per month;
(c) 30% every 5 years;
(d) 10% the first year then 4% the following 2 years.