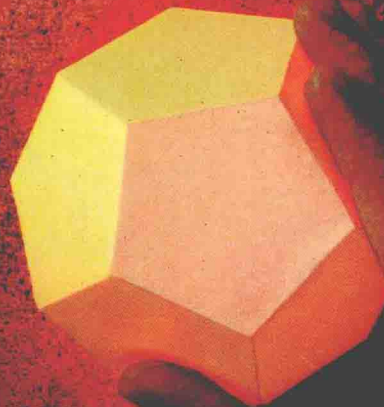


Discovering Geometry

AN INDUCTIVE APPROACH

MICHAEL SERRA



KEY CURRICULUM PRESS

Discovering Geometry™

An Inductive Approach

Second Edition

Michael Serra



KEY CURRICULUM PRESS
Innovators in Mathematics Education

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Foreword to the First Edition

Michael Serra has written a genuinely exciting geometry book. This book is unique in that the students actually create geometry for themselves as they proceed through the activities and the problems. Concepts are first introduced visually, then analytically, then inductively, and, finally, deductively. The spirit of this text is remarkably consistent with recent research on the development of geometric thinking in adolescents, particularly the levels of thinking in the van Hiele theory.

From the beginning, students participate in the construction of definitions. The author makes excellent use of a “nonexamples and examples” approach to encourage students to build their own definitions. As new geometric figures are introduced, activities are structured so that students can discover properties of these figures. Throughout the book, students are asked to make conjectures about figures and about relationships among figures. These conjectures are brought up again for deductive consideration in the chapters on proof.

In addition to a thorough treatment of all the topics anyone has ever wished to cover in a geometry course, Serra provides a number of extra nuggets and superimposes a creative developmental sequence to his topics. Measurement topics such as area and volume appear earlier than in many geometry texts, prior to formal proof. There is an extensive chapter on transformations and tessellations. The tessellations provide ample opportunity to concretely explore the symmetry patterns that make transformations so powerful. Where some texts pay lip service to these topics, Serra provides us with a full and varied menu. Excellent computer activities are sprinkled throughout the text in strategic places, providing an additional environment for student discoveries. Coordinate topics are woven in wherever appropriate. Most of all, however, this book has super problems, written by a master storyteller. Each chapter has one or more cooperative problem solving activities, which encourage interaction and communication among students. Students will have an opportunity to write and talk about their mathematics and thus will have a better chance of understanding their mathematics.

Formal proof does not appear in this book until the last two chapters. However, by the time students are asked to write proofs, they have already made conjectures for months, have formed and tested their own definitions, have solved logic problems, have developed visualization skills through drawings, constructions, and computer activities, and have studied logic, reasoning, and the nature of proof. *They are ready for proofs!*

In the past we have erred in pushing proofs on students too soon, before they had a handle on shapes and their properties. In the past we have asked many of our students to do two things simultaneously—learn geometric concepts and learn deductive reasoning. I applaud Michael Serra’s move to delay proof in his book until students have seen the whole spectrum of geometric concepts. Serra’s book gives students a better chance of learning geometry and of learning about proof.

Finally, this is a book for “doers.” Students constantly *do* things in this book, both alone and with other students. If you want your students and yourself to become actively involved in the process of learning and creating geometry, then this book is for you.

J. Michael Shaughnessy, Ph.D.
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Portland State University
Portland, Oregon

Foreword to the Second Edition

The first edition of Michael Serra's *Discovering Geometry* provided approaches to teaching and learning geometry that were radically different from those of other geometry textbooks. The second edition is even more exciting. Michael Serra continues to make geometry fun and interesting to both students and teachers.

Serra has added new and richer problem settings and projects that attract and hold the interest and attention of students. Added to the many high-interest fantasy problem settings from the first edition, the second edition includes more real-world applications and examples of geometry from many cultures.

The text can be implemented in a variety of instructional settings, providing opportunities for students to think, discuss, and work together as they actually *do* geometry. Students perform investigations to discover and make conjectures about the wonderful properties of geometry. In addition to traditional construction tools, students can use patty papers to streamline many investigations in this edition. Technology in the form of graphing calculators and computers has been integrated as needed and where appropriate.

This edition of *Discovering Geometry* also comes alive in brilliant color with new cartoons, illustrations, artwork, and pictures from real life and from around the world. Students and teachers will benefit from examples of students' work from Michael Serra's and other teachers' classrooms, which are shared throughout the text.

A big plus of this book, as in the first edition, is that Serra puts proofs in the last chapters. This allows students to first be involved in investigating and conjecturing before they are exposed to formal geometric proofs. This suggests a sequence that has proved to be appropriate and effective.

Michael Serra is to be commended for writing such a unique geometry book for teachers and students.

Bettye Forte
Executive Director, Northeast Instructional Support Team
Mathematics Director, Fort Worth Independent School District
Fort Worth, Texas

What Makes *Discovering Geometry* Different?

Features of *Discovering Geometry*

This book was designed so that you and your teacher can have fun with geometry. In *Discovering Geometry* you “learn by doing.” You will learn to use the tools of geometry and to perform geometric **investigations** with them. Your investigations will lead to geometric discoveries. Many of the geometric investigations are carried out in small **cooperative groups** in which you jointly plan and find solutions with other students. I think you will enjoy the **humorous illustrated word problems**. You will help archaeologist Ertha Diggs determine the height of a Mayan jungle temple, help pirate Captain Coldhart bury his treasure, and help Hemlock Bones solve the case of the Belgian stamp theft. I created these problems in the hope of reducing your anxiety about word problems. In the **projects** you will build geometric solids, make kaleidoscopes, design a racetrack, find the height of your school building, and create a mural. There are also several **graphing calculator investigations** and **Geometer’s Sketchpad® projects** in the text. In one Sketchpad™ project you will discover the world of a strange class of geometric shapes called fractals! Each chapter closes with a special **cooperative problem solving** activity set at a lunar colony of the future. I think you’ll enjoy the extra challenges in the Improving Visual Thinking Skills, Improving Reasoning Skills, and Improving Algebra Skills **puzzles** that I have sprinkled throughout the book.

I have designed the projects, the puzzles, and the calculator and computer activities so that you can do them independently, whether or not your class tackles them as a group. Read through them as you proceed through the book.

Chapter Sequence

I begin with a chapter on geometric art to show you that geometry is found in the art of cultures throughout the world. In Chapter 1, you will learn how to reason inductively. Inductive reasoning is the process that you will use to make geometric discoveries in this book. In Chapter 2, you will use inductive reasoning to create definitions of geometric terms. In Chapter 3, you will use tools of geometry to construct geometric figures. These tools include the compass, the straightedge, and patty papers, which you’ll use to make geometric discoveries in the remainder of the book. In Chapter 8, you will learn to create geometric tiling designs similar to some of M. C. Escher’s artwork shown on the chapter opening pages. Finally, in the last three chapters, you will learn about another type of reasoning, which is called deductive reasoning or proof.

Please do not feel overwhelmed by the number of chapters in *Discovering Geometry*. It is not possible to cover *all* of the chapters in one school year. Your teacher will guide you through the book to create one of several different types of geometry courses possible using this text.

Suggestions for Success

It is important to be organized when working with *Discovering Geometry*. Keep a notebook with a section for definitions, a section for new geometric discoveries, and a section for daily notes and exercises. Study your notebook regularly. You will need four tools of geometry for the investigations: a compass, a protractor, a straightedge, and a ruler. Keep a calculator handy.

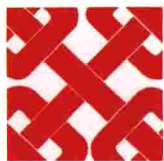
You will find hints and solutions for some key exercises in the back of the book. Exercises that have hints are identified by an asterisk (*). Try to solve the problems on your own without looking at the hints. Refer to the hints to check your method or as a last resort if you can't solve a problem.

Unlike most texts, *Discovering Geometry* will ask you to work cooperatively with your fellow students. This means you should pull your desks together and get to know one another. When you are working together cooperatively, always be willing to listen to one another, to be an active participant, to ask one another questions when you don't understand, and to help one another when asked. When working cooperatively, you can accomplish much more than the sum total of what you can accomplish individually. And best of all, you'll have less anxiety and a whole lot more fun.

Michael Serra
George Washington High School
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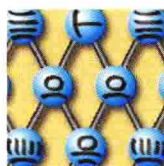
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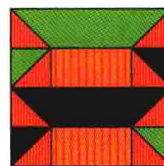
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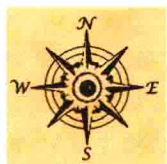
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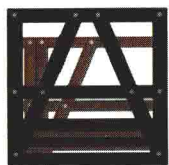
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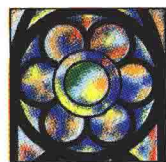
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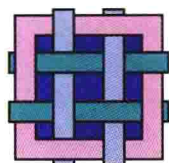
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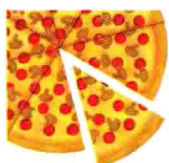
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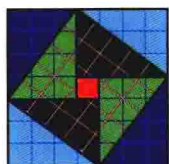
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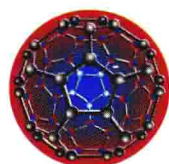
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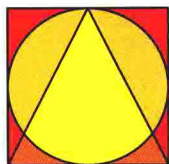
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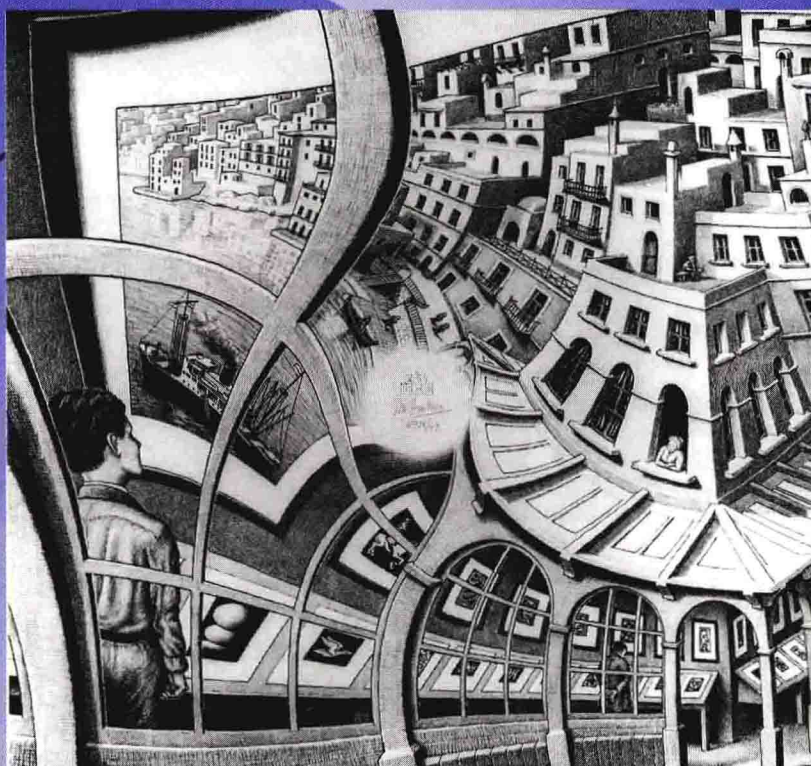
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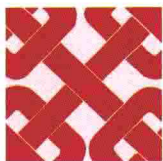
Geometric Art



Print Gallery, M. C. Escher, 1956
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In this chapter you will see how geometry is alive in nature and in the art forms of cultures around the world. Geometry describes the symmetry found in flowers, crystals, and animals and in the geometric designs of artists and architects from every culture. You will see that geometry is not just a college preparatory math class but is also a way of thinking and of seeing the world. In addition, you will practice using two tools of geometry, the compass and the straightedge.

A painting or an etching by Dutch graphic artist M. C. Escher (1898-1972) opens each chapter in this book. Escher used geometry in creative ways to make his sometimes strange and always interesting works of art. In the process he made a number of significant mathematical discoveries, even though he was not a trained mathematician. As you come to each new chapter, see if you can make a connection between the Escher work and the content of that chapter.

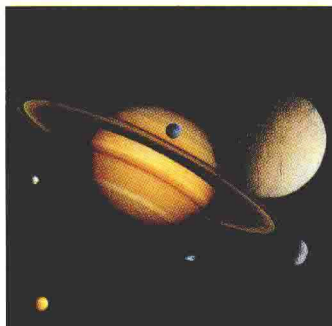


Lesson 0.1

Geometry in Nature and in Art

*There is one art, no more no less,
To do all things with artlessness.*
— Piet Hein

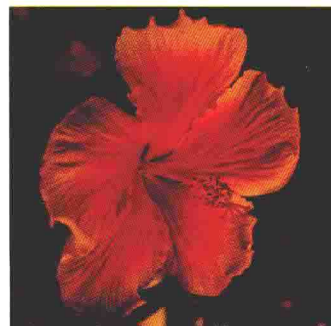
Nature displays an infinite array of geometric shapes, from the small atom to the greatest of the spiral galaxies. Crystalline solids, the honeycomb of the bee, snowflakes, the arrangement of seeds on sunflowers and on pinecones, the spiral of the nautilus shell, the spider's web, and the regular polygons found in the basic shapes of many flowers are just a few of nature's geometric masterpieces.



Circle



Hexagon



Pentagon

Geometry includes the study of the properties of shapes such as circles, hexagons, and pentagons. Outlines of the sun, the moon, and the planets appear as circular shapes. A stone tossed into a still pond and the growth rings in the cross section of a tree exhibit families of circles (concentric circles). The hexagonal shape appears often in inorganic (nonliving) objects of nature. Snowflakes and other crystalline structures have the hexagon as their basic geometric form. The pentagonal shape, which rarely appears in inorganic objects, is one of the primary geometric forms for organic shapes (living organisms). Many flowers and starfish have pentagonal shapes.

People have observed geometric patterns in nature and have incorporated the patterns into a variety of art forms. Mandalas (Lesson 0.5) and knot designs (Lesson 0.6) are geometric art forms that have appeared from early history to the present in many cultures of the world. The circular mandala design, used by the Hindus of India for meditation, is also found in the Aztec calendar stone, in the rose windows of cathedrals of Europe, and in the intricate Islamic designs of the Muslims of North Africa. Knot designs are found in Africa, in northern Scotland (Celtic art), and in Japanese and Chinese lattice designs.

During the Renaissance there was a revival in “realistic” drawing. Artists such as Leonardo da Vinci, Albrecht Dürer, and Raphael of Urbino sought to portray spatial relationships on their canvases as if they were scenes being viewed through a window. Renaissance artists turned to geometry for insight in developing their techniques. You will learn about perspective in Lesson 0.8 and will have a chance to create your own perspective drawings.



Islamic design from
Egypt