ALM 24
Advanced Lectures in Mathematics

# Handbook of Moduli

## (Volume I)

## 模手册 (卷 I)

Editors: Gavril Farkas · Ian Morrison

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The Handbook of Moduli was designed by Ian Morrison in LaTeX using a variant of the standard style files of the Higher Education Press. The text of the Handbook is set in ITC Giovanni and the mathematics in AMS Euler.

Giovanni was designed by Robert Slimbach in 1989 for ITC and was one of the early faces that earned him the Prix Charles Peignot, the Fields Medal of type design awarded "to a designer under the age of 35 who has made an outstanding contribution to type design". It combines the basic proportions of traditional oldstyle designs with the more even color and higher x-height of modern digital fonts to produce an inconspicuous but legible typeface.

Euler was designed in 1981 by Hermann Zapf, a major figure in 20th type design and a pioneer in digital typography, working in close cooperation with Donald Knuth, as an upright, cursive symbol font that would give the effect of mathematics handwritten on a blackboard. In 2008, Zapf reshaped many of the glyphs, with the assistance of Hans Hagen, Taco Hoekwater, and Volker RW Schaa, in order to harmonize the designs and bring them into line with contemporary standards of digital typography.

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## ADVANCED LECTURES IN MATHEMATICS

*The Handbook of Moduli is dedicated to the memory of Eckart Viehweg, whose untimely death precluded a planned contribution, and to David Mumford, who first proposed the project, for all that they both did to nurture its subject; and to Angela Ortega and Jane Reynolds for everything that they do to sustain its editors.*

# Preface

Gavril Farkas and Ian Morrison

The title of these volumes might lead unwary readers to expect an encyclopedic survey for experts in the study of moduli problems in algebraic geometry. What they will discover is rather different. Our aims here are, first, to clarify the audience that we hope the Handbook *will* serve and the approach it *does* takes to its subject and, second, to thank all those who have assisted us in helping it realize these aims.

To begin with, a bit of history. The idea for a Handbook of Moduli originated in a discussion between David Mumford and Lizhen Ji at Michigan in 2006. Lizhen and David produced a draft table of contents that was circulated at the Symposium marking David's retirement from Brown in 2007. The Handbook was originally to have been edited by Ching-Li Chai and Amnon Neeman, but the demands of their work with Takahiro Shiota as editors of the second volume of Mumford's Collected Papers took priority and, at their urging, we agreed to take over editorship in the spring of 2009.

We quickly reached the conclusion that what was needed for many topics was not a discussion of the latest results aimed at specialists, but a survey aimed at a broad community of producers (and even some consumers from cognate areas) of algebraic geometry, most of whom had little prior familiarity with the area. Our goal became a Handbook that would introduce the techniques, examples and results essential to each topic, and say enough about recent developments to prepare the reader to tackle the primary literature in the area. We particularly sought to elicit contributions that illustrated "secret handshakes", yogas and heuristics that experts use privately to guide intuition or simplify calculation but that are replaced by more formal arguments, or simply do not appear, in articles aimed at other specialists.

For many topics, the Handbook succeeds much better than we dared to hope. The credit is due entirely to the hard work of the Handbook's many authors in producing articles that conformed to the goals we had set. Again and again, we were delighted to find that authors, instead of taking the easy course of cutting and pasting from earlier surveys and primary references, had made the substantially greater effort to write the original treatments needed to bridge gaps in the literature and make important problems accessible to a wide audience for the first time.

We expect that they will reap a just reward and that their articles will be widely read and referenced. Here we want to offer them not only our sincerest thanks, but also those of the Handbook's readers, for their exceptional generosity. Many

Handbook articles were also improved by extensive and thoughtful referees' reports. We are grateful for all work that the referees did to improve the Handbook and take this opportunity to thank them collectively on behalf of the contributors.

We must, however, disclaim that the Handbook's coverage is often incomplete, in extreme cases, non-existent. The blame for these gaps is mostly ours. When we solicited contributions to the Handbook, each invitation was accompanied by a suggested topic, and we selected contributors who we thought would be able to cover their topics in the spirit discussed above. The results reflect both our knowledge and taste—of topics and of experts in them—and also, in some cases, our ignorance.

In some areas, we found it easy to produce candidate contributor–topic pairs, and to recruit the contributors we had identified. The Handbook's discussion of, for example, moduli spaces of curves is, therefore, particularly complete—some will say, not without a certain justice, excessive.

In other areas, we had more difficulty both in identifying and in enlisting candidates. A few of the more obvious gaps arose when authors who had accepted our invitation backed out after it was too late to find replacements. A more deeply felt loss—one that impacts the whole subject of moduli—was the untimely death of Eckart Viehweg, who had been one of the first to agree to contribute.

We also omitted a few topics as a courtesy to the authors of monographs devoted to them that we knew to be in preparation, others because papers treating them in the spirit we were seeking had recently appeared, and yet others because we felt that they were developing so rapidly that any contribution dealing with them would have a limited shelf-life. In hindsight, not all of these decisions were well taken.

As a result, the Handbook's treatment of moduli has some major lacunae (mirror symmetry, wall crossing formulae) and there are other topics (moduli of sheaves and bundles) which are discussed but not in the depth that their importance merits. We apologize to readers who may have hoped to find more about these subjects in the Handbook, and (with Lizhen's encouragement) we challenge experts who feel that their areas deserve a fuller exposition to offer him proposals for additional Handbook volumes devoted to them.

The Handbook also benefitted from the efforts of many other colleagues. Amnon Neeman showed considerable doggedness in recruiting us to succeed him and Ching-Li as editors. Scott Wolpert provided valuable advice on the cat-herding elements of the editor's job. Dave Bayer helped enormously in setting up the final production process both to automate complex and error prone operations and to prevent inconsistencies between the LaTeX installations on our home systems and those at Higher Education Press.

Brian Bianchini, International Press' General Manager, made sure that we had the resources we needed throughout the Handbook's growth from the single volume originally projected to the present three. The Advanced Mathematics series editor, Lizhen Ji, was always ready to answer our questions, help with practical difficulties,

and adjust his schedule for the series to adapt to changes in ours. Liping Wang and her production staff at the Higher Education Press were unfailingly accommodating and helpful to us in resolving LaTeX issues—even reTeXing several submissions to bring them into conformity with the Handbook style—and made every effort to ensure that the appearance of the Handbook volumes was up to the standard of their contents.

To all of them, and to many others who provided more informal help, we here offer our sincerest thanks.

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