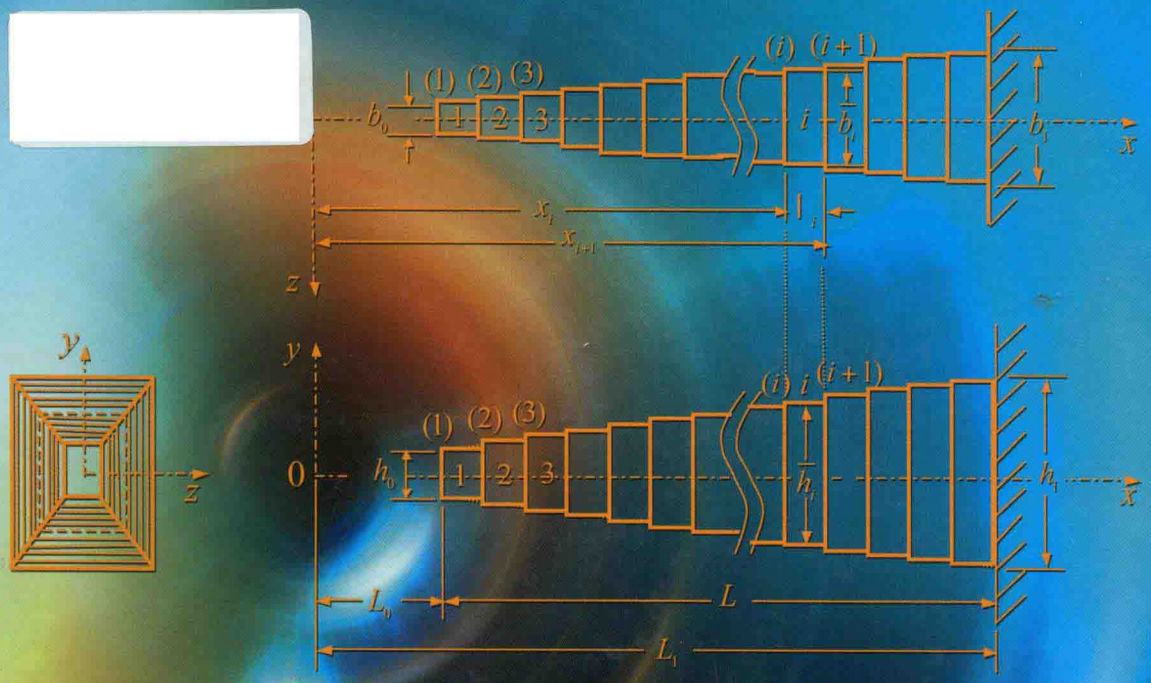

Analytical and Numerical Methods for Vibration Analyses

JONG-SHYONG WU

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ANALYTICAL AND NUMERICAL METHODS FOR VIBRATION ANALYSES

Jong-Shyong Wu
Distinguished Professor of NCKU
Department of Systems and
Naval Mechatronic Engineering
National Cheng-Kung University
Tainan, Taiwan

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ANALYTICAL AND NUMERICAL METHODS FOR VIBRATION ANALYSES

About the Author

Professor Jong-Shyong Wu, obtained his BS and MS degrees from the Department of Mechanical Engineering, National Cheng-Kung University (NCKU), Taiwan, in 1966 and 1969, respectively. After working in a factory for a year and a half, he was recommended to the Department of Naval Architecture and Marine Engineering (DNAME) of NCKU as an instructor, and he has stayed there until now, more than 40 years. During the period in NCKU, he obtained the financial support of the National Science Council (NSC), Taiwan, in 1975, and went abroad to study in the Department of Shipbuilding and Marine Technology, University of Strathclyde, UK, obtaining his Ph. D. degree in 1978. Because his thesis was related to ship hull vibrations in regular waves and in confused seas, he began to give courses on the dynamics of structures and the theory of matrix structural analysis to students in several graduate schools of the Engineering College, NCKU, after he returned to Taiwan in 1978. He has been the Chairman of DNAME of NCKU for 6 years and the head of the Computer and Network Center of NCKU for 6.5 years. He was promoted to professor in 1981 and awarded a distinguished professorship of NCKU in 2004. He has published more than 50 SCI papers in international journals, and obtained the prizes for Outstanding Research (three times) from NSC, the prize of Outstanding Engineering Paper from the Chinese Society of Engineers, Taiwan, and the Medal of Engineering from the Chinese Society of Naval Architects and Marine Engineers, Taiwan. He has been the supervisor of more than 70 MS students and seven Ph.D. students, and a reviewer for more than 15 international journals. This book is the condensation of his multi-year lecture notes and some new approaches that have appeared in his journal papers. It is suitable for undergraduate students, graduate students, engineers or researchers, dependent on the contents of the chapters concerned.

Preface

The author has given courses on the dynamics of structures and the theory of matrix structural analysis to students in several graduate schools (such as Mechanical, Civil, Aeronautical, Marine engineering, etc.) of the Engineering College, National Cheng-Kung University (NCKU), Taiwan, for more than 30 years. In addition to some reference books, lecture notes are the main teaching materials. Since the contents of lecture notes were usually changed annually, some students have asked me to write a book to cover the material in all my lecture notes over the past years. That is the main reason why I have written this book. In addition, it was also important to write a book to introduce the theories and methods presented in some of the author's publications appearing in international journals, despite the much trouble regarding the third-party permissions. The title of this book is *Analytical and Numerical Methods for Vibration Analyses*. It is obvious that the computer is one of the main tools for the solution of vibration problems, using either the analytical or the numerical methods introduced in this book. However, it is hoped that this book can also provide some useful information for readers who are not so familiar with computer languages or programming.

One of the predominant features of this book is that most of the introduced theories and associated mathematical expressions are confirmed by numerical examples. Most of the numerical results are obtained from two or three different methods, and good agreement between the numerical results of different methods is achieved. For example, in Chapter 4, the lowest five natural frequencies and corresponding mode shapes of a uniform or non-uniform beam carrying an arbitrary number of concentrated elements, including lumped masses (with eccentricities and rotary inertias), linear springs, rotational springs and spring–mass systems, are determined by three methods: the lumped-mass model transfer matrix method (LTMM); the continuous-mass model transfer matrix method (CTMM); and the conventional finite element method (FEM). In Chapter 7, for either out-of-plane or in-plane vibrations, the lowest five natural frequencies and associated mode shapes of a circularly curved beam carrying an arbitrary number of concentrated elements are also determined by three methods: the analytical (exact) method; the FEM with curved beam elements, FEM(curved); and the conventional FEM with straight beam elements, FEM(straight). In the other chapters, most of the numerical results are obtained from both the classical analytical (exact) method and the conventional FEM.

Since longitudinal and torsional vibration analyses are also important in the design of the propulsive shafting systems of ships, some attention is paid to the introduction of the axial

vibrations of uniform and conical rods, and the torsional vibrations of uniform and conical shafts by using analytical methods, the TMM and the FEM. In addition, in existing books, the shape functions associated with the 12 degrees of freedom of the three-dimensional Timoshenko beam element are incomplete or neglected. Thus, in Chapter 6, much effort is devoted to their derivation, and then a consistent approach is used to determine the element stiffness matrix and consistent mass matrix by using these shape functions. Furthermore, in the existing literature, the forced vibration response "amplitudes" of a single-degree-of-freedom (SDOF) or a multi-degree-of-freedom (MDOF) system are determined from the "steady-state" responses, and the free vibrating effects appearing in the intermediate steps of forced vibrations are neglected. Since the amplitudes of the last classical "steady-state" responses are much less than the corresponding ones of "total" responses near resonance, in Chapter 8 an efficient technique for determining the amplitudes of "total" forced vibration responses of SDOF and MDOF systems is introduced, in addition to the theory for obtaining the amplitudes of classical "steady-state" responses. Numerical results reveal that the CPU time required by the introduced approach is less than 1% of that required by the conventional FEM. It is noted that a few statements or equations are repeated in this book for the convenience of readers.

At the moment of drinking water, one should think about where that water has come from. This is thus a suitable place for me to say "thanks" to some important people. I came from a farming family, where each member worked very hard. So, first of all, I must thank my father, mother, brothers and sister. It is because of their guidance and care since my childhood that I can overcome most difficulties. I am also greatly indebted to my wife (C.L. Chen) for her long-term support without complaint. Next, I would like to thank Mr. S.Y. Huang, president of Pin-Ho Iron Works, Inc., in whose factory I gained much practical experience. I am also grateful to Professor K.Y. Li, one of my teachers in the Department of Mechanical Engineering, NCKU; at his recommendation I became a teacher and begin my teaching career in the university. Definitely, he is one of my benefactors and will be remembered forever. My thanks also go to Professor C. Kuo in the Department of Shipbuilding and Marine Technology, University of Strathclyde, UK. My first paper in an international journal, "On wave-excited ship vibrations in regular waves and in confused seas", was finished under his supervision. Of course, the financial support of the National Science Council (NSC), Taiwan, leading to my Ph.D. is highly appreciated. I would also like to thank Professor J.R. Maa, one of the former presidents of NCKU – he is a respected scholar and officer, and I learnt a lot from him about many things. Particular thanks go to the following for their help and encouragement: Professors C.I. Weng, C.K. Chen, S.J. Hsieh, T.S. Wang, T.S. Su, P.A. Luh, M.L. Lee, M.J. Huang, H.J. Shaw, C.H. Huang, R.M. Chao, G.P. Too, J.M. Yang, D.S. Hsu, J.Q. Tarn, S.Y. Tsai, H.H. Hwung, Y.L. Chou, T.S. Chen, C.Y. Yang, M.H. Wu, C.C. Liang, T.L. Teng, C. Kao, C. Hsu, W.H. Wang, R.J. Shyu, Y.B. Yang, Y.J. Lee, Y.H. Chen, C.F. Hung, J.H. Kuang and H.Y. Lin, and Messrs. S.P. Lin, T.S. Chen, R.G. Hung, F.S. Wu, J.H. Wu, C. Wu and C.Y. Chou. Finally, from August 1990 to February 1997, I worked in the Computer and Network Center of NCKU, and the people there who helped me very much are highly appreciated: Professors Y.N. Sun, S.R. Tsai, J.H. Chou, J.F. Wang, C.K. Shieh and S.J. Wang, and teachers R.L. Wu, J.W. Yan and N.H. Chiang.

Some materials in this book come from projects supported by the NSC, and this book was finished in the pleasant environment provided by NCKU, both of which are much appreciated. Of course, I must also say "thanks" to many of my other colleagues, friends and students,

although they are not mentioned above because of space limitations. Finally, I would like to give my best regards to the anonymous reviewers for their valuable comments on and kind advice about my publications.

Jong-Shyong Wu
January 2013

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