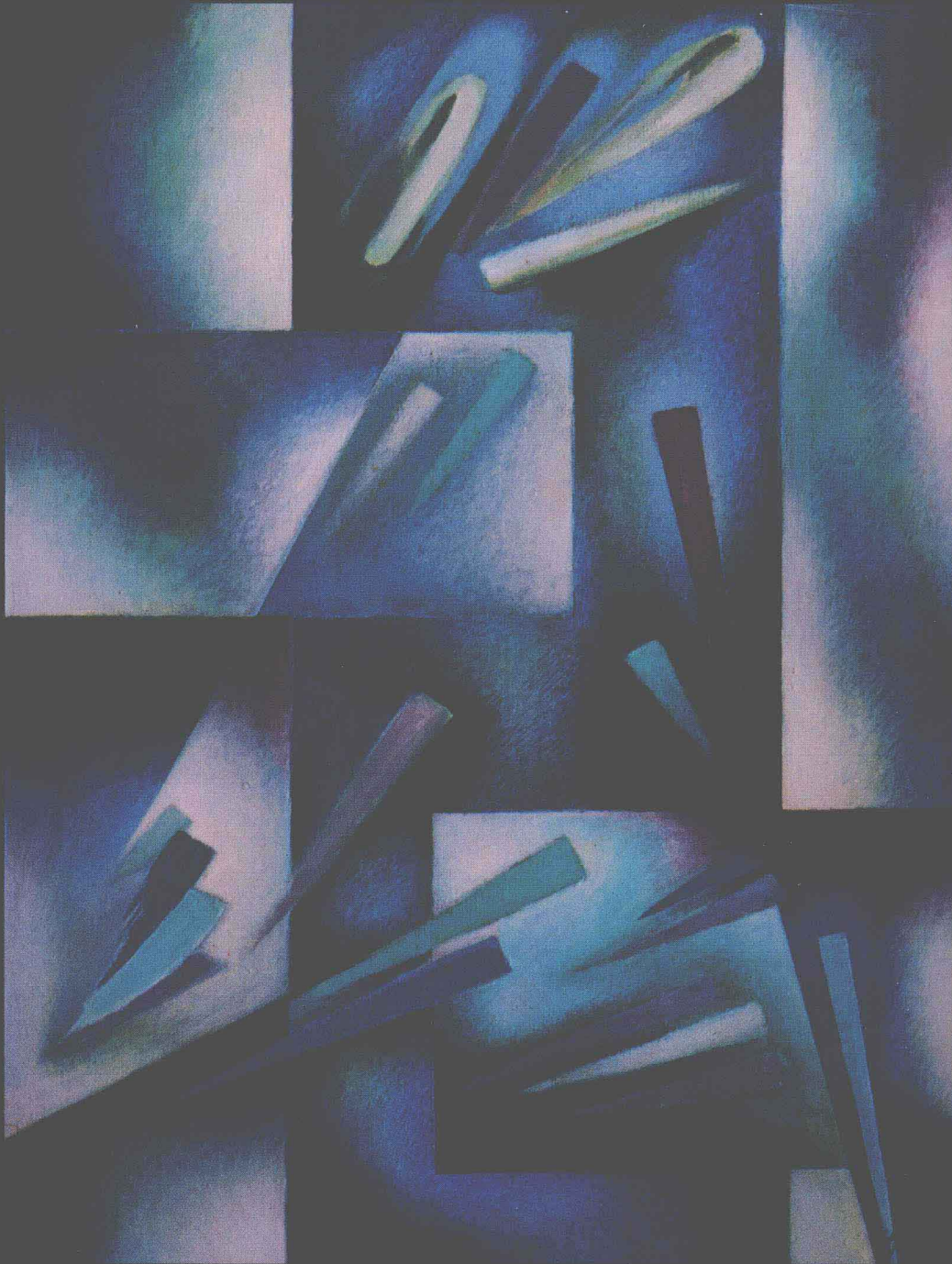


ARITHMETIC and ALGEBRA

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THIRD EDITION



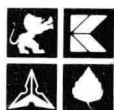
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ROSANNE PROGA

THIRD EDITION

Arithmetic and Algebra

Rosanne Proga



PWS-KENT PUBLISHING COMPANY
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This book is dedicated to Peter.

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Arithmetic and Algebra



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PREFACE

Arithmetic and Algebra, Third Edition, is a practical approach to the fundamentals of arithmetic and elementary algebra for students who elect nonmathematical areas of study. The approach to the subject matter is mature, yet easily understandable, and assumes no prior knowledge of mathematics. Various mathematical techniques are thoroughly explained because knowing *why* a technique works is an aid to remembering *how* to use it.

A main feature of this book is its flexibility. This book is especially suited for a combination arithmetic-elementary algebra course. Both the arithmetic and algebra portions of the book are complete in themselves and could be covered independently in separate arithmetic or elementary algebra courses. A large problem confronting students in an elementary algebra course is that they might have forgotten many arithmetic skills that need to be applied to master algebraic techniques. In this text, examples from arithmetic are used to introduce new algebraic techniques to illustrate the operations being performed. These examples are accompanied by references to sections of the book for the student who might need a more thorough review of a topic. In response to reviewers' comments, I have made the following revisions to this third edition:

- Expanded multiplication table (Chapter 1)
- Examples of rounding whole numbers (Chapter 3)
- Revised income tax problems (Chapter 5)
- Graphs showing greater detail and accuracy (Chapter 5)
- Additional problems involving the areas of composite figures (Chapter 9)
- More diagrams to accompany the solutions to examples (Chapter 9)
- Discussion of consecutive integer problems (Chapter 10)
- Additional examples of polynomial division (Chapter 12)
- Additional examples and exercises involving factoring by grouping (Chapter 13)



TOPICS

The highlights of this book include the following:

- Techniques for estimating answers are presented in Chapter 1 (Whole Numbers) so that students are better able to spot errors in their calculations.
- In Chapter 3 (Fractions), mixed numbers are presented early to achieve an integrated approach to the discussion of basic operations with fractions.
- Although algebraic concepts are introduced in Chapter 4 (Ratio and Proportion) and are used to solve equations in Chapter 5 (Percent), Chapter 10 (Introduction to Algebra) assumes no prior knowledge of algebra.
- Word problems emphasizing consumer applications appear throughout the text and predominate Section 5.4 (Applications of Percent) and Section 5.6 (Reading Graphs).

- Chapter 6 (Positive and Negative Numbers) includes examples of various arithmetic operations on signed integers, decimals, and fractions. This chapter may serve as a starting point for an elementary algebra course.
- Procedures for simplifying square roots are introduced in Chapter 7 (Roots of Numbers) and expanded upon to apply to algebraic expressions in Chapter 17 (Radical Expressions).
- Methods for approximating conversions between the English and Metric systems appear in Chapter 8 (Measurement in the English and Metric Systems) to develop the student's ability to "think metric."
- A thorough treatment of elementary geometry appears in Chapter 9 (Introduction to Geometry).
- Word problems that can be solved using quadratic equations are covered in Chapter 13 (Factoring) in anticipation that some instructors may not be able to cover Chapter 18 (Quadratic Equations).
- A thorough presentation of graphing linear equations and inequalities is presented in Chapter 14 (Graphing), graphing systems of linear equations and inequalities in Chapter 15 (Systems of Linear Equations), and graphing quadratic equations in Chapter 18 (Quadratic Equations).
- Chapter 16 (Rational Expressions) briefly reviews the concepts of mixed numbers and the basic operations with fractions to perform the same operations on algebraic expressions. Section 16.3 (Lowest Common Denominator) is designed to pave the way for students to be able to add and subtract rational expressions having different denominators, a skill that is often difficult to master.

FEATURES

- *Word problems* reinforce each newly learned skill and emphasize applications of mathematics to the real world.
- *Quick Quizzes* appear at the end of each section, enabling students to check their progress while reading the text.
- *Calculator problems* are designated by the symbol , and a brief description of how to use a calculator appears in the appendix. This book may be used with or without a pocket calculator.
- *Warnings* designated by the symbol  alert the student to common algebra mistakes.
- *Chapter Summaries* highlight important definitions and calculations and are followed by review exercises that reinforce the material learned. All items are keyed to the appropriate section of the text for quick referencing.

SUPPLEMENTS

- The *Answer Key* includes answers to even-numbered exercises. The answers to the odd-numbered exercises are found in the back of the book.
- The *Test Bank*, created by Thomas Vanden Eynden of Thomas More College, includes problems similar to those in the book. These can be used to supplement exercises in the text or as quizzes or self-testing tools. Answers are provided.
- A *Computerized Test Bank* available for both IBM PCs and compatibles and the Macintosh, contains problems supplied by Thomas Vanden Eynden.

- A set of developmental math *Videotapes* (VHS 1/2-inch format), which students can use to independently review the key arithmetic and algebra concepts in the text, is available.
- A set of *Expert Algebra Tutor* disks for IBM PCs and compatibles by Sergei Ovchinnikov of San Francisco State University is also available. These disks are tutorial software page-referenced to specific sections of the text. They define the level of tutoring needed by evaluating the user's need for further remediation or advancement in the tutoring session.

ACKNOWLEDGMENTS

I would like to express my appreciation to the many people who contributed to the development of this book. I am especially grateful to the following reviewers for their helpful suggestions:

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My editors at PWS-KENT have each made invaluable contributions during various stages of the project. Tom Orowan did an excellent job of typing the manuscript and Garry Johns of Saginaw Valley State University checked the entire final manuscript for accuracy; I thank them both. Finally, my special thanks to my father for his thoughtful comments and diligent proofreading, and to Peter Marzuk for his continued support and encouragement.

Rosanne Proga

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Whole Numbers

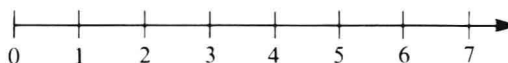
1.1 THE BASE 10 SYSTEM

When we begin to study any new field of knowledge, it is essential that we learn the vocabulary that is used to communicate the fundamental ideas of the subject. Thus, to study mathematics, we must familiarize ourselves with the language of mathematics, whose alphabet consists of not only letters but also numbers.

In this chapter, we will consider the numbers most frequently encountered—the **whole numbers**. These include $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \dots\}$. The braces $\{ \}$ that enclose the list indicate that the whole numbers are considered a set. A *set* is simply a collection or group of objects. The symbol \dots indicates that it is impossible to list all the whole numbers. Since we can add the number 1 to any whole number and obtain another number that is larger, there is no end to the list of whole numbers. We express this idea by saying that the set of whole numbers is *infinite*.

The whole numbers can be represented by a diagram called a number line, shown in Figure 1.1. Notice that larger numbers are located to the right of smaller numbers. The arrow on the number line indicates that there are an infinite number of whole numbers.

FIGURE 1.1



The number system we most frequently use is called the **base 10 system**. In this system, only 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are used to express all numbers. The location of each digit, or **place value**, determines the value of the number.

DEFINITION

The *place value* of a digit is the name of the location of that digit.

The place value increases by a factor of 10 as we move from right to left. The rightmost location is the ones place, the position immediately to the left of it is the tens place, the next place is the hundreds place, and so on, as shown in Table 1.1.

We name numbers according to the location of their digits. For example, the number one thousand has a 1 in the thousands place followed by three zeros: 1,000. After every group of three digits, moving from right to left, we place a comma to make it easier for us to read the number. Each of these three-digit groups is called a **period**. A chart indicating the place values of the first 15 locations included in the first five periods is

- (b) 8,016
 8: thousands place
 0: hundreds place
 1: tens place
 6: ones place
 eight thousand, sixteen

| Thousands | Hundreds | Tens | Ones |
|-----------|----------|------|------|
| 8 | 0 | 1 | 6 |

- (c) 423,701
 4: hundred thousands place
 2: ten thousands place
 3: thousands place
 7: hundreds place
 0: tens place
 1: ones place
 four hundred twenty-three thousand, seven hundred one

| Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones |
|-------------------|---------------|-----------|----------|------|------|
| 4 | 2 | 3 | 7 | 0 | 1 |

- (d) 61,004,072
 6: ten millions place
 1: millions place
 0: hundred thousands place
 0: ten thousands place
 4: thousands place
 0: hundreds place
 7: tens place
 2: ones place
 sixty-one million, four thousand, seventy-two

| Ten Millions | Millions | Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones |
|--------------|----------|-------------------|---------------|-----------|----------|------|------|
| 6 | 1 | 0 | 0 | 4 | 0 | 7 | 2 |

- (e) 7,325,018,640
 7: billions place
 3: hundred millions place
 2: ten millions place
 5: millions place
 0: hundred thousands place
 1: ten thousands place
 8: thousands place
 6: hundreds place
 4: tens place
 0: ones place
 seven billion, three hundred twenty-five million, eighteen thousand, six hundred forty

| Billions | Hundred Millions | Ten Millions | Millions | Hundred Thousands | Ten Thousands | Thousands | Hundreds | Tens | Ones |
|----------|------------------|--------------|----------|-------------------|---------------|-----------|----------|------|------|
| 7 | 3 | 2 | 5 | 0 | 1 | 8 | 6 | 4 | 0 |

Example 2 Write each of the following numbers using digits.

- (a) three thousand, eight
 3,008
- (b) two hundred fifty-seven thousand, seven hundred twenty-two
 257,722
- (c) six million, three thousand, one
 6,003,001

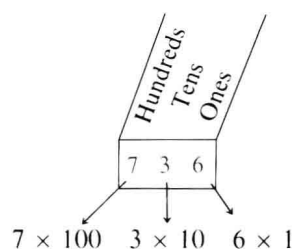
We can also use the concept of place value to rewrite any number in **expanded notation**. This notation will be particularly useful to us when learning how to perform the various arithmetic operations on whole numbers (see box). We put parentheses around

each quantity obtained in step 2 to indicate that the multiplication is to be performed before the addition (step 3).

TO WRITE A NUMBER IN EXPANDED NOTATION

1. Identify the place value of each digit.
2. Multiply each digit by its place value.
3. Add the results found in step 2 together.

For example, the number 736 can be written in expanded notation as follows:



$$736 = 7 \text{ hundreds} + 3 \text{ tens} + 6 \text{ ones} \\ = (7 \times 100) + (3 \times 10) + (6 \times 1)$$

Remember that the parentheses are used to indicate that multiplication is performed before addition.

In some forms of expanded notation, the results in parentheses are simplified further as follows:

$$736 = (7 \times 100) + (3 \times 10) + (6 \times 1) \\ = 700 + 30 + 6$$

We will not perform this extra step when we write numbers in expanded notation.

Example 3 Rewrite each of the following numbers in expanded notation.

(a) 85

$$85 = 8 \text{ tens} + 5 \text{ ones} = (8 \times 10) + (5 \times 1)$$

(b) 409

$$409 = 4 \text{ hundreds} + 0 \text{ tens} + 9 \text{ ones} \\ = (4 \times 100) + (0 \times 10) + (9 \times 1)$$

(c) 350,081

$$350,081 = 3 \text{ hundred thousands} + 5 \text{ ten thousands} + 0 \text{ thousands} \\ + 0 \text{ hundreds} + 8 \text{ tens} + 1 \text{ one} \\ = (3 \times 100,000) + (5 \times 10,000) + (8 \times 10) + (1 \times 1)$$



Certain symbols are used in mathematics to represent the various mathematical operations and to compare numbers. The four basic arithmetic operations are indicated by the symbols in the accompanying box. The letters a and b are used to represent any two whole numbers.

FOUR BASIC ARITHMETIC OPERATIONS AND THEIR SYMBOLS

| | | |
|-----------------|--------------|----------------------------------|
| Addition: | $a + b$ | means to add a and b . |
| Subtraction: | $a - b$ | means to subtract b from a . |
| Multiplication: | $a \times b$ | means to multiply a and b . |
| Division: | $a \div b$ | means to divide a by b . |

Example 4 State what arithmetic operation is indicated by each of the following expressions.

- (a) 2×100
Multiply 2 and 100.
- (b) $8 - 3$
Subtract 3 from 8.
- (c) $4 + b$
Add 4 and b .
- (d) $x \div y$
Divide x by y .

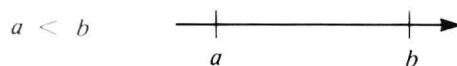


A list of symbols used to compare two numbers is shown in the next box. The letters a and b are again used to represent any two whole numbers.

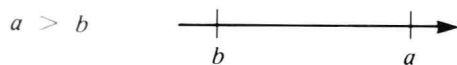
SYMBOLS USED TO COMPARE NUMBERS

| | | |
|-----------------------|-------------|---|
| Equality: | $a = b$ | means a is equal to b . |
| Inequality: | $a \neq b$ | means a is not equal to b . |
| Less than: | $a < b$ | means a is less than b . |
| Greater than: | $a > b$ | means a is greater than b . |
| Approximate equality: | $a \cong b$ | means a is approximately equal to b . |

Notice that in using the $>$ or $<$ symbol, the wide end of the symbol faces the larger number. Also note the respective locations of a and b on the following number lines.

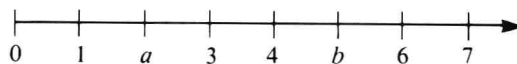


a is less than b indicates that a is to the left of b on a number line.



a is greater than b indicates that a is to the right of b on a number line.

Example 5 Compare each of the following pairs of numbers represented on this number line. Use the $=$, $<$, or $>$ sign to indicate the relationship between them.



- (a) $3 ? 6$
Since 3 is to the left of 6 on the number line, 3 is less than 6.
 $3 < 6$