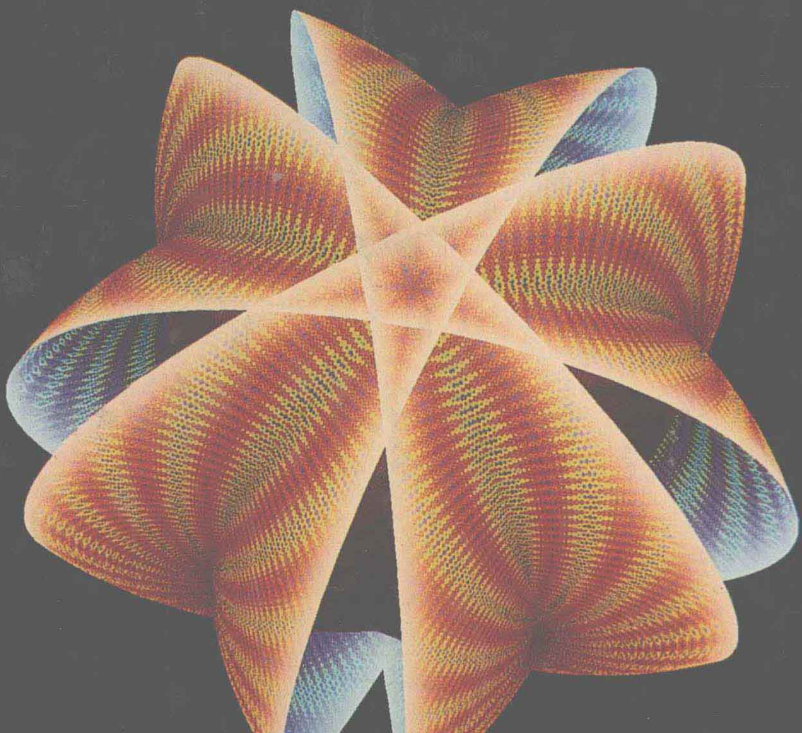
FINITE MATHEMATICS WITH CALCULUS

An Applied Approach

SECOND EDITION



Zitarelli • Coughlin

Finite Mathematics with Calculus: An Applied Approach

Second Edition

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Preface

In the summer of 1991 a committee of the National Research Council, which is part of the National Academy of Sciences, issued a report called *Moving Beyond Myths*.* The report describes many serious problems in undergraduate mathematics education and provides an action plan for attacking them. One of the myths that this committee identified (see page 12) is

Myth: *Only scientists and engineers need to study mathematics.*

Reality: Mathematics is a science of patterns that is useful in many areas. Indeed, the most rapid areas of growth in applications of mathematics have been in the social, biological, and behavioral sciences. Financial analysts, legal scholars, political pollsters, and sales managers all rely on sophisticated mathematical models to analyze data and make projections. Even artists and musicians use mathematically based computer programs to aid in their work. No longer just a tool for the physical sciences, mathematics is a language for all disciplines.

The goal of this book is to reverse this myth. Although the table of contents is standard, the means of achieving the goal differs from other texts in two notable ways. First, in every section the Referenced Exercises refer to areas outside mathematics and are accompanied by footnotes so the student can pursue them in greater depth or the instructor can assign them as part of special projects. Because such applications deal with real-life situations, calculators are frequently necessary to carry out the computations.

We hope to show students through these exercises that mathematics is everywhere. No matter what major a student chooses, mathematics can play a key role in solving interesting problems in that discipline. Here is a representative list of selected Referenced Exercises, along with the section where each can be found.

Application	Source	Section
stereo speakers	Teledyne Acoustic Research	1–1
archaeological deposits	*American Antiquity*	2–1
crop rotations	*American Journal of Agricultural Economics*	3–3
dairy farming	*Journal of the Operational Research Society*	4–1
athletes' contracts	text *Financial Management*	5–2

*William E. Kirwan (Editor), *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*, Washington D.C., National Academy Press, 1991.

Application	Source	Section
pitch classes	*Journal of Music Theory*	6–1
homicide and deterrence	*Southern Economics Journal*	7–5
economic forecasts	*Management Science*	8–3
urban status	*American Journal of Sociology*	9–1
auditing procedures	*The Accounting Review*	10–2
stocks and bonds	*Forbes Magazine*	11–3
income tax	*Wall Street Journal*	12–3
telecommunications	*Forecasting Public Utilities*	13–2
regeneration of trees	*Ecology*	14–5
learning curves	*Decision Sciences*	15–2
pollution control	*American Journal of Agricultural Economics*	16–4
gifted students	*Journal of Mathematical Psychology*	17–1
cancer treatment	*Physics in Medicine and Biology*	18–2

In addition, every chapter has at least one Case Study that treats a particular application in greater detail than the Referenced Exercises. The topics have been chosen not only to illustrate areas where mathematics has played a crucial role in solving an important problem, but to highlight areas of particular human interest.

Even if the instructor does not have time to cover all the Referenced Exercises or Case Studies, it is hoped that their relevance and wealth (in both diversity and number) will make a lasting impression.

Changes in the Second Edition

This second edition of the book differs from the first edition in several ways.

1. The exercises are divided into three sets. The first set consists of the standard assortment of problems, usually numbering between 50 and 70. The first 20 or so generally reflect the examples in the text and are routine, while the later problems are a bit more challenging. The second set, titled ''Referenced Exercises,'' contains problems from areas outside mathematics; these exercises are accompanied by complete references to the literature. The third set, ''Cumulative Exercises,'' contains problems whose solutions call on material from the preceding sections in that chapter. These problems often require different skills than do the usual problems. The authors have found this feature very useful in their own classes as a way to continually help students review old material as they study the current section. In total, there are over 4,000 exercises in this book, about 20% of them new to this edition. Although the book continues to contain application exercises associated with many subjects, many simple drill problems have been added as well to help students master concepts before applying them.
2. There are many new and updated Referenced Exercises.
3. There are two new Case Studies, one on ''Trial by Jury'' and the other on ''Lenin as Statistician.''

4. The normal exercise sets and the cumulative exercise sets include two types of problems that several users of the first edition requested: those that are stated in words and require the translation from the ordinary language into English, and those that require a geometric interpretation.
5. At the end of each chapter is a set of exercises for Programmable Calculators. An Appendix discusses such calculators. The programmable, or graphing, calculator is an important new tool in the teaching of applied mathematics. However, instructors who do not use this calculator in class can skip the programmable calculator material without any loss of continuity. This edition, like the first, also includes references to ordinary scientific calculators in key places.
6. Many instructors who used the first edition felt that four important topics should have their own sections. Section 6–5 is now exclusively devoted to the binomial theorem, Section 7–8 to expected value, Section 12–4 to continuous and differentiable functions, and Section 18–6 to volume.

Organization

The text covers two distinct areas, finite mathematics (Chapters 1 through 10) and calculus (Chapters 11 through 18). Chapter R presents a brief review of those topics whose mastery is necessary beforehand. The first four sections of Chapter R present background material for finite mathematics, and the remaining four sections for calculus.

The material on finite mathematics can be divided into three parts: linear mathematics (Chapters 1 through 4), probability and statistics (Chapters 6, 7, and 8), and applications (Chapters 5, 9 and 10).

Linear Mathematics

Chapter 1 introduces the Cartesian coordinate system and linear equations. Systems of two equations in two variables are presented separately from systems of three equations in three variables for those instructors who want to proceed directly to the geometric solution of linear programming problems in Chapter 3.

Chapter 2 introduces matrices and their connection to systems of linear equations. Section 1–4 on Gaussian elimination is a prerequisite for Section 2–3 on the Gauss-Jordan method. The matrix inverse method is applied to input-output analysis.

Chapters 3 and 4 deal with linear programming. Chapter 3 illustrates the geometric method of solving linear programming problems with two variables. In Chapter 4 the simplex method is used for solving linear programming problems with more than two variables.

Probability and Statistics

Chapter 6 lays the foundation for this part. It reviews the basic aspects of set theory, including Venn diagrams and tree diagrams, and presents counting techniques, permutations, and combinations. In the second edition we have expanded our coverage of the Binomial Theorem by devoting a new section to it. In Chapter 7 the basic aspects of probability are motivated by a study of auto accidents on a major highway in Boston. After the set-theoretic foundations are described, laws governing the addition, subtraction, multiplication, and division of probabilities are derived. We have also expanded our coverage of the expected value and binomial experiments by devoting a section to each in this edition. Chapter 8 presents the rudiments of statistics, including graphical methods, measures of central tendency, measures of spread from the central tendency, the normal curve, and binomial experiments.

Applications

Chapter 5 is independent of the rest of the book. It deals primarily with the mathematics of finance, but it also treats inflation, cost of living, and population growth.

Chapters 9 and 10 apply matrices and probability to Markov chains and game theory. The last section in the book ties this material to linear programming, and the epilog discusses the historical meeting that produced the confluence of these seemingly unrelated fields.

The second part of the text can also be divided into three parts: differential calculus (Chapters 11 through 15), integral calculus (Chapters 16 and 17), and functions of several variables (Chapter 18).

Differential Calculus

Chapter 11 covers the algebraic techniques needed for the subsequent material. Functions are defined and studied, and various properties of polynomial functions and rational functions are illustrated. Chapter 12 explains the limit of a function by discussing velocity, rate of change of a moving object, and the tangent to a graph. The definition of the derivative is used to calculate the derivatives of several functions.

Chapter 13 presents various techniques for computing the derivatives of functions. Chapter 14 applies these techniques in a wide variety of problems.

Chapter 15 introduces exponential and logarithmic functions. Their derivatives are obtained by calculator experiments aimed at suggesting general rules. Those rules are then proved rigorously.

Integral Calculus

Chapter 16 introduces integration via antidifferentiation. After indefinite integrals have been defined, the Fundamental Theorem of Calculus relates the derivative to the integral. Then the integral is used to compute areas bounded by curves.

Chapter 17 presents two techniques of integration, the use of tables of integrals, and numerical integration.

Functions of Several Variables

Chapter 18 extends the definition of the derivative and the definition of the integral to functions of more than one variable. Partial derivatives are defined and applied to the sketching of surfaces, while double integrals are evaluated for functions of two variables and used to compute volumes.

Format

We have tried to keep the length of each section to what can be covered in a typical 50-minute class. Sometimes, however, the topic has dictated more extensive coverage. Each numbered section has been partitioned into subsections to help the instructor prepare lectures and to help the student organize the material.

There are two kinds of examples. One explains a new skill which is being encountered for the first time. It is labeled simply EXAMPLE. The other illustrates a skill which was explained beforehand. It uses the following format:

EXAMPLE

Problem

Solution

The student should be able to make a good attempt at solving the problem independently before reading its solution.

Each chapter ends with a review of the terms, notation, and formulas that were introduced in the chapter. It also contains review problems that can be used to review for tests.

Supplements

A number of supplements are available for use with this text.

The **Instructor's Manual**, by Richard Shores, Lynchburg College, contains the partially worked-out solutions to all exercises in the text. (Answers to the odd-numbered exercises are also in the back of the text.) Also contained are additional teaching hints and aids for the instructor.

A **Student Solutions Manual and Study Guide**, also by Richard Shores, contains fully worked-out solutions to every other odd-numbered problem. This manual is designed to help students with their problem-solving skills: these solutions can be used as models in solving similar problems.

The software package **MathPath** by George Bergeman, Northern Virginia Community College, is available free to users of this text. MathPath has been revised for this edition. It supplies graphical and computational support for many of the important topics in each chapter. The software requires an IBM or IBM-compatible computer with at least 512K. Prospective users might want to consult an article that compared the software packages that accompany six books: Joan Wyzkoski Weiss, ''IBM Software for Finite Mathematics, Part I,'' *The College Mathematics Journal* **22**(3) (May, 1991), 248–254.

The test bank, by Robert Kurtz and Pao-sheng Hsu, of the University of

Maine at Orono, contains about 1,800 questions, all new to this edition. "Writing Across the Curriculum" problems, which apply mathematics to different fields, make this manual unique. Even instructors who use their own test banks will find this one useful.

The test bank is also available in a computerized format for IBM and IBM-compatible computers. This computerized version allows an instructor to custom-design tests and to sort the questions by several different categories. It requires 256K and two disk drives or 384K and a hard drive, graphics card and monitor, and a printer capable of handling graphs. Full instructions are included.

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David E. Zitarelli

Raymond F. Coughlin

Philadelphia, PA
December 1991

Algebra Review

R

CHAPTER OUTLINE

Mathematics and high finance: the New York Stock Exchange. (H. Armstrong Roberts)

Chapter Overview

The material in this chapter is a review in the sense that you have learned most of it previously. Some of it will be fresh in your mind, but other concepts will have to be learned anew. Mastering this material makes it much easier to understand the rest of the text.

The first four sections can be regarded as a review of algebraic topics that are used in finite mathematics. The last four sections cover some precalculus techniques.

The first two sections deal with equations and inequalities that contain only one variable. Later in the text these ideas are expanded to include equations and inequalities with more than one variable. There are many uses of exponents in finite mathematics and calculus. They are reviewed in Section R–3. The absolute value of numbers and expressions containing absolute values are covered in Section R–4.

Polynomials are building blocks of many branches of mathematics, especially calculus. They are introduced in Section R–5 by studying those polynomials with one variable. Factoring polynomials is an important skill that is reviewed in Section R–6. Solving equations is necessary for a thorough understanding of many concepts in calculus as well as in finite mathematics. In Section R–7 we cover the solution of quadratic equations. The last section deals with rational expressions that are ratios of polynomials.

R–1 Equalities in One Variable

One of the fundamental building blocks of mathematics is the study of equations. An equation states that one expression is equal to another, such as

$$4 + 6 = 10 \qquad x + 6 = 10 \qquad 4 + 2K = 10$$

The first example is always true while the other two equations are true for some values of the **variable,** which is what the unknown quantity is called. Although most of the time the variables used are x and y, any letter will suffice. The second equation uses x while the third uses K.

An equation that can be written in the form $ax + b = c$, for real numbers a, b, and c with $a \neq 0$, is called a **linear equation in one variable.** The variable can be represented by any letter. For example, $x + 6 = 10$ and $4 + 2K = 10$ are linear equations in x and K, respectively. Examples of equations that are **non-linear** (not linear) are $x^2 = 1$ and $x^{1/2} = 4$.

The equation $x + 6 = 10$ is true when $x = 4$ and is false for any other value of x. We say that $x = 4$ is the **solution of the equation.** To solve the equation $4 + 2K = 10$, isolate K on the left-hand side of the equation by first subtracting 4 from each side of the equation and then dividing the resulting equation by 2.

$$\begin{aligned} 4 + 2K &= 10 \\ 4 + 2K - 4 &= 10 - 4 \\ 2K + 4 - 4 &= 10 - 4 \\ 2K &= 6 \\ K &= 3 \end{aligned}$$

This means that the solution of the equation is $K = 3$. The solution should always be checked by substituting it into the original equation. In this case substitute $K = 3$ into the equation $4 + 2K = 10$ and check whether the resulting statement is true.

$$4 + 2(3) \stackrel{?}{=} 10$$

The left-hand side equals the right-hand side, meaning that the solution checks.

The following properties of real numbers are used to solve equations. A variable represents a real number so that it also obeys the properties.

> **Properties of Real Numbers Used in Solving Equations**
>
> Let a, b, and c represent real numbers or expressions that represent real numbers.
>
> 1. If $a = b$, then $a + c = b + c$
> 2. If $a = b$, then $ac = bc$
> 3. $a + b = b + a$ **(the commutative law of addition)**
> 4. $a(b + c) = ab + ac$, $(a + b)c = ac + bc$ **(the distributive laws)**

Sometimes an equation will have more than one term containing the variable, such as $2x + 4x$. In such a case use property 4, the distributive rule, to "gather like terms." That is, $2x + 4x = (2 + 4)x = 6x$. Example 1 demonstrates how to solve a linear equation in one variable when more than one term contains the variable.

EXAMPLE 1

Problem

Solve (a) $6x + 9 = 2x - 3$ (b) $2(3p - 1) + 2p = 3$

Solution (a) First isolate x on the left-hand side by subtracting $2x$ and 9 from each side, and then gathering like terms and dividing by the appropriate number; that is, use rule 2 with $c = \frac{1}{4}$.

$$\begin{aligned} 6x + 9 &= 2x - 3 \\ 6x + 9 - 2x - 9 &= 2x - 3 - 2x - 9 \\ 4x &= -12 \\ x &= -3 \end{aligned}$$

The solution is $x = -3$.

(b) First multiply 2 times the expression in the parentheses according to property 4 and then proceed as in part (a).

$$\begin{aligned} 2(3p - 1) + 2p &= 3 \\ 6p - 2 + 2p &= 3 \\ 8p &= 5 \\ p &= \tfrac{5}{8} \end{aligned}$$

The solution is $p = \frac{5}{8}$.

Example 2 shows how to solve a linear equation in one variable when some additional arithmetic is needed.

EXAMPLE 2

Problem

Solve $\dfrac{6x - 1}{5} = \dfrac{3x + 10}{4}$

Solution First eliminate the fractions by multiplying through the equation by the least common multiple of the denominators, 20. This yields

$$20\,\frac{(6x - 1)}{5} = 20\,\frac{(3x + 10)}{4}$$

$$4(6x - 1) = 5(3x + 10)$$

Use property 4.

$$24x - 4 = 15x + 50$$

Isolate x on the left-hand side by subtracting $15x$ and adding 4 to each side, and then gathering like terms and dividing by the appropriate number.

$$\begin{aligned} 24x - 4 &= 15x + 50 \\ 24x - 4 - 15x + 4 &= 15x + 50 - 15x + 4 \\ 9x &= 54 \\ x &= 6 \end{aligned}$$

These techniques can be used to solve equations that are not linear.

EXAMPLE 3

Problem

Solve $7 - 3/x = -2$.

Solution First multiply each side of the equation by x.

$$\begin{aligned} (7 - 3/x)x &= -2x \\ 7x - 3 &= -2x \end{aligned}$$

Isolate x on the left-hand side by adding $2x + 3$ to each side, and then gathering like terms and dividing by the appropriate number:

$$\begin{aligned} 7x + 2x &= 3 \\ 9x &= 3 \\ x &= \tfrac{1}{3} \end{aligned}$$

EXERCISE SET R–1

In Problems 1 to 24 solve the linear equation.

1. $3x + 5 = 4x - 2$
2. $4y - 3 = 5 - 2y$
3. $3 + 4x = 4 - 2x$
4. $1 - 3z = 5z - 2$

5. $3(s - 5) + 3 = 2s$
6. $4r - 5(3 - 2r) = r + 3$
7. $3q - 4(23 - 21q) = 15q + 30$
8. $2 - 4(2.5 - 5.1t) = 2(1.5t + 2.2)$
9. $3(x - 1) + 3 = 2(1 + x)$
10. $4y - 5(3 - y) = 2(2y + 3)$
11. $2x/5 - 3/8 = 5/8 - 7x/20$
12. $3y/4 - 1/8 = 5y/12 - 7y/8$
13. $1 + 2(1.5 - 1.1y) = 2(0.5 + 1.2y)$
14. $2 + 3(1.5 - x) = 4(1.5 + 0.5x)$
15. $t/5 - 3t/2 = 1/10 - t/20$
16. $y/2 + y/8 = 5y/2 - 7$
17. $x/2 - 4x/3 = 3/10 - 7x/20$
18. $y/4 + 3/8 = 5y/8 - 7y/4$
19. $\dfrac{x - 1}{5} = \dfrac{x + 3}{7}$
20. $\dfrac{x + 2}{5} = \dfrac{x + 3}{6}$
21. $\dfrac{3x + 2}{4} = \dfrac{4x + 3}{5}$
22. $\dfrac{2x - 1}{3} = \dfrac{5x - 2}{4}$
23. $\dfrac{7x + 2}{3} = \dfrac{2x + 8}{5} + 1$
24. $\dfrac{2x - 2}{3} = \dfrac{3x - 1}{6} - 1$

In Problems 25 to 44 solve the equation.

25. $3/x + 1 = 2$
26. $2 - 3/x = -1$
27. $4/x + 1 = 10$
28. $5 - 1/x = 1$
29. $4/x - 3/x = 1$
30. $5/x + 7/x = 19$
31. $2/3x + 1 = 1/x$
32. $3 - 3/2x = 1/x$
33. $2(1 - 1/x) = 1$
34. $3(1 + 1/x) = 6$
35. $4(2/x - 1) = 4$
36. $3(3/x + 1) = 12$
37. $1 + 2(1 - 2/x) = 2$
38. $3 - 2(2 + 4/x) = 7$
39. $1/x = 2(1 + 1/x)$
40. $2/3x = 4(1 - 1/x)$
41. $\dfrac{1}{(x - 1)} = 1$
42. $\dfrac{2}{(3 - x)} = 1$
43. $1 + \dfrac{3}{(x + 1)} = 2$
44. $2 - \dfrac{5}{(1 + 2x)} = 1$

R–2 Linear Inequalities in One Variable

An **inequality** states that one expression is unequal to another. Sometimes an equals sign is included with the **inequality sign** to indicate that the expressions can be equal. There are four inequality signs.

$<$ represents "is less than"
$>$ represents "is greater than"
$\leq$ represents "is less than or equal to"
$\geq$ represents "is greater than or equal to"

Examples of inequalities without a variable are

$$3 < 4 \qquad 3 \leq 5 \qquad -1 > -2 \qquad -3 \leq -3$$

These represent statements about real numbers. They are all true but a linear inequality can also be false, such as $3 < 0$, and $-1 < -2$. If a variable is present, the inequality is sometimes true and sometimes false, depending on the value assigned to the variable. Examples of inequalities in one variable are

$$3x < 4 \qquad t + 3 \leq 5 \qquad 3x + 4 > 10 \qquad 6 + 2y \geq 10$$

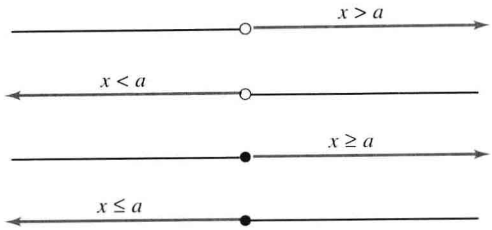

FIGURE 1

A **linear inequality in one variable** is one that can be expressed as $ax < b$, where a and b are real numbers, or expressed with any of the other three inequality signs and any letter for the variable. Thus $3x < 4$ is a linear inequality with $a = 3$ and $b = 4$. The properties of real numbers used to solve inequalities are similar to those properties used to solve equations. The important difference is when an inequality is multiplied by a negative number, property 3 here.

Properties of Real Numbers Used in Solving Inequalities

Let a, b, and c represent real numbers or expressions that represent real numbers. The properties are stated for "less than" ($<$), but they also hold for the other three inequality signs.

1. If $a < b$, then $a + c < b + c$
2. If $a < b$, and $c > 0$, then $ac < bc$
3. If $a < b$, and $c < 0$, then $ac > bc$

Property 3 states that the sign of the inequality changes if the inequality is multiplied by a negative number. For example, $3 > 2$ but $3(-4) < 2(-4)$ because $-12 < -8$. Dividing by a negative number also changes the sign of the inequality because division is the same as multiplying by the multiplicative inverse of the number. For example, $6 > 2$ but $6(-1/2) < 2(-1/2)$ because $-3 < -1$.

The **solution of a linear inequality** is expressed as $x < a$, $x > a$, $x \leq a$, or $x \geq a$. These expressions represent the set of all real numbers less than a or greater than a, where a is not included in the set for the first two expressions but a is included in the set for the other two expressions. The graph of these expressions is shown in Figure 1. The graphs use a shaded line to indicate the solution set with an open circle to indicate that the endpoint is not included and a shaded circle if it is included.

Example 1 illustrates how to solve a linear inequality in one variable.

EXAMPLE 1

Problem

Solve $3x - 5 < 7$

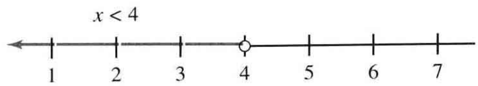

FIGURE 2

Solution Isolate x on the left-hand side of the inequality and then divide by 3.

$$
\begin{aligned}
3x - 5 &< 7 \\
3x - 5 + 5 &< 7 + 5 \\
3x &< 12 \\
x &< 4
\end{aligned}
$$

The graph of the solution is in Figure 2. The solution is not one number; it consists of infinitely many numbers.

Example 2 uses more properties to solve a linear inequality.

EXAMPLE 2

Problem

Solve $3(1 - 2x) > 3x + 15$

Solution First use the distributive law and then proceed as in Example 1.

$$
\begin{aligned}
3 - 6x &> 3x + 15 \\
3 - 6x - 3 - 3x &> 3x + 15 - 3 - 3x \\
-9x &> 12 \\
x &< -\tfrac{4}{3}
\end{aligned}
$$

The last step used property 3 for inequalities with $c = -\frac{1}{9}$ so that the sense of the inequality is reversed. The solution is graphed in Figure 3.

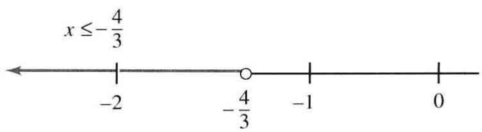

FIGURE 3

Example 3 solves an inequality that includes an equal sign in the inequality.

EXAMPLE 3

Problem

Solve $2(3 + 5x) \le 6(x - 3)$