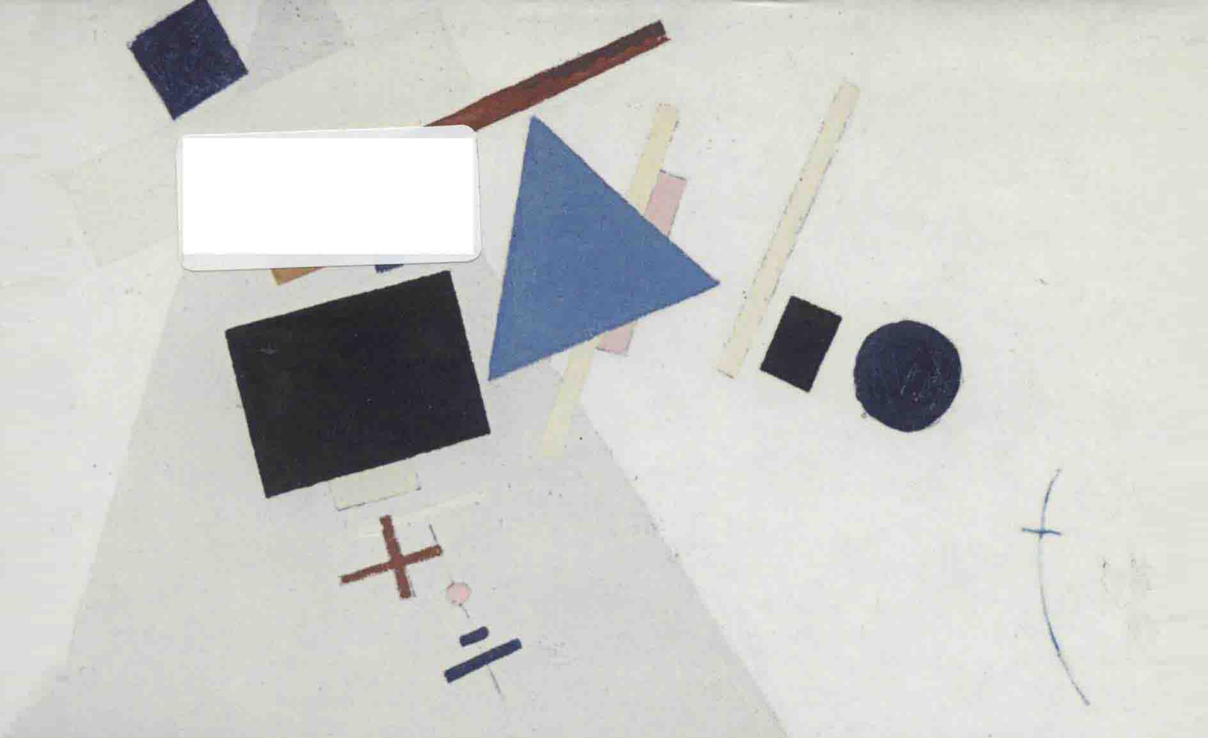

New Foundations for Physical Geometry

The Theory of Linear Structures

TIM MAUDLIN

OXFORD

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New Foundations for Physical Geometry

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To V
Volim te

I criticize by creation, not by finding fault.

Cicero

Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.

Galileo

Acknowledgments

Composing this manuscript has been the most exhilarating intellectual adventure of my life. Perhaps, in the end, I have been tilting at windmills. Even so, every day for several years I have felt as if I were battling giants, and with some success. The aim of these volumes is nothing less than the creation of a new mathematical language, designed to represent the physical structure of the universe. The language exists. And as with any new language, it takes experimentation and imagination and fastidious care to learn how to express concepts within it. Many friends and students and colleagues have been enthusiastic about the project. The result is immeasurably better for their criticisms, insights, and contributions.

First and foremost are the students who attended a graduate seminar at Rutgers in the spring of 2010. Their sharp analytical skills and exceptional meticulousness in attacking problems revealed many errors and shortcomings in the manuscript. It was a unique experience: we were all working together, trying out possibilities and exploring new territories of thought. I owe a particularly profound debt to Z. Perry, Olla Solmyak, Thomas Blanchard, Martin Glazier, and especially Justin Bush, who quickly settled several questions that I had not been able to resolve.

Doug Kutach commented extensively on the whole draft, with many valuable comments on both content and presentation. No one has gone through the manuscript with so much precision and care, and I am deeply in his debt. Doug has constructed a web site, Project Line at sagaciousmatter.org, for discussion of this project.

Bert Sweet once again ferreted out many infelicities. Adam Elga made the case that problem sets would be pedagogically useful (and it was only through poorly constructed problems that many errors in definition and analysis came to light in spring 2010).

Very early on, Ned Hall became a fellow explorer, and many of his ideas and insights are woven into the text. Detlef Dürr's positive reaction to the project was profoundly heartening and inspiring. The interest of Frank Arntzenius and Cian Dorr, although perhaps more skeptical, has also been a source of encouragement. The rest of the Mirror Lake Institute—Shelly Goldstein, Nino Zanghì, Rodi Tumulka, David Albert, Barry Loewer, and Jim Pryor—have as ever created and continue to create the ideal climate for conceptual investigation. Without all of this support, over many, many years, none of this would have come to be.

A different sort of support, irreplaceable in its own way, came from the John Simon Guggenheim Memorial Foundation. I was privileged to be chosen as a Guggenheim Fellow, which allowed me to devote the whole academic year of 2008–09 to this project. At the time, I vastly underestimated the extent of the task before me. Without the Foundation's support it would have been many more years (if ever) before the project came to fruition. Rutgers University generously supplied the additional resources to make the critical year possible.

My deepest and most inexpressible debt is to my family. And most of all to Vishnya, to whom it all is dedicated.

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Introduction

The thesis of these books is both simple and audacious. It is so simple that the basic claims can be reduced to two sentences. First: the most fundamental geometrical structure that organizes physical points into a space is the line.[1] Second: what endows spacetime with its geometry is time. The remainder of these two volumes does nothing but elucidate those sentences. Everything flows from them in such a straightforward way that I am almost convinced that the reader could stop reading forthwith and, with sufficient patience and diligence, reconstruct most of what follows from these two propositions.

As for the audacity, acceptance of either of these propositions demands the rejection of widely held and deeply entrenched alternatives. Consider a collection of objects that we wish to regard as forming not merely a set (which it does automatically) but as forming a *space*. Organizing the set into a space requires something more than the set-theoretic structure. What, at the most primitive level, is this "something else"?

For over a century, the mathematical subject devoted to this question has been *topology*. In topological theory, the fundamental structure that organizes a set into a space—organizes it so that notions such as the continuity of a function and the boundary of a set can be defined—is the *open set*. One specifies the topology of a space by specifying which of its subsets are open sets. All the topological characteristics of the space are then determined by the structure of its open sets.

Topology is sometimes called "rubber sheet geometry" because it describes geometrical characteristics of a space that are preserved under "stretching" the space without tearing or pasting. It is not obvious what should be meant by "tearing" or "pasting" a space, but the salient point is that topology concerns some sort of geometrical structure that is independent of distances. Intuitively, stretching can change the distances between points, but will not change, for example, whether one point is enclosed by another set of points. As we will say, topology concerns the *submetrical* structure of a space. Standard topology asserts that the specification of the open sets confers this structure on the space.

I will offer an alternative mathematical tool—a different way of understanding submetrical structure. This requires constructing a competitor to standard topology,

[1] The word "line" sometimes connotes only *straight* lines. The intention here is a usage that covers both straight and curved lines, since we will be considering a level of abstraction at which the distinction between straight and curved does not exist.

which I call the *Theory of Linear Structures*. Simply put, specifying the Linear Structure of a space amounts to specifying which sets of points in the space are lines. In terms of the lines, notions such as the continuity of a function, the boundaries of a set, and the connectedness of a space are defined. These definitions sometimes render different verdicts than the standard topological definitions and have a wider sphere of natural application. My burden is to show that we do better, when considering the geometrical structure of a physical space, by thinking in terms of the Linear Structure of the space than in terms of its open sets. If I am right, then the standard mathematical tools used for analyzing physical geometry have, for over a hundred years, been the wrong tools.

So the first task to be tackled is purely mathematical: to present, from its foundations, a new method of analyzing submetrical structure. Anyone familiar with the tremendous scope and complexity of topology will appreciate the audacity mentioned previously. Topology is the subject of hundreds of books and many thousands of papers in mathematics. Recovering or recasting the results of standard topological analysis in terms of Linear Structures would be the work of several lifetimes. So all that can be done here is to lay the foundations, to show how the most basic concepts defined in standard topology can be given alternative definitions in the Theory of Linear Structures. This first volume of *New Foundations* is devoted to this task, and will not cover even as much territory as the most elementary introduction to standard topology. If I am able to convince the reader of the value of this new approach, it will not be by seeing farther than with the standard theory, but by looking deeper. I will try to show that the definitions and analyses available in the Theory of Linear Structures offer a better understanding of geometrical structure, and allow for definitions that more closely capture the intuitive notions we are trying to explicate, than do the standard definitions. We *understand* geometrical structure better if we think in terms of lines rather than open sets.

Even if one comes to share this assessment, still the magnitude of the task I am suggesting may render the undertaking slightly absurd. It is rather like noticing that the Empire State Building would have been better situated had it been built a few blocks over and turned on an angle. One may agree with the appraisal, but still be reluctant to go to the trouble to reconstruct on better foundations. Maybe standard topological theory is not the best way to understand physical geometry, but it is still *good enough*. Thomas Kuhn observed: "As in manufacture, so in science—retooling is an extravagance to be reserved for the occasion that demands it" (Kuhn, 1996, p. 76). Persuasive arguments that such an occasion has arisen are hard to come by, and the more extensive the retooling, the more persuasive they must be. Following common practice when confronting such problems, I will resort to both a carrot and a stick.

The stick consists in a critique of standard topology. Of course, the issue is not a *mathematical* one: standard topology is a perfectly well-defined mathematical subject with rigorous and wide-ranging results. Rather, the critique is conceptual. A formalized mathematical subject such as topology is devised in the first place to capture, in a clear and precise language, certain informal concepts already in use. It is only because we begin with some grasp of a subject like geometrical structure that we seek strict definitions in the first place. Those formalized definitions can do a better or worse job of capturing the informal concepts whose names they inherit. It may be tempting to think that this is a purely *semantic* debate, in the pejorative sense of that term: after all, if someone wants to *define* a word like "continuous" or "connected" or "boundary" using the resources of standard topology, who is to object? As long as the definition is given, one can regard the term as nothing but an abbreviation, a concise way to refer to the defined concept. Such an approach makes the whole project of criticizing formal definitions appear wrong-headed.

But the situation is subtler than that. Certain mathematical terms are not chosen arbitrarily, but are used because we already have some understanding of them. Long before the formal theory of topology was developed, mathematicians had something in mind when they characterized a function as continuous or a space as connected. Their concepts may have been somewhat imprecise, but everyone would have accepted some clear instances of continuous and discontinuous functions. For example, the sine function is a continuous function and the step function is not. And beyond these particular examples, notions such as continuity would be explicated by informal definitions. So when the topologist seeks to define "continuity" in her proprietary technical language, she is not entirely free. The definition must be shown to correspond—to the extent that a formally defined notion can correspond to a more informal and fuzzy one—with the concept with which one began. If it does not, then the formal theory has failed in its aim.

In the first chapter of the *Physics*, Aristotle characterized the method of science as starting from those things that are clearer and more knowable to us and proceeding to those things that are clearer and more knowable in themselves. The mathematical elucidation of geometrical structure must proceed in the same way: one starts with the familiar, though somewhat obscure, and proceeds to the clearly and exactly defined. The fundamental axioms and definitions are presented in a more rigorous technical vocabulary, and then the initial notions are defined, and illuminated, by means of the technical notions. One returns to the starting point with a deeper understanding. But if one of the tasks is to explicate those initial concepts, then one should carefully consider whether the technical definitions have done justice to the original concepts, at least where their application was clear and uncontroversial.

Different readers will probably have wildly divergent reactions to these criticisms of standard topology. In particular, readers already familiar with the standard definitions—especially mathematicians or physicists who commonly use the standard theory—will have so internalized the standard definitions that *those definitions express what they now mean by terms such as "continuous"*. These readers will have to make an effort to recall the original, somewhat amorphous, concepts that stood in need of clarification. And given the utility of the formalized notion, such readers are likely to see no point in trying to capture some more naïve notion. On the other hand, readers with little background in standard topology have the double task of learning the standard definitions and evaluating criticisms of them at the same time. They may be more open to accepting the critique, but also less concerned about it in the first place. So I will not place too much weight on these shortcomings of the standard theory; though I will point them out nonetheless.

Perhaps a more effective line of attack concerns the scope of application of the standard theory. Topology was initially developed as a tool for describing certain spaces—a central example being Euclidean space. In particular, the spaces most naturally suited for topological treatment are *continua* (we leave to later sections the discussion of exactly what this means!). But the single most important object of which we need a geometrical account is *physical space* (or *spacetime*), and there is no guarantee that physical space is a continuum. Indeed, many physicists believe that at a sufficiently fine scale physical space is discrete rather than continuous. If standard topology is not an effective tool for articulating the geometrical structure of discrete spaces, then it may not be well suited for the primary requirements of physics. It would, in any case, be preferable to have an account of geometrical structure that can be applied with equal ease to discrete and continuous spaces. The Theory of Linear Structures can be so applied.

Sticks, however, will never be enough to drive mathematicians and physicists out of the precincts of standard topology. Even if the standard approach is somehow flawed, they will reasonably demand a viable alternative. So the onus of persuasion must rest with the carrot: the Theory of Linear Structures must be sufficiently intriguing in its own right to attract interest. I cannot claim unbiased judgment here, but I can attest that playing with the theory is a tremendous amount of fun. One is given a set of primitives (the lines), and then one has to try to fashion reasonable definitions of other geometrical notions in terms of them. Often it is not obvious how to do this, and many alternative strategies present themselves. For example, once the set of lines in a space has been specified, how can one define what it means for a set of points to be open, or closed, or for one set of points to be the boundary of another, or for a set to be connected, or for a function from one space to another to be continuous? There is no mechanical algorithm for producing such definitions, nor any indisputable standard by which a proposed definition can be evaluated. One wants the definitions to be natural and to yield intuitively correct results, but one also wants the definitions to *lead to interesting theorems*. That is, the properties invoked in the definitions need to be exactly those properties from which other interesting results can be derived. But the fecundity of definitions is only established by the production of proofs. A fascinating dialectic therefore develops: one proposes a definition and then sees whether interesting proofs using the defined properties are forthcoming. If the proofs require slightly different properties, then the definitions can be adjusted.[2] Given the nature of the dialectic, one is always left uncertain

[2] For a delightful discussion of this dialectic in the search for formal definitions of informal concepts, see Imre Lakatos's *Proof and Refutations* (1976). My own experience in trying to formulate definitions in terms of the Linear Structure corresponds exactly to Lakatos's description.

whether better definitions are not possible: one needs the definitions to generate the proofs, but one only gets a sense of how fecund the definitions are once the proofs are available. If the foregoing description seems too abstract, I can recommend only that the reader try it: once the basic axioms of a Linear Structure have been specified, try to construct definitions of terms like "open set" or "continuous function". I hope that especially mathematicians and physicists will give this a shot, and see how easy it is to become hooked. The feeling of productive conceptual play is the ultimate carrot that I have to offer.

The first feat of audacity, then, is to contend that the most well-entrenched approach to the formal analysis of geometrical structure should be forsaken, in some contexts, for a completely new one. In the spirit of fair play, the second thesis should be as outrageous to physicists as the first is to mathematicians. For if one accepts the use of Linear Structures to articulate submetrical geometry, then the foremost *physical* question that confronts us is: what accounts for the Linear Structure of physical spacetime? I claim that the geometry of spacetime is produced by time.

Why should such a claim be considered audacious? Because it reverses the common wisdom about the theory of Relativity. Relativity is often taken to imply that time is "just another dimension" like a spatial dimension, so the notion that there is anything physically special about time (as opposed to space) is outmoded classical thinking. Relativity is said to postulate a "four-dimensional block universe" which is "static", and in which the passage of time is just an illusion. Einstein himself wrote, after the death of his great friend Michele Besso, that "[f]or those of us who believe in physics, this separation between past, present, and future is only an illusion, however tenacious" (Einstein, 1972, p. 258). In short, Relativity is commonly characterized as having *spatialized time*; that is, of having put the temporal dimension on an equal physical footing with the spatial dimensions, and of having thereby robbed time of any fundamental difference from space.

My contention is just the opposite: the theory of Relativity shows, for the first time in the history of physics, how to *temporalize space*. In Relativity, but not in any preceding classical theory, one can regard time as the basic organizing structure of spacetime. *In a precise sense, spacetime has geometrical structure only because it has temporal structure, and insofar as there is spatial geometry at all, it is parasitic on temporal structure.* The argument to this conclusion is straightforward: the (submetrical) geometry of spacetime is determined by its Linear Structure, and the Linear Structure of a Relativistic spacetime is determined by its temporal structure. So rather than somehow demoting time from its position in classical physics, Relativity promotes time to a more central position. This thesis will be the topic of the second volume of *New Foundations*, which will begin with a short recap of the basic mathematical results, so readers more interested in the physics than the mathematics may prefer that volume to this. For mathematicians, the opposite preference may hold.

Having touted the outrageousness of these books' central claims, let me now calm the waters. Regarding the physical thesis, we should immediately note that the special geometrical role of time in structuring spacetime is not, at a technical level, at all contentious. The standard account of spacetime structure in Relativity permits a simple temporal characterization of time-like lines while no parallel characterization of space-like lines exists. The only real bone of contention here will be the significance of that fact.

With regard to the mathematical claim, let me reiterate that there is nothing wrong *per se* with standard topology as a tool of mathematical analysis. Many questions can be properly and insightfully addressed by standard topological analysis. The weakness of standard topology emerges chiefly when treating the specific subject of *geometrical space*. But what exactly do I mean by that term?

Metaphorical and Geometrical Spaces

In the right context, almost any collection of objects can be considered to form a "space". For example, if one is studying Newtonian mechanics, such as Newton's theory of gravity applied to point particles, it is natural to speak of "the space of solutions" of Newton's equations of motion. Each "point" in this space, each individual element, describes the motions of a set of particles governed by Newtonian gravity. There is an intuitive sense—which can be made technically precise—in the which various solutions can be "closer" or "farther" from one another, and hence an intuitive sense in which the whole set of solutions can be thought of as having a "geometry". But this sort of talk of a "space" is evidently not literal. This "space" is, in an obvious sense, a *metaphorical* space; it is just a way of talking about the solutions and a measure of *similarity* among them. Analogously, philosophers are wont to speak of "logical space" as the set of all possible worlds. But this set also only forms a "space" in a metaphorical sense: space talk is just a picturesque means of discussing various ways and degrees that individual possible worlds are similar to one another.

In contrast, consider Euclidean space, the subject matter of Euclidean geometry.[3] Euclidian space is an abstract object in the way that all mathematical objects are abstract. But Euclidian space is not just metaphorically a space. When we say that one point in Euclidian space is "closer" to another than it is to a third, we are not suggesting that the first point is more similar to the second than to the third in any way. Indeed, intrinsically the points of Euclidian space are all exactly alike: they are all, in themselves, perfectly identical. The points of Euclidian space, unlike the "points" of the space of solutions to Newton's equations, really are points: they have no internal structure. The "points" of the space of solutions form a (metaphorical) "space" only because they are highly structured and different from one another.

[3] Just exactly what this means is not perfectly clear! Euclid thought he was studying the structure of physical space, but we now take that view to be mistaken. Still, Euclidean geometry seems to have a subject matter: for example, certain constructible figures in the Euclidean plane. Our understanding of what is meant by "the Euclidean plane" seems to be sharp enough to pick out an abstract structure fairly precisely.

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