SIXTH

COLLEGE ALGEBRA

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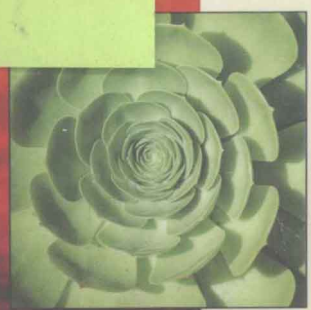

Michael Sullivan

College Algebra

SIXTH EDITION

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Preface to the Instructor

As a professor at an urban public university for over 30 years, I am aware of the varied needs of college algebra students who range from having little mathematical background and a fear of mathematics courses to those who have had a strong mathematical education and are highly motivated. For some of your students, this will be their last course in mathematics, while others may decide to further their mathematical education. I have written this text for both groups. As the author of precalculus, engineering calculus, finite math and business calculus texts, and, as a teacher, I understand what students must know if they are to be focused and successful in upper level mathematics courses. However, as a father of four college graduates, I also understand the realities of college life. I have taken great pains to insure that the text contains solid, student-friendly examples and problems, as well as a clear, seamless, writing style. I encourage you to share with me your experiences teaching from this text.

The Sixth Edition

The Sixth Edition builds upon a solid foundation by integrating new features and techniques that further enhance student interest and involvement. The elements of previous editions that have proved successful remain, while many changes, some obvious, others subtle, have been made. A huge benefit of authoring a successful series is the broad-based feedback upon which improvements and additions are ultimately based. Virtually every change to this edition is the result of thoughtful comments and suggestions made from colleagues and students who have used previous editions. I am sincerely grateful for this feedback and have tried to make changes that improve the flow and usability of the text.

New to the Sixth Edition

Real Mathematics at Motorola

Each chapter begins with  Field Trip to Motorola, a brief description of a current situation at Motorola, followed by  Interview at Motorola, a biographical sketch of a Motorola employee. At the end of each chapter is  Project at Motorola, written by the Motorola employee, that contains a description, with exercises, of a problem at Motorola that relates to the mathematics found in the chapter. It doesn't get more REAL than this.

Preparing for This Section

Most sections now open with a referenced list (by section and page number) of key items to review in preparation for the section ahead. This provides a just-in-time review for students.

Chapter R Review

This chapter, a revision of the old Chapter 1, has been renamed to more accurately reflect its content. It may be used as the first part of the course or as a just-in-time review when the content is required in a later chapter. Specific references to this chapter occur throughout the book to assist in the review process.

Content

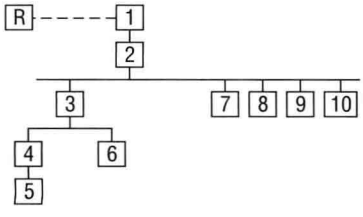
The Appendix, Graphing Utilities, has been updated and expanded to include the latest features of the graphing calculator. While the graphing calculator remains an option, identified by a graphing icon, references to the Appendix occur at appropriate places in the text for those inclined to use the graphing calculator features of the text.

Organization

- The discussion on Rational Functions now appears in two sections, Rational Functions I and Rational Functions II: Analyzing Graphs. This division should allow the sections to be covered in one teaching period each.
- The discussion of Polynomial and Rational Inequalities now appears after Polynomial and Rational Functions. This allows us to use information obtained about the graphs to solve the inequalities. Students and instructors will appreciate how easy this usually tough concept is now handled.
- Zeros of a Polynomial Function now appears in a separate chapter following Polynomial and Rational Functions to provide more flexibility in teaching and testing.
- Separate chapters on Sequences; Induction; the Binomial Theorem and Counting and Probability also provide more flexibility in coverage.

Features in the 6th Edition

- Section ***OBJECTIVES*** appear in a numbered list to begin each section.
- NOW WORK PROBLEM XX appears after a concept has been introduced. This directs the student to a problem in the exercises that tests the concept, insuring that the concept has been mastered before moving on. The Now Work problems are identified in the exercises using yellow numbers and a pencil icon.
- Optional Comments, Explorations, Seeing the Concept, Examples, and Exercises that utilize the graphing calculator are clearly marked with a calculator icon. Calculator exercises are also identified by the icon and green numbers.
- References to Calculus are identified by a calculus icon.
- Historical Perspectives, sometimes with exercises, are presented in context and provide interesting anecdotal information.
- Varied applications are abundant both in Examples and in Exercises. Many contain sourced data.
- Discussion, Writing, and Research problems appear in each exercise set, identified by an icon and red numbers. These provide the basis for class discussion, writing projects, and collaborative learning experiences.
- An extensive Chapter Review provides a list of important formulas, definitions, theorems, and objectives, as well as a complete set of Review Exercises, with sample test questions identified by blue numbers.



Using the 6th Edition Effectively and Efficiently with your Syllabus

To meet the varied needs of diverse syllabi, this book contains more content than expected in a college algebra course. The illustration shows the dependencies of chapters on each other.

As the chart indicates, this book has been organized with flexibility of use in mind. Even within a given chapter, certain sections can be skipped without fear of future problems.

Chapter R Review

This chapter, a revision of the old Chapter 1, has been renamed to more accurately reflect its content. It may be used as the first part of the course or as a just-in-time review when the content is required in a later chapter. Specific references to this chapter occur throughout the book to assist in the review process.

Chapter 1 Equations and Inequalities

Primarily a review of Intermediate Algebra topics, this material is prerequisite for later topics. For those who prefer to treat complex numbers and negative discriminants early, Section 5.3 can be covered at any time after Section 1.3.

Chapter 2 Graphs

This chapter lays the foundation. Sections 2.5 and 2.6 may be skipped without adverse effects.

Chapter 3 Functions and Their Graphs

Perhaps the most important chapter. Section 3.6 can be skipped without adverse effects.

Chapter 4 Polynomial and Rational Functions

Topic selection is dependent on your syllabus.

Chapter 5 The Zeros of a Polynomial Function

Topic selection is dependent on your syllabus. Section 5.1 is not absolutely necessary, but its coverage makes some computations easier.

Chapter 6 Exponential and Logarithmic Functions

Sections 6.1–6.5 follow in sequence; Sections 6.6, 6.7, and 6.8 each require Section 6.3.

Chapter 7 Conics

Sections 7.1–7.4 follow in sequence.

Chapter 8 Systems of Equations and Inequalities

Sections 8.1–8.2 follow in sequence; Sections 8.3–8.8 require Sections 8.1 and 8.2, and may be covered in any order. Section 8.9 depends on Section 8.8.

Chapter 9 Sequences; Induction; the Binomial Theorem

There are three independent parts: Sections 9.1–9.3, 9.4, and 9.5.

Chapter 10 Counting and Probability

Sections 10.1–10.3 follow in order.

ACKNOWLEDGMENTS

Textbooks are written by authors, but evolve from an idea into final form through the efforts of many people. Special thanks to Don Dellen, who first suggested this book and the other books in this series. Don's extensive contributions to publishing and mathematics are well known; we all miss him dearly.

I would like to thank Motorola and its people who helped make the projects in this new edition possible. Special thanks to Iwona Turlik, Vice President and Director of the Motorola Advanced Technology Center (MATC), for providing the opportunity to share with students examples of their experience in applying mathematics to engineering tasks.

I would also like to thank the authors of these projects:

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- Bill Oslon, Andrew Skipor, John St. Peter, Tom Tirpak, and George Valliath, Motorola Laboratories, Motorola Advanced Technology Center (MATC)
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Michael Sullivan

Preface to the Student

As you begin your study of College Algebra, you may feel overwhelmed by the number of theorems, definitions, procedures, and equations that confront you. You may even wonder whether or not you can learn all of this material in the time allotted. These concerns are normal. Keep in mind that many elements of College Algebra are all around us as we go through our daily routines. Many of the concepts you will learn to express mathematically, you already know intuitively. For many of you, this may be your last math course, while for others, just the first in a series of many. Either way, this text was written with you in mind. I have taught college algebra courses for over thirty years. I am also the father of four college students who called home from time to time, frustrated and with questions. I know what you're going through. So I have written a text that doesn't overwhelm, or unnecessarily complicate College Algebra, but at the same time it gives you the skills and practice you need to be successful.

This text is designed to help you, the student, master the terminology and basic concepts of College Algebra. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of the material easier and more rewarding. This book is meant to be a "machine for learning," one that can help you focus your efforts and get the most from the time and energy you invest.

How to use this Book Effectively and Efficiently

First, and most important, this book is meant to be read—so please, begin by reading the material assigned. You will find that the text has additional explanation and examples that will help you. Also, it is best to read the section before the lecture, so you can ask questions right away about anything you didn't understand.

Many sections begin with "Preparing for This Section," a list of concepts that will be used in the section. Take the short amount of time required to refresh your memory. This will make the section easier to understand and will actually save you time and effort.

A list of ***OBJECTIVES*** is provided at the beginning of each section. Read them. They will help you recognize the important ideas and skills developed in the section.

After a concept has been introduced and an example given, you will see NOW WORK PROBLEM XX. Go to the exercises at the end of the section, work the problem cited, and check your answer in the back of the book. If you get it right, you can be confident in continuing on in the section. If you don't get it right, go back over the explanations and examples to see what you might have missed. Then rework the problem. Ask for help if you miss it again.

If you follow these practices throughout the section, you will find that you have probably done many of your homework problems. In the exercises, every "Now Work Problem" number is in yellow with a pencil icon . All the odd-numbered problems have answers in the back of the book and worked-out solutions in the Student Solutions Manual supplement. Be sure you have made an honest effort before looking at a worked-out solution.

At the end of each chapter is a Chapter Review. Use it to be sure you are completely familiar with the equations and formulas listed under "Things to Know." If you are unsure of an item here, use the page reference to go back and review it. Go through the Objectives and be sure you can answer "Yes" to the question "I should be able to. . . ." If you are uncertain, a page reference to the objective is provided.

Spend the few minutes necessary to answer the "Fill-in-the-Blank" items and the "True/False" items. These are quick and valuable questions to answer.

Lastly, do the problems identified with blue numbers in the Review Exercises. These are my suggestions for a Practice Test. Do some of the other problems in the review for more practice to prepare for your exam.

Please do not hesitate to contact me, through Prentice Hall, with any suggestions or comments that would improve this text. I look forward to hearing from you.

Best Wishes!

Michael Sullivan

MOTOROLA PROJECTS

Everyone seems to have a cell phone or pager... Focusing on this type of product, we visit the Motorola Corporation. **"Field Trip to Motorola"** highlights an individual's use of mathematics on the job at Motorola. **"Interview at Motorola"** is a short biography chronicling that individual's educational and career path. The **"Project at Motorola"** concludes the chapter, leading you through an assignment like the one described at the beginning of the chapter.

CHAPTER 3

Functions and Their Graphs

Outline

Field Trip to Motorola

During the past decade the availability and usage of wireless Internet services has increased manyfold. The industry has developed a number of pricing proposals for such services. Marketing data have indicated that subscribers of wireless Internet services have tended to desire flat rate fee structures as compared with rates based totally on usage.

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Interview at Motorola

Jocelyn Carter-Miller is Corporate Vice President and Chief Marketing Officer (CMO) for Motorola, Inc., an over $30 billion global provider of integrated communications and embedded electronics solutions. As CMO she has helped build the Motorola brand and image and has developed high-performance marketing organizations and processes. Jocelyn also heads motorola.com, the Motorola electronic commerce and information Web site. In this new role, she and her team have developed a strategy for serving Motorola's broad and diverse constituencies offering a full range of electronic services.

In her previous roles as Vice President–Latin American and Caribbean Operations and Director of European, Middle East and African Operations, Jocelyn headed international wireless data communications operations for Motorola, creating profitable opportunities through strategic alliances, value-added applications, and new product and service launches. She also developed skills in managing complex, high-risk ventures in countries like Brazil and Russia, setting standards for her company's practices in emerging markets.

Prior to her career at Motorola, Jocelyn served as Vice President, Marketing and Product Development for Mattel, where she broke new ground, driving record sales of Barbie and other toys using integrated product, entertainment, promotional, and licensing programs.

Jocelyn builds strong relationships and new opportunities through her involvement on outside boards and community organizations. She serves on the board of the Principal Financial Group and on the nonprofit boards of the Association of National Advertisers, the University of Chicago Women's Business Group Advisory Board, and the Smart School Charter Middle School.

Jocelyn holds a Master of Business Administration degree in marketing and finance from the University of Chicago and a Bachelor of in accounting from the University bana–Champaign and is a Certified tant. She is married to Edward Mill Edventures, an educational reform de and has two daughters, Alexis and K

Jocelyn has won numerous award in national publications, and regularly ness and community groups. She has c Melissa Giavagnoli the book *Netwo Relationships and Opportunities for* was published in June 2000 by Jossey- Through their Web site Networlding.c Melissa facilitate meaningful connec ally beneficial opportunities for no Networlders alike.

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Project at Motorola

During the past decade the availability and usage of wireless Internet services have increased. The industry has developed a number of pricing proposals for such services. Marketing data have indicated that subscribers of wireless Internet services have tended to desire flat fee rate structures as compared with rates based totally on usage. The Computer Resource Department of Indigo Media (hypothetical) has entered into a contractual agreement for wireless Internet services. As a part of the contractual agreement, employees are able to sign up for their own wireless services. Three pricing options are available:

Silver Plan: $20/month for up to 200 K-bytes of service plus $0.16 for each additional K-byte of service

Gold Plan: $50/month for up to 1000 K-bytes of service plus $0.08 for each additional K-byte of service

Platinum Plan: $100/month for up to 3000 K-bytes of service plus $0.04 for each additional K-byte of service

You have been requested to write a report that answers the following questions in order to aid employees in choosing the appropriate pricing plan.

(a) If C is the monthly charge for x K-bytes of service, express C as a function of x for each of the three plans.

(b) Graph each of the three functions found in part (a).

(c) For how many K-bytes of service is the Silver Plan the best pricing option? When is the Gold Plan best? When is the Platinum Plan best? Explain your reasoning.

(d) Write a report that summarizes your findings.

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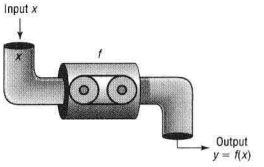
CLEAR WRITING STYLE

Sullivan's **accessible writing style** is apparent throughout, often utilizing various approaches to the same concept. An author who writes clearly makes potentially difficult concepts intuitive, making class time more productive.

Sometimes it is helpful to think of a function f as a machine that receives as input a number from the domain, manipulates it, and outputs the value. See Figure 6.

Figure 6



The restrictions on this input/output machine are as follows:

1. It only accepts numbers from the domain of the function.
2. For each input, there is exactly one output (which may be repeated for different inputs).

For a function $y = f(x)$, the variable x is called the **independent variable,** because it can be assigned any of the permissible numbers from the domain. The variable y is called the **dependent variable,** because its value depends on x.

Pages 208-209

PREPARING FOR THIS SECTION

Before getting started, review the following:

- ✓ Intervals (Section 1.5, pp. 125–126)
- ✓ Evaluating Algebraic Expressions, Domain of a Variable (Review, Section 2, pp. 19–21)
- ✓ Intercepts (Section 2.2, pp. 157–158)
- ✓ Scatter Diagrams; Linear Curve Fitting (Section 2.5, pp. 185–189)

3.1 FUNCTIONS

OBJECTIVES
1. Determine Whether a Relation Represents a Function
2. Find the Value of a Function
3. Find the Domain of a Function
4. Identify the Graph of a Function
5. Obtain Information from or about the Graph of a Function

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PREPARING FOR THIS SECTION

The **"Preparing for this Section"** feature provides you and your instructor with a list of skills and concepts needed to approach the section, along with page references. You can use the feature to determine what you should review before tackling each section.

STEP-BY-STEP EXAMPLES

Step-by-step examples ensure that you follow the entire solution process and give you an opportunity to check your understanding of each step.

EXAMPLE 5 Analyzing the Graph of a Rational Function with a Hole

Analyze the graph of the rational function: $R(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$

Solution We factor R and obtain

$$R(x) = \frac{(2x - 1)(x - 2)}{(x + 2)(x - 2)}$$

In lowest terms,

$$R(x) = \frac{2x - 1}{x + 2}, \qquad x \neq -2$$

STEP 1: The domain of R is $\{x \mid x \neq -2, x \neq 2\}$.

STEP 2: The graph has one x-intercept: $1/2$. The y-intercept is $R(0) = -1/2$.

STEP 3: Because

$$R(-x) = \frac{2x^2 + 5x + 2}{x^2 - 4}$$

we conclude that R is neither even nor odd. Thus, there is no symmetry with respect to the y-axis or the origin.

STEP 4: The graph has one vertical asymptote, $x = -2$, since $x + 2$ is the only factor of the denominator of $R(x)$ *in lowest terms*. However, the rational function is undefined at both $x = 2$ and $x = -2$.

STEP 5: Since the degree of the numerator equals the degree of the denominator, the graph has a horizontal asymptote. To find it, we either use long division or form the quotient of the leading coefficient of the numerator, 2, and the leading coefficient of the denominator, 1. Thus, the graph of R has the horizontal asymptote $y = 2$. To find out whether the graph of R intersects the asymptote, we solve the equation $R(x) = 2$.

$$\begin{aligned} R(x) &= \frac{2x - 1}{x + 2} = 2 \\ 2x - 1 &= 2(x + 2) \\ 2x - 1 &= 2x + 4 \\ -1 &= 4 \qquad \text{Impossible} \end{aligned}$$

The graph does not intersect the line $y = 2$.

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REAL-WORLD DATA

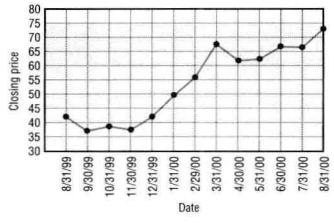
Real-world data is incorporated into examples and exercise sets to emphasize that mathematics is a tool used to understand the world around us. As you use these problems and examples, you will see the relevance and utility of the skills being covered.

TABLE 1

Date	Closing Price ($)
8/31/99	41.09
9/30/99	37.16
10/31/99	38.72
11/30/99	38.34
12/31/99	41.16
1/31/00	49.47
2/29/00	56.50
3/31/00	65.97
4/30/00	63.41
5/31/00	62.34
6/30/00	66.84
7/31/00	66.75
8/31/00	74.88

Courtesy of A.G. Edwards & Sons, Inc.

Figure 8
Monthly closing prices of Intel stock 8/31/99 through 8/31/00



We can see from the graph that the price of the stock was rising rapidly from 11/30/99 through 3/31/00 and was falling slightly from 3/31/00 through 5/31/00. The graph also shows that the lowest price occurred at the end of September, 1999, whereas the highest occurred at the end of August, 2000. Equations and tables, on the other hand, usually require some calculations and interpretation before this kind of information can be "seen."

Look again at Figure 8. The graph shows that for each date on the horizontal axis there is only one price on the vertical axis. Thus, the graph represents a function, although the exact rule for getting from date to price is not given.

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"NOW WORK" PROBLEMS

Many examples end with the phrase **"Now Work Problem —."** Sending you to the exercise set to work a similar problem provides the opportunity to immediately check your understanding. The corresponding "Now Work" problem is easily identified in the exercise sets by the pencil icon and yellow exercise number.

63. **Suspension Bridge** A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center. (Assume that the road is level.)

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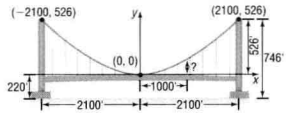
EXAMPLE 10 The Golden Gate Bridge

The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its 746-foot-tall towers are 4200 feet apart. The bridge is suspended from two huge cables more than 3 feet in diameter; the 90-foot-wide roadway is 220 feet above the water. The cables are parabolic in shape and touch the road surface at the center of the bridge. Find the height of the cable at a distance of 1000 feet from the center.

Solution We begin by choosing the placement of the coordinate axes so that the x-axis coincides with the road surface and the origin coincides with the center of the bridge. As a result, the twin towers will be vertical (height $746 - 220 = 526$ feet above the road) and located 2100 feet from the center. Also, the cable, which has the shape of a parabola, will extend from the towers, open up, and have its vertex at $(0, 0)$. As illustrated in Figure 14, the choice of placement of the axes enables us to identify the equation of the parabola as $y = ax^2$, $a > 0$. We can also see that the points $(-2100, 526)$ and $(2100, 526)$ are on the graph.

Figure 14



Based on these facts, we can find the value of a in $y = ax^2$.

$$y = ax^2$$
$$526 = a(2100)^2 \quad y = 526;\ x = 2100$$
$$a = \frac{526}{(2100)^2}$$

The equation of the parabola is therefore

$$y = \frac{526}{(2100)^2}x^2$$

The height of the cable when $x = 1000$ is

$$y = \frac{526}{(2100)^2}(1000)^2 \approx 119.3 \text{ feet}$$

The cable is 119.3 feet high at a distance of 1000 feet from the center of the bridge. ■

NOW WORK PROBLEM 63.

FITTING A QUADRATIC FUNCTION TO DATA

5 In Section 2.5 we found the line of best fit for data that appeared to be linearly related. It was noted that data may also follow a nonlinear relation. Figures 15(a) and (b) show scatter diagrams of data that follow a quadratic relation.

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SOLUTIONS

Solutions, both algebraic and graphical, are clearly expressed throughout the text.

EXAMPLE 1 **Graphing a Quadratic Function Using Trasformations**

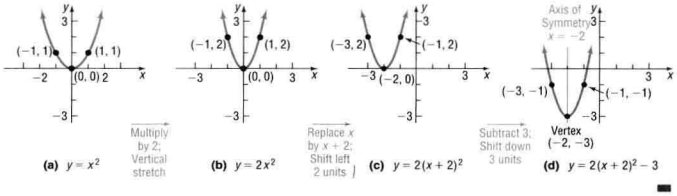
Graph the function $f(x) = 2x^2 + 8x + 5$. Find the vertex and axis of symmetry.

Solution We begin by completing the square on the right side.

$$f(x) = 2x^2 + 8x + 5$$
$$= 2(x^2 + 4x) + 5 \qquad \text{Factor out the 2 from } 2x^2 + 8x.$$
$$= 2(x^2 + 4x + 4) + 5 - 8 \qquad \text{Complete the square of } 2(x^2 + 4x). \text{ Notice that the factor of 2 requires that 8 be added and subtracted.}$$
$$= 2(x + 2)^2 - 3 \qquad (2)$$

The graph of f can be obtained in three stages, as shown in Figure 6. Now compare this graph to the graph in Figure 5(a). The graph of $f(x) = 2x^2 + 8x + 5$ is a parabola that opens up and has its vertex (lowest point) at $(-2, -3)$. Its axis of symmetry is the line $x = -2$.

Figure 6



(a) $y = x^2$ (b) $y = 2x^2$ (c) $y = 2(x + 2)^2$ (d) $y = 2(x + 2)^2 - 3$

Check: Graph $f(x) = 2x^2 + 8x + 5$ and use the MINIMUM command to locate its vertex.

NOW WORK PROBLEM 17.

The method used in Example 1 can be used to graph any quadratic function $f(x) = ax^2 + bx + c, a \neq 0$, as follows:

$$f(x) = ax^2 + bx + c$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c \qquad \text{Factor out } a \text{ from } ax^2 + bx.$$
$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) \qquad \text{Complete the square by adding and subtracting } a(b^2/4a^2). \text{ Look closely at this step!}$$
$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$
$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \qquad c - \frac{b^2}{4a} = c \cdot \frac{4a}{4a} - \frac{b^2}{4a} = \frac{4ac - b^2}{4a}$$

Based on these results, we conclude the following:

GRAPHING UTILITIES

Graphing utilities are optional in this text and their use is clearly identified by the use of the graphing utility icon.

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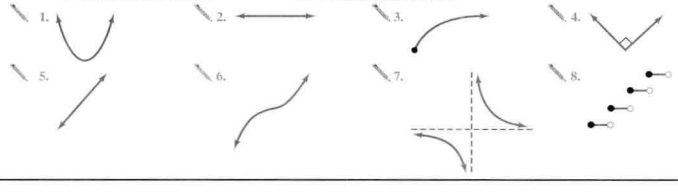
3.3 EXERCISES

In Problems 1–8, match each graph to the function listed whose graph most resembles the one given.

A. *Constant function* B. *Linear function* C. *Square function*
D. *Cube function* E. *Square root function* F. *Reciprocal function*
G. *Absolute value function* H. *Greatest integer function*



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END-OF-SECTION EXERCISES

Sullivan's exercises are unparalleled in terms of thorough coverage and accuracy. Each **end-of-section exercise** set begins with visual- and concept-based problems, starting you out with the basics of the section. Well-thought-out exercises better prepare you for exams.

MODELING

Many examples and exercises connect real-world situations to mathematical concepts. Learning to work with **models** is a skill that transfers to many disciplines.

with his or her age. The following table shows the median income I of individuals of different age groups within the United States for 1996. For each age group, the class midpoint represents the independent variable, x. For the age group "65 years and older," we will assume that the class midpoint is 69.5.

Age	Class Midpoint, x	Median Income, I
15–24 years	19.5	$21,438
25–34 years	29.5	$35,888
35–44 years	39.5	$44,420
45–54 years	49.5	$50,472
55–64 years	59.5	$39,815
65 years and older	69.5	$19,448

Source: U.S. Census Bureau

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) The quadratic function of best fit to these data is

$$I(x) = -44.8x^2 + 4009x - 41392$$

determine the age at which an
t to earn the most income.
predict the peak income earned.
to verify that the function given
dratic function of best fit.
ty, draw a scatter diagram of the
he quadratic function of best fit
m.

ports of crude oil (1000 barrels per day) for the years 1980–1997.

Year, x	Imports, I	Year, x	Imports, I
1980	5263	1989	5843
1981	4396	1990	5894
1982	3488	1991	5782
1983	3329	1992	6083
1984	3426	1993	6787
1985	3201	1994	7063
1986	4178	1995	7230
1987	4674	1996	7508
1988	5107	1997	7996

Source: U.S. Energy Information Administration

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) The quadratic function of best fit to these data is

$$I(x) = 18.04x^2 - 71,495.6x + 70,831,298$$

Use this function to determine the year in which imports of crude oil were lowest.
(c) Use the function found in part (b) to predict the number of barrels of imported crude oil in 1998.
(d) Use a graphing utility to verify that the function given in part (b) is the quadratic function of best fit.
(e) With a graphing utility, draw a scatter diagram of the data and then graph the quadratic function of best fit on the scatter diagram.

Page 335 **Pages 298-299**

The new graphs reflect the behavior produced by the analysis. Furthermore, we observe two turning points, one between 0 and 1 and the other to the right of 4. Rounded to two decimal places, these turning points are (0.52, 0.07) and (11.48, 2.75).

Figure 45

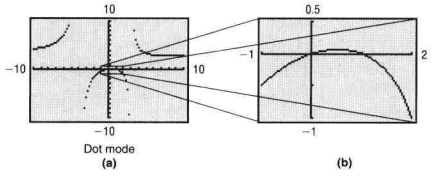
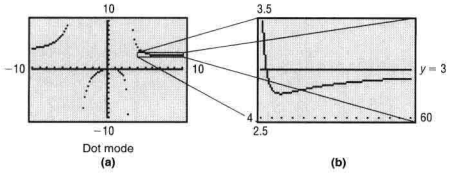


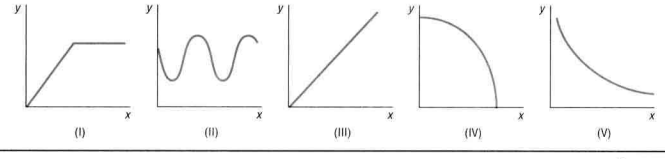
Figure 46



GRAPHING UTILITIES AND TECHNIQUES

Increase your understanding, visualize, discover, explore, and solve problems using a **graphing utility.** Sullivan uses the graphing utility to further your understanding of concepts, not to circumvent essential math skills.

73. Match each function with the graph that best describes the situation.
(a) The cost of building a house as a function of its square footage
(b) The height of an egg dropped from a 300-foot building as a function of time
(c) The height of a human as a function of time
(d) The demand for Big Macs as a function of price
(e) The height of a child on a swing as a function of time



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DISCUSSION WRITING AND READING EXERCISES

These exercises, clearly identified by the notebook icon and/or red numeration, are designed to get you to "think outside the box." These exercises seek to foster an intuitive understanding of key mathematical concepts. It is easy to find these exercises as they are highlighted by the book icon and red exercise number.

LINKS TO CALCULUS

This icon draws attention to the underpinnings of calculus.

If we know the function $C(t)$ that relates the year t to the cost C of tuition and fees, then the average rate of change from 1991 to 1992 may be expressed as

$$\text{Average rate of change} = \frac{C(1992) - C(1991)}{1992 - 1991} = \frac{251}{1} = \$251/\text{year}$$

Expressions like this occur frequently in calculus.

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CHAPTER REVIEW

The Chapter Review helps check your understanding of the chapter materials in several ways. **"Things to Know"** gives a general overview of review topics. The **"How To"** section provides a concept-by-concept listing of operations you are expected to perform. The **"Review Exercises"** then serve as a chance to practice the concepts presented within the chapter. Several of the Review Exercises are numbered in blue. These exercises can be combined to create the Chapter Test. Since these problems are odd numbered, you can check your answers in the back of the book. The review materials are designed to make you, the student, confident in knowing the chapter material.

CHAPTER REVIEW

Library of Functions

Linear function (p. 235)
$f(x) = mx + b$
Graph is a line with slope m and y-intercept b.

Constant function (p. 235)
$f(x) = b$
Graph is a horizontal line with y-intercept b.
See Figure 22.

Identity function (p. 235)
$f(x) = x$
Graph is a line with slope 1 and y-intercept 0.
See Figure 23.

Cube function (p. 236)
$f(x) = x^3$
See Figure 25.

Reciprocal function (p. 236)
$f(x) = 1/x$
See Figure 27.

Square function (p. 235)
$f(x) = x^2$
Graph is a parabola with intercept at $(0, 0)$.
See Figure 24.

Square root function (p. 236)
$f(x) = \sqrt{x}$
See Figure 26.

Absolute value function (p. 236)
$f(x) = |x|$
See Figure 28.

Greatest integer function (p. 237)
$f(x) = \text{int}(x)$
See Figure 29.

Things To Know

Function (p. 206)

A relation between two sets of real numbers so that each number x in the first set, the domain, has corresponding to it exactly one number y in the second set. The range is the set of y values of the function for the x values in the domain.

x is the independent variable; y is the dependent variable.

A function f may be defined implicitly by an equation involving x and y or explicitly by writing $y = f(x)$.

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Objectives

You should be able to:

Determine whether a relation represents a function (p. 205)

Find the value of a function (p. 209)

Find the domain of a function (p. 211)

Identify the graph of a function (p. 212)

Obtain information from or about the graph of a function (p. 213)

Find the average rate of a change of a function (p. 224)

Use a graph to determine where a function is increasing, is decreasing , or is constant (p. 227)

Use a graph to locate local maxima and minima (p. 228)

Determine even or odd functions fr... a graph (p. 229)

Identify even or odd functions from the equation (p. 230)

Graph the functions listed in the library of functions (p. 235)

Graph piecewise-defined functions (p. 238)

Graph functions using horizontal and vertical shifts (p. 242)

Graph functions using compressions and stretches (p. 246)

Graph functions using reflections

Fill-in-the-Blank Items

1. If f is a function defined by the equation $y = f(x)$, then x is calle... _____ variable.
2. A set of points in the xy-plane is the graph of a function if and only ... graph in at most one point.
3. The average rate of change of a function equals the _____ graph.
4. A(n) _____ function f is one for which $f(-x) = f(x)$ for e... function f is one for which $f(-x) = -f(x)$ for every x in the domain ...
5. Suppose that the graph of a function f is known. Then the graph of $y =$ ...

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True/False Items

T F **1.** Every relation is a function.

T F **2.** Vertical lines intersect the graph of a function in no more than one point.

T F **3.** The y-intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.

T F **4.** A function f is decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

T F **5.** Even functions have graphs that are symmetric with respect to the origin.

T F **6.** The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.

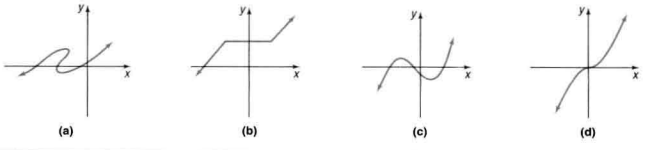
T F **7.** $f(g(x)) = f(x) \cdot g(x)$.

T F **8.** The domain of the composite function $(f \circ g)(x)$ is the same as that of $g(x)$.

Review Exercises

Blue problem numbers indicate the author's suggestions for use in a Practice Test.

1. Given that f is a linear function, $f(4) = -5$ and $f(0) = 3$, write the equation that defines f.
2. Given that g is a linear function with slope $= -4$ and $g(-2) = 2$, write the equation that defines g.
3. A function f is defined by $f(x) = \dfrac{Ax + 5}{6x - 2}$. If $f(1) = 4$, find A.
4. A function g is defined by $g(x) = \dfrac{A}{x} + \dfrac{8}{x^2}$. If $g(-1) = 0$, find A.
5. Tell which of the following graphs are graphs of functions.



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