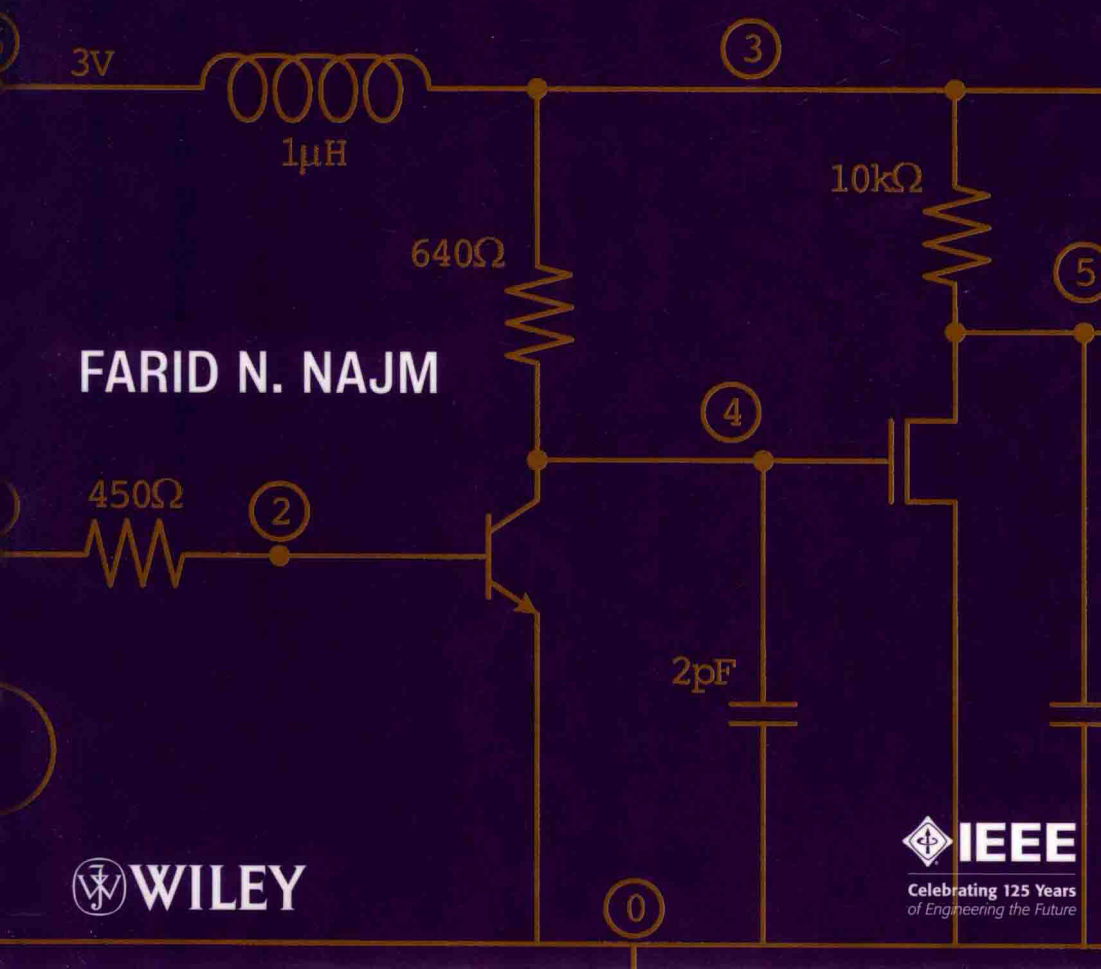




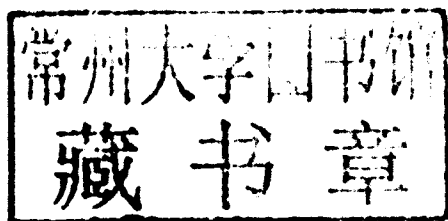
CIRCUIT SIMULATION

FARID N. NAJM



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Farid N. Najm



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CIRCUIT SIMULATION

To my wife, *Diana*,
and to our two daughters,
Lily Marie and *Tanya Kristen*.

PREFACE

This text describes in detail the numerical techniques and algorithms that are part of modern circuit simulators, with a focus on the most commonly used simulation modes: DC Analysis and Transient Analysis. After a general introduction in chapter 1, network equation formulation is covered in chapter 2, with emphasis on modified nodal analysis (MNA). The coverage also includes the network cycle space and bond space, element stamps, and the question of unique solvability of the system. Solving linear resistive circuits is the focus of chapter 3, which gives a comprehensive treatment of the most relevant aspects of linear system solution techniques. This includes the standard methods of Gaussian elimination (GE) and LU factorization, as well as some in-depth treatment of numerical error in floating point systems, pivoting for accuracy, sparse matrix methods, and pivoting for sparsity. Indirect solution methods, such as Gauss-Jacobi (GJ) and Gauss-Seidel (GS) are also covered. As well, some discussion of node tearing and partitioning is given, in recognition of the recent trend of increased usage of parallel software on multi-core computers.

Solving nonlinear resistive circuits is covered in chapter 4, with a focus on Newton's method. A detailed study is given of Newton's method, including its links to the fixed point method and the conditions that govern its convergence. A rigorous treatment is then provided of how this method applies to circuit simulation, leading up to the notion of companion models for nonlinear resistive elements, with coverage of multiterminal elements. As well, a coverage of quasi-Newton methods in simulation is provided, which includes the three commonly used homotopy methods for DC Analysis: source stepping, Gmin stepping, and pseudo-transient. Simulation of dynamic circuits, both linear and nonlinear, is covered in chapter 5. This chapter gives a detailed treatment of methods for solving ordinary differential equations (ODEs), with a focus on those methods that have been found useful for circuit simulation. Issues of accuracy and stability of linear multistep methods are covered in some depth. These methods are then applied to circuit simulation, illustrating how the companion models of dynamic elements are derived. Here too, multiterminal elements are addressed, as well as other advanced topics of time-step control, variable time-step, charge conservation, and the use of charge-based models in simulation.

My aim throughout has been to produce a text that has two key features: 1) sufficient depth and breadth so that it can be used in a graduate course on the topic, and 2) enough detail so as to allow the reader to write his/her own basic circuit simulator. I hope that I have succeeded. Indeed, the book has already

been tested for this dual purpose, as I have used it to teach a graduate course on circuit simulation at the University of Toronto. As part of this course, students write a rudimentary circuit simulator, in a sequence of five computer projects, all of which are included in the problem sets in this text. The first project is simply to develop a parser; the second is to develop code that builds the MNA system of equations for any linear resistive circuit, using element stamps; the third requires the implementation of an LU factorization capability to solve the MNA system. The fourth project implements a Newton loop around the MNA solver, allowing the simulation of basic nonlinear resistive circuits. The fifth and final project builds a time-domain simulation loop around the Newton loop, using the trapezoidal rule. The result is a basic simulator that can simulate circuits containing MOSFETs, BJTs, and diodes (using the simplest first-order models for these devices), along with the standard linear elements. With problem sets and computer projects at the end of every chapter, this text is suitable as the main textbook for a course on the topic. As well, the text has sufficient depth that I hope it would serve as a reference for practicing design engineers and computer-aided design practitioners.

Throughout the text, detailed coverage is given of the mathematical and numerical techniques that are the basis for the various simulation topics. Such a theoretical background is important, I feel, for a full understanding of the practical simulation techniques. However, this theoretical background is given piecemeal, as the need arises, and is never presented as an end in itself; it is scattered throughout the text and paired up with the various simulation topics. Furthermore, and in order to maintain the focus on the end-goal of practical simulation methods, I have found it necessary to *state all theorems without proof*. Ample references are provided, however, which the interested reader can consult for a deeper study.

Finally, the reader is encouraged to consult the following web site, where I hope to maintain various resources that are relevant to this book, including an up-to-date list of any known errors:

<http://www.eecg.utoronto.ca/~najm/simbook>

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Farid N. Najm
January 2010
Toronto, Canada

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