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GROWTH THEORY

VOLUME III

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Growth Theory
Volume III

Equilibrium Growth Theories

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Growth Theory
Volume III

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Introduction

This three-volume work provides a comprehensive selection of the most important articles on growth theory. The readings in Volume I address theories that attempt to explain the stylized facts of growth. Volume II focuses on normative models of the growth process. Volume III integrates the positive analysis found in the first volume with the welfare approach found in the second volume. Taken together, the volumes depict the development of growth models from the early aggregative theory without explicitly optimizing agents to the current practice of formulating growth models with an explicit microeconomic foundation for consumption and investment decisions. Both the questions and methods of the new equilibrium approach to growth theory are adapted from optimal growth theory. In this sense the descriptive and normative theories are intertwined and elements of both points of view may be found in each of the three volumes.

Many modern developments in the theory of competitive equilibrium over time stem from the seminal contributions of Ramsey, von Neumann, and Solow that are embodied in the neoclassical growth theory. Modern theories of financial markets, real business cycles, capital taxation, growth, and development all have used the neoclassical growth model as their frame of reference. The distinguishing characteristic of *equilibrium models* is the new competitive dynamics paradigm in which household and producer decisions have an explicit foundation in optimizing behaviour. This fundamental characteristic is common to all the readings selected for this volume.

In this volume we present some selections of the models and applications of this dynamic equilibrium approach to growth theory. Two alternative implementations of the microeconomic foundations for household consumption-savings decisions are represented in the readings. The majority of the contributions utilize the infinitely lived household assumption first introduced by Ramsey. The overlapping generations model associated with Allais and Samuelson is also given brief mention.

Volume III is divided into four parts. Part I covers one-sector theories, while Part II is devoted to models with two or more sectors. In both these parts, the models employ a representative agent to carry out the consumption-savings decisions in the model economy. Heterogeneous agent models are included in Part III, and applications of the various models to problems in asset pricing and real business cycle theory are found in Part IV.

The fundamental theorems of classical welfare economics state conditions for a competitive equilibrium to be a Pareto optimal allocation and conversely. Models of dynamic competitive equilibrium over an infinite horizon are general equilibrium theories cast in an infinite dimensional commodity space framework. A recurring theme in literature on dynamic equilibria is to understand whether or not the competitive dynamics of a model are equivalent to the dynamics realized by an omniscient central planner. Put differently, the infinite dimensional commodity space framework opens the possibility that the welfare theorems could fail in an intertemporal equilibrium setup. The connection, if any, between equilibria and optimal programme forms the unifying thread between this and the other two volumes.

The exploration of one-sector equilibrium models represented in Part I reveals many of the issues revolving around model specification and the general validity of the welfare theorems. The paper by Shell summarizes the major finding of Samuelson's (1958) overlapping generations model: a dynamic competitive equilibrium is not necessarily a Pareto optimal allocation. Shell argues that the basis for this result is the 'double infinity' of trades and dates characteristic of the overlapping generations model. Aiyagari's contribution is to show that *efficient* accumulation programmes in one-sector overlapping generations models generate a time series for aggregate capital stocks that is indistinguishable from the series found by solving some classical Ramsey optimal planning model with an infinitely lived central planner. In this sense, the two competing paradigms agree on the efficient paths of the economy's development, even though the microeconomic foundations for the consumption-savings decision are fundamentally different.

The infinitely lived household approach to equilibrium dynamics in one-sector models is illustrated first by the Epstein and Hynes study and second by Becker's article. In both papers, the dynamic general competitive equilibrium is cast in terms of a perfect foresight equilibrium path in which expectations of future profiles of prices turn out to be self-fulfilling or correct. The Epstein and Hynes contribution is to show how postulating a flexible rather than a fixed time preference rate on the part of households may be a crucial assumption for deriving interesting properties of equilibria. Becker suggests that a capital tax distorted equilibrium model may, in some cases, be analysed as if the resulting equilibrium solved an artificially constructed Ramsey problem. In the special case of a zero tax rate, the equilibrium path and Pareto optimal programmes coincide; an equivalence between the central planned optimum and decentralized equilibrium obtains. Moreover, given Aiyagari's equivalence result, efficient programmes in the overlapping generations are indistinguishable from equilibrium programmes in the undistorted representative infinitely lived agent model.

Part II is devoted to several of the themes found in Part I as well as to some new issues arising from the multiplicity of capital goods or assets when there are two or more sectors. The presence of alternative assets for storing wealth has profound implications for both the determinancy of equilibrium as well as for the potential qualitative properties of a dynamic equilibrium solution.

The duality between a perfect foresight equilibrium and optimum growth solution is central to dynamic economic analysis. Becker's paper in Part II develops this linkage for representative agent multisector models under the perfect foresight hypothesis. The key to demonstrating the possible duality holds is that the competitive economy imposes a transversality condition on itself in addition to the usual Euler equations characteristic of the first order conditions for an optimum or equilibrium programme. The Eurler equations say that the own rate of return on any capital good plus anticipated capital gains equals the endogenously determined interest rate at each moment of time. In effect, there are no profitable short-run arbitrages for an investor along an equilibrium profile. The transversality condition is an expression of the lack of profitable long-run arbitrages extended over the entire planning horizon. In this sense, the competitive equilibrium transversality condition resolves the Hahn problem (see Volume I) regarding the determinancy of intertemporal equilibrium.

Samuelson's paper in Part II first showed the importance of the intertemporal efficiency conditions for linking ideal market solutions and centrally planned optimal programmes. Following Hahn, he stressed the importance of convergence of the equilibrium profile to the golden-rule as key to the determination of equilibrium. This, in turn, required the path to be *permanently efficient*. Samuelson then posed a fundamental question: in a competitive economy, what determines the

profile of future prices that will be self-fulfilling? In effect, he asks whether or not the market can live on its dreams, or must equilibrium prices be determined by market fundamentals based on expected future-asset-returns alone. His discussion on this issue is the starting point of the modern literature both on price bubbles and on the possibility of multiple perfect foresight paths. This raises a deeper aspect of the indeterminacy problem for equilibrium profiles than mere convergence to a long-run equilibrium. This new indeterminacy question pervades the rational expections macrodynamics literature and can be seen to have its roots in the discussion about the Hahn problem.

The validity of the fundamental welfare theorems for a representative, infinitely lived agent model does not carry a presumption that the resulting equilibrium path is stable. For example, the Hopf bifurcation essay by Benhabib and Nishimura (Volume II) fits into the framework of Becker's multisector equivalence principle. Consequently, a perfect foresight multisector model may exhibit competitive cycles. The Benhabib and Nishimura work reprinted in Part II of this volume demonstrates that equilibrium cycles in discrete time can arise in a model with two sectors. The existence of a second sector is critical: a one-sector discounted Ramsey (equilibrium) model gives rise to a monotonic path of capital accumulation. The introduction of a second sector opens the possibility of capital intensity reversals along the optimum-equilibrium programme. Given discounting, the incentives to arbitrage across the perfectly foreseen oscillations and thereby smooth this path may not be viable. If the discount rate is sufficiently high, then the Boldrin and Montrucchio results from Volume II may be invoked to conclude that competitive chaos may result. In short, the dynamics of models with more than one sector may be substantially richer than for the one-sector case and therefore offer a potential base for building models of real business cycles.

Rational expectations equilibrium is the stochastic analogue of the perfect foresight equilibrium hypothesis. Prescott and Mehra introduced the concept of a *recursive competitive equilibrium* as a way to solve dynamic stochastic equilibrium problems. The crux of their idea is to mimic the dynamic programming policy function approach to solving the corresponding optimum problem. Put differently, a recursive competitive equilibrium generates a current pricing function of the momentary state of the economy, an optimal value function, a policy function for the consumer's consumption decision, a period production plan for each producer, and a law of motion governing the (stochastic) evolution of the system. The homogeneous agents case of their paper corresponds to the representative agent model previously discussed. They demonstrate a type of equivalence between Pareto optimal programmes and recursive competitive equilibrium programmes. The latter are also rational expectations equilibrium plans in this model. Applications of their framework will be taken up below in our remarks on the readings in Part IV.

The selections in Part II also include Brock's monetary perfect foresight equilibrium paper. He recasts the problem of monetary growth first encountered in Volume I in an explicit maximizing setup in order to provide an equilibrium in which all variables, including the price level, are determined as functions of the exogenously specified quantity of money. The consequence of this specification is that Brock is able to address comparative dynamics responses of an equilibrium profile to *perfectly foreseen* changes in the money supply. Moreover, he shows that multiple perfect foresight equilibria may exist for some specifications of preferences and growth rates of the money supply process. He also derives conditions for the 'real sector' decisions to be determined separately from the decision regarding real balance holdings – a classical dichotomy obtains along an equilibrium path.

Ramsey formulated a steady state model of a heterogeneous agent equilibrium process in addition to his well-known representative agent study of optimum growth. The papers chosen

for Part III all have their origins in Ramsey's heterogeneous agent ideas. He argued that if there are two classes of agents differing only with respect to the rate at which they discount future utilities, then the most patient agents will enjoy 'bliss' consumption and the less patient agents will consume at the minimum level to sustain life in a stationary state. This theme is reworked in all three papers found in Part III.

Bewley examines the existence and stability properties of a perfect foresight equilibrium with complete markets for borrowing and lending. He postulates fixed discount rates for agents future utilities. Along equilibrium paths, the relatively impatient agents consumption tends to zero – a strong form of the Ramsey solution. In the case where all agents have a common discount rate, Bewley demonstrates a turnpike theorem for equilibrium plans using a novel development of ideas originally found in the multisector turnpike literature. Lucas and Stokey examine equilibria with transfer payments but allow for agents to have flexible rates of time preference as embodied in a general recursive utility postulate for agents' characteristics. Their model also assumes complete markets. In a steady state (for an exchange economy) they demonstrate that the distribution of consumption across agents need not be as dramatic as in Ramsey's setting of the problem. A similar point is made by Epstein and Hynes in their article on flexible time preference.

Becker and Foias examine a one-sector model with agents ranked according to their fixed rates of impatience from most to least patient. They postulate *incomplete markets* in contrast to the other studies: agents are not allowed to borrow against future labour income. This implies that the analogue of Ramsey's steady state has the most patient agent holding all the capital and consuming a wage and capital income, whereas the other agents only consume their wage income. Becker and Foias also demonstrate that equilibrium paths are stable provided the underlying elasticity of substitution in the one-sector production function exceeds or equals one. In the alternative case, they show that an equilibrium cycle of period two is a possible outcome in this model. In effect, perfectly foreseen oscillations in capital income cannot be smoothed due to the interaction of the borrowing constraint and the discounting of future utilities.

The readings in Part IV, together with the articles by Epstein, Hynes, Becker, and Brock from Parts I and II, are designed to illustrate the range of applications employing the new equilibrium dynamic methods. Uncertainty is a common thread running through the selections collected in this section. *Intrinsic uncertainty* is emphasized based on the underlying shocks to preferences, technology, or endowments (as opposed to *extrinsic uncertainty* such as 'sunspots'). A homogeneous agent assumption is another common element of the papers found in Part IV.

Lucas' paper is the first attempt to model asset pricing as a recursive competitive equilibrium. He examines a simplified exchange economy in which assets are claims to a stochastic dividend stream. He utilizes dynamic programming techniques to derive the existence of a unique stationary asset pricing function for the various assets. Equilibrium prices 'reflect all available information', an idea consistent with the rational expectations hypothesis. Moreover, the asset prices are explicitly tied to the primitive characteristics of preferences, endowments, and the dividend generating technology in the model economy. In this way, conditions sufficient for asset prices to exhibit the martingale property signifying *market efficiency* as used in the finance literature may be explored within an explicit microeconomic foundation. He draws the important conclusion that the presence of a diminishing marginal rate of substitution of future for current consumption may be inconsistent with the martingale property.

Brock formulates an asset pricing model that also reflects an underlying productive technology based on real capital. In effect, he combines elements of financial asset theory with a Ramsey style

growth theory model. Like Lucas, Brock shows there is a unique stationary asset pricing function in this model and finds sufficient conditions for equilibrium programmes to have interpretations related to the arbitrage theory of asset pricing as formulated by Ross and the capital-asset pricing formulas proposed by Sharpe and Lintner.

The purpose of the Donaldson and Mehra article is to derive equilibrium prices, firm value, the market and risk-free returns, and the market risk premium in another model connecting financial market theory and stochastic optimal growth theory with a one-sector technology. One novel aspect of their paper is that production shocks may be correlated over time in contrast to the more usual assumption of independent and identically distributed shocks. Their paper also reveals the possibilities for comparative dynamics analysis in the recursive competitive equilibrium framework. Specifically, they investigate the responsiveness of the market risk premium as underlying parameters governing the productive technology and tastes of the representative consumer undergo alteration. For example, given their calibration of the model, they show that greater risk aversion on the part of the representative agent leads to an increase in the risk premium. While this result is not surprising, they do find that market return on capital first rises and then declines with greater risk aversion. This in turn reflects a fundamental trade-off between reduced variation in consumption and increased variation in output as the risk parameter changes. Average market returns to capital may decline since the range of steady state capital stocks expands as the household becomes more risk averse.

Real business cycle theory represents another important application of equilibrium dynamics. The paper by King, Plosser, and Rebelo presents research in this vein. Their paper examines both theory and empirical observations on fluctuations in order to argue that the stochastic neoclassical growth model has a place in understanding the observed time series of key real macroeconomic variables. One important point of departure from the usual growth theory literature is the need to model labour–leisure decisions on the part of the household sector. This reflects the importance of linking variations in work effort with intertemporal substitution made possible in equilibrium by the process of capital accumulation. Their model does not incorporate financial variables, but rather concentrates only on the real variables in the economy. In this sense, their work shows how the process of economic growth and business cycles may become intertwined and grounded in the underlying structure of tastes, endowments, and technology taken as the primitives in the model economy.

The equilibrium theories of Volume III may also be tied to the 'new growth theory' papers of Lucas and Romer found in Volume I. The process of optimal accumulation, equilibrium growth, real and financial asset pricing, and business cycles have a common foundation in growth theory. The models and techniques of the growth theorist open the door to framing new questions as well as reexamining old questions at the heart of economic theory. The readings in all three volumes are woven together as one line of thought from the seminal papers of Ramsey, von Neumann, and Solow to the current dynamic equilibrium theories and applications.

It is evident that growth theory continues to evolve, both by elaborating and refining old themes and by pushing off into new directions. Even as these words are being written, with each new journal issue results appear that deal with questions such as credit rationing and market incompleteness, the role of the banking sector and binding credit constraints, the importance of derivative securities, and a host of other questions. Such new developments continue to blur even more the traditional distinctions between macroeconomics, monetary economics, and finance. Volume IV, should it ever appear, might well be entitled *Integrated Growth Theories*.

Contents

Part I
One-Sector Models

[1]

Notes on the Economics of Infinity

Karl Shell

University of Pennsylvania

This is an attempt to expose the essence of Samuelson's consumption-loan paradox. It is maintained that the double infinity of traders and dated commodities allows for competitive equilibria that are not Pareto-optimal. While such models are most interesting in the dynamic setting, the fact that generations do not meet is not essential. The chain-letter aspect of the model reminds us that the appropriate form of the budget constraint is not obvious for the potentially infinitely long-lived economic entity (such as the corporation or the family). The analysis is related to recent contributions in the theories of general equilibrium, economic planning, and decentralization.

1. Paul Samuelson's (1958) paper on consumption loans is to my mind one of the most original and stimulating contributions to modern economic theory. In each period, there are assumed to be a finite number of individuals and one homogeneous commodity (say chocolate). Individuals are assumed to live for three periods: first, as dependent youths; second, as breadwinning[1] adults; and third, as retirees. There are no externalities. Samuelson shows that—in a world without end—the competitive equilibrium allocation is not necessarily Pareto-optimal. Some imposed reallocation can be found, making no individual worse off while making at least one individual better off.

2. A variety of attempts at explaining this "Samuelson paradox" have been made. Most such attempts concentrate on one of two general points:

A. In an economy with births and deaths, all souls cannot meet in a single market. Since Spiro Agnew cannot haggle over chocolate with George Washington and Buck Rogers, the usual assumptions of general equilibrium theory are violated.

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[1] To be precise, I should say "chocolate-winning adults."