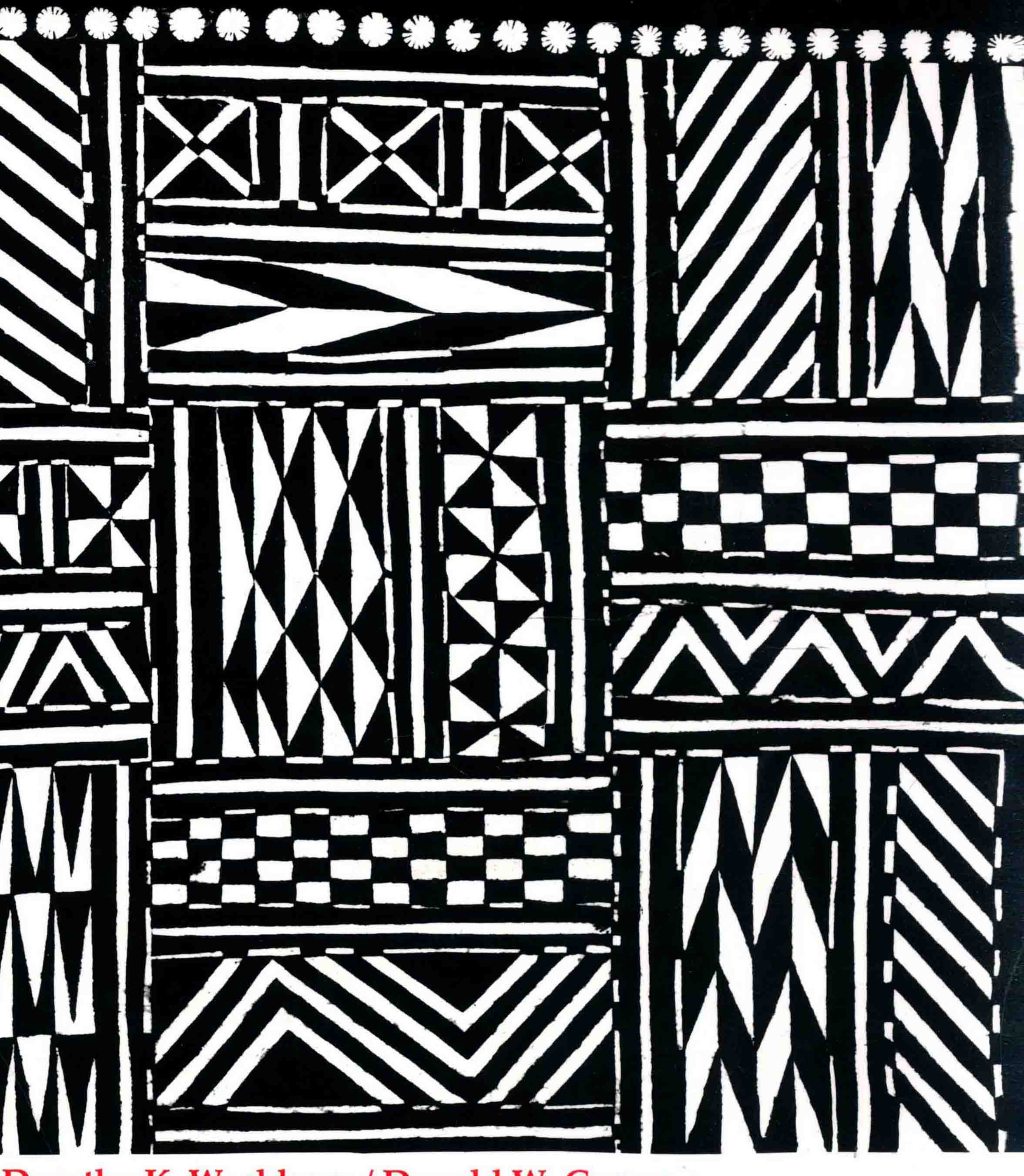
Symmetries of Culture

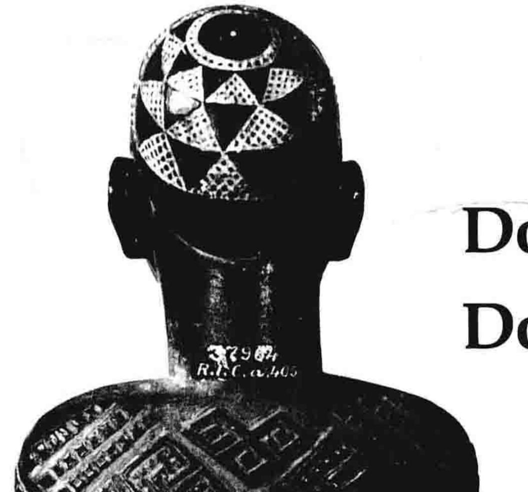
Theory and Practice of Plane Pattern Analysis



Dorothy K. Washburn / Donald W. Crowe

Symmetries of Culture

Theory and Practice of Plane Pattern Analysis



Dorothy K. Washburn
Donald W. Crowe

University of Washington Press *Seattle and London*

Third printing (pbk.), 1998
Designed by Veronica Seyd
Printed and bound in Hong Kong

The paper used in this publication meets the minimum requirements of American National Standard for Information Sciences—Permanence of Paper for Printed Library Materials, ANSI Z39.48–1984. ∞

Publication of this book was made possible in part by the assistance of the J. Paul Getty Trust, and by a grant from the Division of Research Programs of the National Endowment for the Humanities.

Library of Congress
Cataloging-in-Publication Data
Washburn, Dorothy Koster.
Symmetries of culture.
Bibliography: p.
Includes index.
1. Repetitive patterns (Decorative arts).
2. Pattern perception.
3. Symmetry (Art).
I. Crowe, Donald W.
(Donald Warren),
1927– . II. Title.
NK1570.W34 1987
745.4 87–27924
ISBN 0-295-97084-7

Stirrup, Edo (Fig. 5.286)

Symmetries of Culture

Above, bronze *yu* (Fig. 6.9); *facing*, wood figurine, Yombe (Fig. 5.291)

Acknowledgments

THE AUTHORS began this endeavor separately, Washburn in her 1977a monograph and Crowe in his several articles (1971, 1975, 1982). We joined forces realizing that we could contribute substantially to each other's knowledge and understanding of cultural and mathematical aspects from our respective disciplines. Washburn gratefully thanks the Miller Institute for Basic Research in Science, University of California, Berkeley, for an unrestricted two-year Fellowship (1976–78) which supported the developmental phases of this work. Crowe is indebted to Claudia Zaslavsky, whose invitation to contribute to her book *Africa Counts* led to his first study of symmetry in cultural objects; to the 1978 article by Doris Schattschneider, which was the inspiration for the flow charts; and to California State University, Chico, for a Visiting Professorship which gave him the time to dig in Ghana and begin his collaboration with Washburn. We also thank Branko Grünbaum and G. C. Shephard, pioneers in the analysis of patterns, who kindly made available sections of their work, *Tilings and Patterns* (1987), before its publication.

An endeavor of this magnitude could not have been successful without the assistance of many people. We thank especially all the museums that made available photographs of objects from their collections. These are acknowledged separately with each illustration. Photographs of the many objects from the California Academy of Sciences were taken by Christopher Thomas. Copy photographs from books were made by Susan Middleton and Judith Steiner. Coleen Sudekum and Amy Pertschuk redrew objects for which photographs were unobtainable. The computer-generated schematic drawings of the two-dimensional motion classes were made by Mike Case of Beloit College. Melanie Herzog-Stockwell of the Cartographic Laboratory of the University of Wisconsin added the reflection lines and centers of rotation. Funding for the purchase of museum photographs and the preparation of illustrations was provided by the Irving Fund and research funds of the California Academy of Sciences, and the College of Letters and Sciences, University of Wisconsin, Madison. Finally, we are especially grateful to Marian Bock and Veronica Seyd of the University of Washington Press for their careful attention to the difficult task of polishing the rough spots in the manuscript while preparing it for final publication.

Introduction

THE USE of the geometric principles of symmetry for the description and understanding of decorated forms represents the union of two normally separate disciplines—mathematics and design. The only limitation to the types of designs which can be described by these principles is that they must consist of regularly repeated patterns. That is, they must be designs with parts moved by rigid geometric motions.

In this book we demonstrate how to use the geometric principles of crystallography to develop a descriptive classification of patterned design. Just as specific chemical assays permit objective analysis and comparison of objects, so too the description of designs by their geometric symmetries makes possible systematic study of their function and meaning within cultural contexts.

This particular type of analysis classifies the underlying structure of decorated forms; that is, the way the parts (elements, motifs, design units) are arranged in the whole design by the geometrical symmetries which repeat them. The classification emphasizes the way the design elements are repeated, not the nature of the elements themselves. The symmetry classes which this method yields, also called motion classes, can be used to describe any design whose parts are repeated in a regular fashion. On most decorated forms such repeated design, properly called pattern, is either planar or can be flattened (e.g., unrolled), so that these repeated designs can be described either as bands or strips (one-dimensional infinite) or as overall patterns (two-dimensional infinite) in a plane.

This is essentially a handbook for the nonmathematician. There are a number of geometry textbooks which derive the pattern classes, but they usually offer more detail than is needed by a user in the humanities. Conversely, there have been a number of articles and monographs written by and for social scientists, but these are usually specific to one body of data. We have attempted to offer a more comprehensive survey of patterns occurring on decorated objects from cultures all over the world and to systematically show how to classify the finite, one-dimensional, and two-dimensional one- and two-color designs with the use of flow charts and other detailed descriptions of the symmetry motions and colors present.

Chapter 1 presents a short historical review of the discovery and enumeration of the plane pattern classes, the importance of symmetry for form identification and classification, and theoretical issues in the application of this type of analysis to designs found on cultural material. Chapters 2 through 6 show how to classify patterned design with the aid of flow charts, schematic drawings of each class, and photographs and drawings of actual objects decorated with such patterns. These chapters treat the one- and two-color, one- and two-dimensional patterns, and the common finite designs, because these are the most frequently encountered.

The mathematical background needed to understand and perform symmetry analysis is summarized in Chapters 2 and 3. The reader must be conversant with these principles before reading Chapters 4, 5, and 6, which describe the one-dimensional, two-dimensional, and finite designs respectively. Each of the chapters on one- and two-dimensional designs begins with a section which describes the flow charts and leads the reader through the use of the charts with an example from each motion class. The last section of each of these two chapters presents a more detailed explanation of each motion class and shows actual examples of this class. Chapter 7 discusses special problems which may be encountered in symmetry classification. Three Appendixes present in detail the mathematical proofs for the existence of the four motions and the seven one-dimensional designs, and correlate the several other nomenclatures with the standard crystallographic nomenclature which we use in this book.

We suggest that the user first grasp the principles of geometry outlined in Chapters 2 and 3 and become familiar with the flow charts, before embarking on a study of a specific body of data. Then, in order to classify any particular pattern, use the flow charts in Sections 4.2 and 5.1 and the schematic drawings in 4.3 and 5.2 to key out the pattern. As a final check, turn to the examples shown in 4.3 and 5.2 for comparison with patterns already identified.

There are computer-generated schematic drawings of every one- and two-color, one- and two-dimensional design. Indicated in each drawing are the mirror (solid) and glide (dashed) lines and centers of rotation: ◑, ○ for twofold rotation, with and without color reversal; ▽ for threefold rotation; ◪, □ for fourfold rotation, with and without color reversal; and ⬢, ⬡ for sixfold rotation, with and without color reversal. In these drawings, right triangles or trapezoids represent the asymmetrical fundamental building blocks of repeated patterns.

A number of patterns have been illustrated for classes where the symmetry is particularly difficult to see, where it frequently occurs on a number of different kinds of media, or where there are problems and deviations from the symmetry which must be considered. Further discussion of problem situations which repeatedly occur is contained in Chapter 7.

Contents

Loincloth, Chancay style (Fig. 5.1, detail)

Symmetries of Culture

Plate, Deruta, Italy, A.D. 1520–25 (Fine Arts Museum of San Francisco, no. 54.45.3)

1 History and Theory of Plane Pattern Analysis

1.1 Introduction

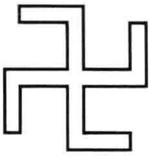
IN THIS CHAPTER we first present a short mathematical history (1.2) of the plane pattern crystallographic motion classes and their initial applications to nonmathematical fields (1.3) as a preliminary to our discussion (1.4) of the appropriateness of this methodology, in terms of the perceptual process of pattern recognition and the theoretical needs of style analysis, for the study of designs in material culture.

1.2 Crystallographic and Mathematical History

In his brief prehistory of group theory, A. Speiser (1927) suggested that the origin of higher mathematics (at that time thought to be in Greece about 500 B.C.) should be pushed back a thousand years to the Egyptian use of one-dimensional and two-dimensional patterns (see Jones 1856 for illustration of such designs). In his view, the creation of certain of these two-dimensional patterns, with many complicated symmetries, was a mathematical discovery of the first magnitude.

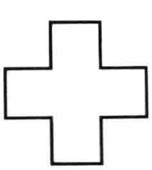
In contrast to these Egyptians, the later Greeks, who otherwise studied geometry in a most profound way, seem to have had less interest in such infinite patterns. However, they developed the theory of finite designs—in the form of regular polygons, especially the equilateral triangle, square, regular pentagon, and regular hexagon—to a high level. This theory included, in Euclid, a detailed analysis of the five regular polyhedra and, later, the thirteen Archimedean polyhedra. Both of these classes of polyhedra can be interpreted as patterns on the surface of a sphere. However, the Greeks apparently did not emphasize the analogous (infinite) patterns in the plane: the three regular tessellations, and the eight semiregular tessellations.

Following these Greek constructions there is little record of purely mathematical studies for hundreds of years. But the work of Byzantine artisans of Ravenna and Constantinople and their successors in Venice, and the Islamic pattern makers throughout the Mediterranean and east to India, carried on what we must think of as mathematical work. Although they did not call themselves mathematicians, in retrospect (cf. Müller 1944) we see their methods and results as having important geometric content.



1.1a Swastika with rotation symmetry

During the Renaissance, Italian artists and architects made much use of finite designs. It is thought that Leonardo da Vinci consciously studied the symmetries of finite designs and determined all of them so as to be able to attach chapels and niches without destroying the symmetry of the whole. His conclusion, now called Leonardo's Theorem (Martin 1982:66), was that the only possible (one-color) finite designs are those which have rotational symmetry alone, like a swastika (Figure 1.1a), and those which have both rotational and reflection symmetry, like a Greek cross (Figure 1.1b) or a square.



1.1b Cross with rotation and reflection symmetry

We will use the symbol *c4* to denote designs with fourfold rotational symmetry but no reflection symmetry, like the swastika. More generally, *cn* denotes any design having only *n*-fold rotational symmetry, such as a swastika with n arms. For the corresponding designs with reflection symmetry, we use the symbol *dn*. Thus a Greek cross and a square have type *d4* symmetry and a regular hexagon has type *d6* symmetry.

Other regular polygons, in the form of fountains, are ubiquitous in Italy. A typical example, the Maggiore fountain in Perugia, has two tiers, the upper one having 12-fold (i.e., *d12*) rotational symmetry and the lower one 25-fold rotational symmetry. The rose window of Santa Chiara in Assisi has 15-fold and 30-fold symmetry (Munari 1966:63). The pulpit of St. Stephen's in Vienna alternates threefold (*c3*) and fourfold (*c4*) symmetries (Weyl 1952:67). An unusual assemblage of examples of *c3*, *c4*, *c5*, *c6*, and *c7* symmetries, carved in wood to imitate Gothic windows, is found in the St. Johannes church in Lüneburg.

Dürer's book on geometry (1525) carried this and other information about regular polygons to Germany for the use of artists. A hundred years later, Kepler made careful studies of regular polyhedra, and in 1611 wrote a monograph on the snowflake in which he considered the packing of circles in a plane and spheres in space. Kepler's work can be thought of as the forerunner of crystallography, the study of which in the nineteenth century led to almost all the mathematical information we have on repeated patterns until very recent times.

In the early nineteenth century Hessel found the thirty-two main classes of crystals (i.e., three-dimensional repeated patterns) which are still used today. The names of Bravais, Jordan, Sohncke, Barlow, and Schoenflies figure prominently in the massive effort which culminated in the complete list of all 230 three-dimensional repeated patterns published by Fedorov in 1891. Fortunately, these 230 patterns and the 4,783 four-dimensional patterns recently enumerated by Brown, Bülow, Neubüser, Wondratschek, and Zassenhaus (1978) do not directly concern the student of plane patterns.

One-Color Patterns

The enumeration of the seventeen two-dimensional (one-color) patterns was also published by Fedorov in 1891. Because this paper appeared only in Russian and was of little interest for crystallography, it was not until the 1920s that the classification of one- and two-dimensional patterns became generally known, through the papers of Niggli (1924, 1926) and Pólya (1924).

The second edition (1927) of Speiser's group theory text first called the attention of mathematicians to these results. Speiser adopted the notation used by Niggli, but unfortunately interchanged two of the Niggli symbols. The mathematical literature of the next fifty years was infected by the consequences of this error. It was finally corrected by Schattschneider (1978). Fortunately, the crystallographers continued on their own path, so this error does not appear in their work.

It is to Speiser's student, Edith Müller, that we owe credit for perhaps the first systematic use of these tools in the analysis of material culture. Her 1944 thesis is a detailed study of the patterned art of the Alhambra. Although Müller is often credited with having found all seventeen one-color plane patterns there, she refers explicitly to only eleven; two others have been documented elsewhere. (In the notation explained in Chapter 2, the eleven recorded by Müller are *p1, pmg, cm, pmm, cmm, p4, p4m, p4g, p3, p6,* and *p6m*. Grünbaum, Grünbaum, and Shephard [1986] record *p31m* and *pm* and Jones [1856:Pl. 41, 5] records *pg* [ambiguous].) Not until 1987 did the combined efforts of Spanish mathematicians and topologists provide documentation of the presence of all seventeen one-color plane patterns in the Alhambra.

Discussions of the one-color band and plane patterns began to appear more frequently in mathematical texts following Speiser, particularly in Coxeter (1961), Fejes Tóth (1964), Burckhardt (1966), Guggenheimer (1967), Cadwell (1966), and O'Daffer and Clemens (1976). The recent books of Lockwood and Macmillan (1978) and Martin (1982) deal extensively with these topics. Of these references, the last four are perhaps the most accessible to the non-mathematician. The Schattschneider (1978) paper is also recommended.

Two-Color Patterns

The history of the two-color and more highly colored patterns is well described in the treatise of Grünbaum and Shephard (1987), which can be taken as the definitive text for the mathematical theory of patterns in general. The following briefly summarizes the extensive "Notes and References" at the end of their Chapter 8. (They refer to our one-dimensional patterns as "strips," and to our two-dimensional patterns as "periodic patterns.")

The first complete, explicit, and deliberate enumeration of the two-color one- and two-dimensional patterns is found in the papers of the textile physicist H. J. Woods (1935, 1936). The mosaic representations of all forty-six two-color patterns in our Chapter 3 are reproduced from Woods 1936. Other mosaics were used in 1957 by Belov and Belova (excerpted in Shubnikov and Belov 1964), who were presumably unaware of Woods's earlier work (Crowe 1986).

Until very recently the most active study of two-color patterns was conducted by the Soviet school of crystallography, following the lead of Shubnikov (e.g., Shubnikov and Koptsik 1974; Shubnikov and Belov 1964). For reasons related to crystallographic situations (of no importance for the study of patterns as such), they introduced the concept of "gray" patterns to accompany the one-color and two-color (black-white) patterns, and these gray patterns were often pictured along with the others. Since the gray patterns are exactly like the one-color patterns, modern treatments often omit them. However, their inclusion means that in many earlier papers the totals 24 (= 7 one-color

+ 17 two-color) for one-dimensional patterns and 63 (= 17 one-color + 46 two-color) for two-dimensional patterns, which are customary in the most recent literature (including the present handbook), appear as 31 (= 7 one-color + 7 gray + 17 two-color) and 80 (= 17 one-color + 17 gray + 46 two-color).

For the reader who wants to know more about the mathematics of two-color patterns than is in the present handbook, we recommend Woods (1936), Belov and Tarkhova (1956, reprinted in Shubnikov and Belov 1964), Loeb (1971), Lockwood and Macmillan (1978), or Grünbaum and Shephard (1987). A particularly elementary discussion is found in Schattschneider (1986).

The notation we have adopted for the two-color two-dimensional patterns is the "rational symbol" of Belov and Tarkhova. (Some of their misprints are corrected in our Section 3.4.) For the two-color one-dimensional patterns we use the convenient notation of Belov (in Shubnikov and Belov 1964:225). This latter notation is explained in our Section 3.4.

Multicolored Patterns

For patterns with more than two colors, it is only recently that there has been agreement concerning the most reasonable definitions, and there is still no generally accepted nomenclature. The following studies are the most recent statements which may be used to classify multicolored patterns. For detailed bibliographic and historical notes see Schwarzenberger (1984) and Chapter 8 of Grünbaum and Shephard (1987).

Jarratt and Schwarzenberger (1981) determined the number of colorings of one-dimensional patterns with n colors, for all values of n. Wieting (1982) enumerated all possible colorings of two-dimensional patterns by n colors, for values of n up to sixty. He gives detailed drawings for the ninety-six ways of coloring two-dimensional patterns with four colors. Finally, Grünbaum and Shephard (1987) illustrate all twenty-three ways of coloring two-dimensional patterns with three colors.

1.3 Applications of Geometry to Design: Historical Precedents

Interest in the application of the principles of geometric symmetry to fields other than crystallography has been sporadic. "Reinventing the wheel" pervades the literature as a number of individuals have separately discovered that symmetry can be a useful analytical tool. Many authors appear unaware of other similar work and there appears to have been, until the last decade, little follow-up on these isolated pioneering introductions.

Our short historical discussion here can be merely an introduction to the literature in this important field. We make no guarantee to present an exhaustive listing of these efforts. Publication in obscure serials and the isolated, singular nature of many of these studies make omissions almost certain.