

Derek I. Bloomfield

BASIC MATHEMATICS

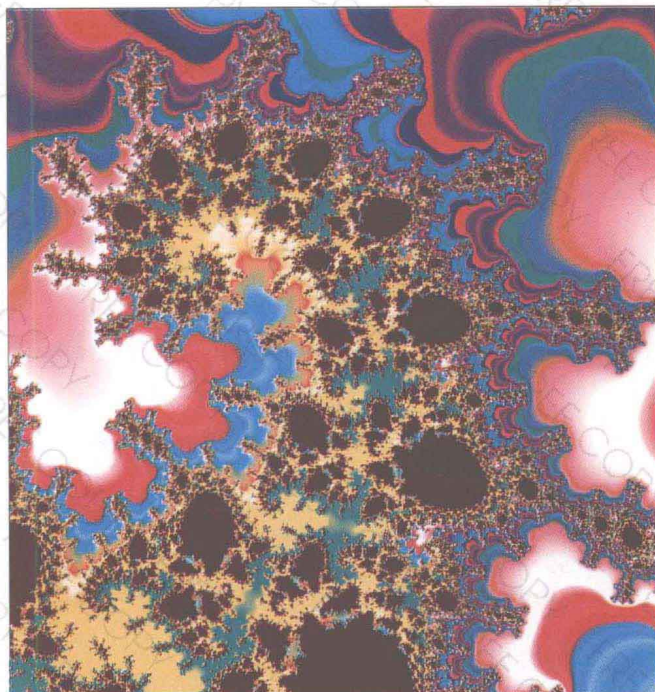
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BASIC MATHEMATICS

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Orange County Community College
Middletown, New York



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DEDICATED TO MY WONDERFUL CHILDREN JENNIFER, MAX, DEREK JR, AND DAVID



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About the cover and chapter opener images

More than eye-catching computer graphics, these images are known as fractals. Fractals are mathematical characterizations of the complexities found in nature. Jagged coastlines, fluffy clouds, the bark of trees, and snowflakes all exhibit these intricate patterns. These fractals are from the Mandelbrot and Julia sets. They are named for Benoit B. Mandelbrot, who introduced the concept of fractals in the early 1970s, and French mathematician Gaston Julia, who studied self-similar boundaries with Pierre Fatou during the first world war. An interesting property of fractals is their iterative nature. No matter how much you enlarge the fractal, it looks uncannily similar to lesser enlargements. The simple yet complex beauty of nature and our universe is represented in the colorful rings, whorls, and spirals.

PREFACE



Purpose and Style

This book is intended for anyone who needs to learn the fundamental skills of mathematics. This audience consists of students who didn't do as well in high school as they wish they had, adults interested in returning to school to better themselves or to change careers, people wanting to take high school equivalency examinations, people needing more mathematics to advance in their work, and a long list of others. This book is written with these people in mind. Its sole purpose is to bring such students up to a level of competence in mathematics that will enable them to take additional mathematics courses or to take courses in other areas in which a knowledge of basic mathematical skills is required.

This book is intended for use in any of three instructional modes:

1. a conventional lecture-type class;
2. a mathematics laboratory;
3. a self-study program in which the student works at his or her own pace.

The book consists of 12 chapters, and the material in each chapter is presented in the following manner: A concise explanation of the fundamental concepts for the particular topic is given; examples illustrating these fundamental concepts are worked out step-by-step; and an ample number of similar problems are given in the exercise sets enabling the student to master the concepts involved. The exercises progress from simple to more difficult in an effort to give the student confidence in the ability to solve problems. The answers to odd-numbered exercises are given at the end of the text.

The major goal of this method of presentation is a correct understanding of the topics and maximum skill in performing the mathematical operations.

The material is presented in measured amounts so that the student can complete a topic before moving on to the next concept. An achievement test is given at the end of each chapter with answers to all exercises given in the answer section.

Outstanding Features

- **Number Knowledge:** At the beginning of each chapter is a "human interest" mathematical interlude.
- **How to be Successful at Mathematics:** Hints to help students are scattered throughout the first two chapters.
- **Warning Signs:** These help students avoid frequently committed errors.
- **Critical Thinking Exercises:** Students are required to find errors in worked-out solutions to problems.
- **Mental Mathematics Exercises:** Practice exercises that are to be done without paper and pencil; these help students to master certain important fundamental skills as well as help to learn the lost art of doing mathematics "in their head."
- **In-Your-Own-Words Exercises:** Students are required to think through



and to write out definitions and procedures in their own words. This lets them know whether they really understand a concept or not.



- **Living-in-the-Real-World Exercises:** These represent real-world applications of mathematics in everyday situations.
- **Calculators:** Chapter 12, the last chapter in the book is about how to use a calculator. The instruction and exercises stand alone and may be included at the beginning of the semester, at the end, or not at all, at the individual instructor's discretion.
- **Chapter Summary:** At the end of each chapter is a list of definitions, rules, and procedures, each of which is accompanied by a worked-out example illustrating that procedure.
- **Chapter Review Exercises:** A set of exercises, keyed to the appropriate section in the chapter, is included at the end of each chapter.
- **Achievement Test:** A test is included at the end of each chapter with answers to all exercises to show the student whether or not he or she has mastered the concepts of the chapter.
- **Cumulative Reviews:** At the end of every third chapter is a cumulative set of exercises to ensure that students remember previous material.
- **Pedagogical Use of Color:** It's not just decoration.
- **Workbook Style with Perforated Pages:** These may be handed in if the instructor so desires.
- Over 800 worked-out examples with step-by-step explanations are presented, with important steps highlighted in color.
- Over 5,000 exercises have been carefully chosen to clarify explanations and to provide necessary drill.
- Sufficient space is provided for working out the exercises right in the text. This also provides a good reference when it comes time to review.

Ancillaries

- **Westest 3.0:** A computer-generated testing program. Problems may be selected and mixed and matched to suit the instructor's needs. There are versions for both MacIntosh and IBM PCs or compatibles running DOS or Windows 3.0.
- **Instructor's Manual with Test Bank** containing a test bank, review sheets, and chapter tests.
- **Instructor's Solutions Manual** containing worked-out solutions for all even-numbered problems in the test.
- **Student's Solutions Manual** containing worked-out solutions for all odd-numbered problems in the text.
- **Annotated Instructor's Edition** containing answers to all exercises written adjacent to the problem for easy instructor reference.
- **Mathens Tutorial Software** containing multiple examples of the basic problem types in *Basic Mathematics*. The program is easy to use, interactive with the student, keyed to each section in the text, and versions are available for both the MacIntosh and IBM PCs and compatibles.
- **Video Tapes** keyed to the text section-by-section.

I would like to thank the reviewers for their many helpful and constructive suggestions. These include:

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A Note from the Author to the Student

There is a lot of truth in the old adage, “There’s no success like success.” I learned a long time ago that the first problem on a math exam should be one that *everyone* can do. It leads to confidence and success. Rearranging the same problems on the same exam so that the first problem is an especially difficult one can so destroy a student that you can’t even spell your own name. Your anxiety level goes up, your confidence level plummets.

So, in my writing, I have attempted to alleviate anxiety, build confidence, and ensure mathematical success in the following ways.

1. I speak personally to you, the student, letting each of you know that I care about you and about your success.
2. I explain how to solve problems in a way that you can understand.
3. I very, very carefully make sure that the exercise sets progress slowly from simple to the more difficult ones. If you start at the beginning and work through the set in sequence, you will learn a tremendous amount, sometimes subtly, without even knowing what is happening to you. And, as on my exams, *everybody* can do the first few problems.
4. Nothing in the book is incidental. There are no fillers. In that regard, it’s like a Mozart symphony—every note means something. In my book, every word counts.
5. My style is convincing and encouraging. In general I have more confidence in you as a student than perhaps you have in yourself. I speak to that throughout.

This book is written for you and I’m genuinely interested in what you think about it. Please write to me and let me know what you like about it or how you think it can be improved.

Thank you.

Derek Bloomfield
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BASIC MATHEMATICS



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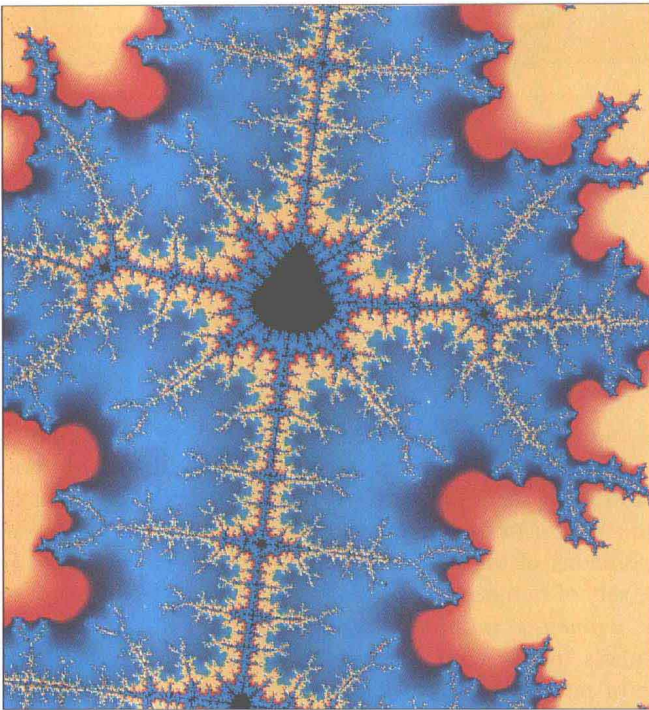
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Whole Numbers



INTRODUCTION

I asked my daughter, Jennifer, what her favorite word was. She answered “candy,” which came as no great surprise to me. What does this have to do with mathematics? you ask. As far as Jennifer is concerned, the only thing mathematical about candy is, “How many pieces?” As she grows a little older, she will probably become more concerned with, “How much does it cost?” These two questions, *how many* and *how much*, account for much of what mathematics deals with. Of course, these two questions can become much more complicated if we consider all the processes involved in a large candy-manufacturing business: recipes, costs, profits, payrolls, sales statistics, and many, many more.

Everyone who uses this book would like to understand the mathematical concepts involved in these processes. The reasons why people don’t understand them are many. At this point, however, it is too late to worry about why you didn’t learn many of these skills. Therefore, your immediate concern must be how to learn them now. One point must not be overlooked: all of you are capable of learning these skills no matter how bad you think you are at math.

This knowledge will open up the doors to new educational goals. Work hard, be persistent, and success will follow.

In this first chapter we will take a look at the numbers we are most familiar with: 0, 1, 2, 3, 4, We will learn how to read and write them properly and also how to combine them in various ways.

CHAPTER I—NUMBER KNOWLEDGE



It is difficult to imagine how we could get along without using numbers. For example, how could we make the following comparisons of different countries if we could not use numbers?

Country	Area (square miles)	Population	Literacy Rate	Annual Income per Person	Life Expectancy	
					Males	Females
United States	3,540,939	246,800,000	96%	\$16,444	71	78
Japan	143,572	123,200,000	99%	\$21,820	75	81
India	1,229,737	853,000,000	36%	\$ 250	53	52

1.1 Naming, Reading, and Writing Whole Numbers



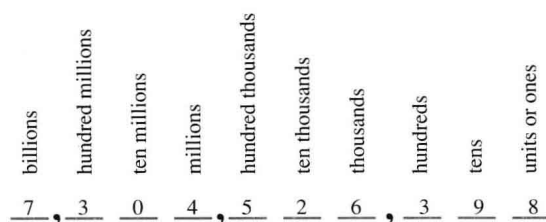
The numbers that we all know the most about are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and so on. These numbers are called the *whole numbers*. They form perhaps the most important set of numbers in mathematics because all other numbers used in mathematics can be defined in terms of the whole numbers. For this reason, the student should master all the operations of the whole numbers before going on to study other, more complicated, sets of numbers.

Technically speaking, a *number* is an abstract concept and a *numeral* is a symbol or group of symbols that is used to represent or name a number. One rather well known group of numerals used to represent numbers is the system of *Roman numerals*—I, II, III, IV, V, VI, and so on. The most common group of numerals used to represent numbers is 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, which is used today throughout much of the world. This group is called the *Hindu-Arabic system* because it was invented in India and transmitted to the Western world by the Arabs.

To write any whole number, we use only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If you want to write a whole number larger than 9, you must combine two or more of these digits.

For the number eight hundred fifty-two, we write the numeral 852. For the number eighty-seven, we write the numeral 87. Note that the 8 means something different in each numeral. In 852, the 8 means 8 hundreds. In 87 the 8 means 8 tens. For this reason, our system of numeration is called a *place-value* system. A digit takes on a specific value depending on what place it occupies in a given numeral.

In the following figure, look at the place value of each digit in the numeral 7,304,526,398:



In this number, the 8 represents 8 units or 8 ones. A digit in this far-right position is in the *units place*. The 9 immediately to the left of the 8 is in the *tens place* and represents 9 tens. Moving to the left, we have 3 in the hundreds place, meaning 3 hundreds; 6 in the thousands place, meaning 6 thousands, and so on.

How many hundred thousands are represented in the figure? What does the 4 represent? How many 10 millions do we have?

EXAMPLE 1 Name the place value of the underlined digit: 526,342.

SOLUTION Hundred thousands

EXAMPLE 2 Name the place value of the underlined digit: 3,862,124.

SOLUTION Tens

You will note that as we go from right to left we separate the digits into groups of three digits each by using commas. With the use of these groups let us now read our numeral. We read 7,304,526,398 as seven billion, three hundred four million, five hundred twenty-six thousand, three hundred ninety-eight. Note that in the last group on the right we say three hundred ninety-eight instead of three hundred ninety-eight ones.

EXAMPLE 3 Write the numeral 826,217 in words.

SOLUTION Eight hundred twenty-six thousand, two hundred seventeen.

EXAMPLE 4 Write the numeral 5,006,308 in words.

SOLUTION Five million, six thousand, three hundred eight.

EXAMPLE 5 Write the numeral 60,000,066 in words.

SOLUTION Sixty million, sixty-six.

Not only must we be able to write numerals in words, but we must also be able to go the other way and write words as numerals.

EXAMPLE 6 Write six hundred seventy-eight as a numeral.

SOLUTION 678

EXAMPLE 7 Write forty-two million, six hundred eighteen thousand, seven as a numeral.

SOLUTION 42,618,007

Expanded Notation

The number 428 consists of the digit 4, the digit 2, and the digit 8. As we know, each of these digits has a specific value, depending on which place it occupies. The 4 represents 4 hundreds, the 2 represents 2 tens, and the 8 represents 8 ones. We could write this as

$$428 = 4 \text{ hundreds} + 2 \text{ tens} + 8 \text{ ones}$$

or, more simply,

$$428 = 400 + 20 + 8$$

This way of writing a numeral according to the place value of each digit is called writing the numeral in **expanded form** or **expanded notation**. We shall find this notation extremely useful when we perform operations with whole numbers, so it is important to learn the process well.

EXAMPLE 1 Write 5,382 in expanded notation.

SOLUTION $5,382 = 5,000 + 300 + 80 + 2$

EXAMPLE 2 Write 26,318,524 in expanded notation.

SOLUTION $20,000,000 + 6,000,000 + 300,000 + 10,000 + 8,000 + 500 + 20 + 4$

EXAMPLE 3 Write 20,006 in expanded notation.

SOLUTION $20,000 + 6$

1.1 Exercises

▼ Name the place value of the underlined digit in each of the following

1. 3,824,127
ten thousands

2. 5,182
tens

3. 23,612,894
ones

4. 52,860
ones

5. 3,821
thousands

6. 516
tens

7. 30,004
hundreds

8. 382,114
thousands

9. 682,148,362
hundred millions

10. 508,362,785,510
ten billions

11. 701,400,216
thousands

12. 300,000,000,000
billions

▼ Name the place value of the 3 in each of the following:

13. 1,037 tens

14. 3,127 thousands

15. 7,003 ones

16. 43 ones

17. 30 tens

18. 30,576
ten thousands19. 1,073,522
thousands20. 3,516,285
millions

▼ Write the following numerals in words, then read them.

21. 52 fifty-two

22. 316 three hundred sixteen

23. 5,328 five thousand, three hundred twenty-eight

24. 62,531,218
sixty-two million, five hundred thirty-one thousand,
two hundred eighteen

25. 702 seven hundred two

26. 50,500 fifty thousand, five hundred

27. 308,000 three hundred eight thousand

28. 500,000,005 five hundred million, five

29. 60,606,060,606
sixty billion, six hundred six million, sixty thousand,
six hundred six.30. 21,300,096
twenty-one million, three hundred thousand, ninety-six

31. 4,001 four thousand, one

32. 246,820
two hundred forty-six thousand, eight hundred twenty

▼ Write each of the following written statements as a numeral.

33. Thirty-six 36

34. Five hundred sixteen 516

35. Seventy-four thousand, nine hundred ninety-seven
74,99736. Twenty-five million, twenty-five thousand,
twenty-five 25,025,025

37. Seventy million 70,000,000

38. Seventy million, seven 70,000,007

39. Seventy million, seven thousand 70,007,000

40. Five hundred twenty-six thousand, four hundred
eight-five 526,48541. Seventy-four billion, sixty million, four hundred
sixty 74,060,000,460

42. Three hundred thousand, six 300,006

43. Six hundred thousand 600,000

44. Six hundred thousand, six hundred six 600,606

▼ Write each of the following numerals in expanded notation.

45. 26 $20 + 6$ 46. 518 $500 + 10 + 8$ 47. 508 $500 + 8$ 48. 3,186 $3,000 + 100 + 80 + 6$ 49. 20,038 $20,000 + 30 + 8$ 50. 500,005 $500,000 + 5$