MINIMUM ESSENTIALS OF STATISTICS

As Applied to Education and Psychology

BY

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To

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PREFACE

This volume is intended as a textbook in statistics for students in education and psychology. Sufficient questions, exercises, drill materials, and true-false statements have been included to obviate the necessity of using a classroom manual or workbook. Since it has gone through three complete experimental, mimeographed editions, its practicability as a textbook has been tested. It has been used in whole or in part by the author in his statistics classes at Peabody College for the past six years.

In the preparation of this textbook much consideration has been given to the needs of school superintendents, principals, supervisors, and classroom teachers in the solution of their practical statistical problems. It is believed that the classroom teacher will be able to use this text with a marked degree of facility. The compact nature and practical treatment of the material should make the book particularly valuable to school officials and teachers in service.

All the essential and most widely used statistical measures and processes have been treated. But only the *minimum essentials* have been included. The extraneous and purely academic materials in statistics have been omitted, and only those processes used by the typical student in education and psychology have been presented. As a general rule, only one procedure (the most widely accepted and generally used) is given for computing each statistical measure. The student has not been confused with a multiplicity of processes for determining the same statistical values. The minimum amount of space has been devoted to process, the major emphasis being placed on the interpretation of statistical measures.

An attempt has been made to present the material in the simplest form possible and still not cause the student to have a distorted conception of statistical measures in general. The language, in so far as possible, is non-technical. The book is written for the *non-mathematical student.* A knowledge of high-school algebra should be all the mathematics necessary to comprehend the measures treated. At the same time the book has not been overly simplified. A thorough knowledge of the contents of this volume should enable the graduate student in education and psychology to do the statistical phases of practically all the research problems which he encounters.

Page references have been made in Appendix A to the treatment of the various statistical measures in 26 textbooks in statistics. A study of these references will give the student different points of view regarding the material treated in this textbook. Experience and research indicate that the student in educational statistics should not use the reading method (that is, restricting his study of statistics solely to reading statistical literature) or the problems method (confining his study to working statistical problems) exclusively, because the combination method (a combination of reading statistical literature and working problems) appears to give slightly better results. It has been suggested that an approximately equal distribution of the student's time between working statistical problems (as given at the end of each chapter) and reading statistical literature (as given in Appendix A) is slightly more profitable than giving a larger proportion of his time either to reading or to working problems.[1]

The answers to the problems are given in Appendix D. They have been checked and rechecked. The author will appreciate having his attention called to any errors discovered.

[1] Dennis H. Cooke, "Two Experiments in Learning Educational Statistics," *Journal of Educational Research, XXVI*, 674–78, May, 1933.

The author wishes to give due acknowledgment to many of his former graduate students in statistics who have criticized the material and made many valuable suggestions regarding the book in its several experimental editions. He is especially indebted to Dr. E. R. Enlow and Professor W. A. Cordrey, each of whom read the manuscript carefully, made many helpful suggestions, and assisted in checking all the calculations and formulas. He is grateful to Dr. Susan B. Riley who read the manuscript from the standpoint of English construction. To the publishers of many statistical books and educational and psychological journals from which material has been borrowed and permissions to quote granted, the author wishes to express his sincere appreciation.

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MINIMUM ESSENTIALS OF STATISTICS

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ORGANIZATION AND CLASSIFICATION OF DATA

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Given a group of data in numerical form, before very many statistical facts can be determined therefrom it is necessary that the data be organized and classified in a manner that is most appropriate to these facts, and in such a way that succeeding calculations will be facilitated. It is frequently necessary to know whether a given score, say 30, is high, average, or low within a given group of scores. To determine this fact one must know something about the nature of the distribution; that is, the number of students scoring above and below 30, and how far above and below. It is in this respect that many teachers fail to achieve the best interpretations of pupils' marks and test scores; that is, they do not interpret them in terms of the marks and scores of the entire group. Unless the group is quite small, it is desirable that numerical data be organized in such a manner that we can see readily how many students scored between 70 and 80, between 80 and 90, etc. Such organization of data requires the construction of a frequency table or distribution, which will be described in the following section.

How to Distribute Data into a Frequency Table

The data in Table I will be used in illustrating the construction of a frequency distribution. These data are the scores of 72 fifth-, sixth-, seventh-, and eighth-grade pupils in verbal intelligence.[1]

TABLE I

SCORES OF 72 FIFTH-, SIXTH-, SEVENTH-, AND EIGHTH-GRADE PUPILS IN VERBAL INTELLIGENCE

Pupil No.	Score	Pupil No.	Score	Pupil No.	Score	Pupil No.	Score	Pupil No.	Score	Pupil No.	Score
1	30	13	30	25	37	37	63	49	63	61	86
2	39	14	37	26	39	38	47	50	70	62	83
3	64	15	49	27	81	39	54	51	62	63	95
4	45	16	46	28	50	40	66	52	80	64	64
5	36	17	34	29	48	41	75	53	67	65	88
6	25	18	38	30	64	42	77	54	81	66	103
7	24	19	33	31	67	43	53	55	59	67	74
8	57	20	17	32	45	44	30	56	88	68	74
9	23	21	39	33	38	45	28	57	84	69	123
10	35	22	46	34	54	46	45	58	85	70	76
11	32	23	52	35	56	47	56	59	76	71	69
12	38	24	31	36	53	48	61	60	81	72	78

The steps in making the distribution of scores are as follows:

1. Determine the number and size of class intervals. It is usually desirable to have not fewer than 10 or 12 and not more than 18 or 20 class intervals or sub-units of the total range in grouping test scores or other measures for further statistical treatment. There is no exact rule for determining this number, but the nature of the data must be taken into consideration. As a general rule, the class groupings in distributions containing fewer than 10 or 12 steps are so coarse that subsequent calculations from these distributions will likely involve an appreciable error due solely to the coarseness of grouping. On the other hand, in the majority of distributions the point of diminishing returns, in relation to additional accuracy, is reached when the number of intervals exceeds 18 or 20; that is, the added accuracy in increasing the number of intervals beyond 18

[1] See Table XXIII, p. 136.

or 20 does not justify the expenditure of the additional time and effort involved.

As a general rule, when the scores group themselves at points on the scale that are equally distant from each other, the width of the class interval should be the distance from one of these points to another or some multiple thereof. If there are periodic clusters of measures, such as those produced by the concentration of teachers' marks around multiples of five, the class interval should be adjusted so as to have such bunches of measures in the middle of the successive classes (class intervals).

The difference of 106 (or 123 − 17 = 106) between the largest and the smallest scores in Table I suggests adopting a grouping by 10's, which obviously will provide at least the minimum number of classes. Grouping the scores of Table I naturally suggests establishing class limits at the multiples of 10. Since intelligence scores do not tend to cluster around multiples of 10, there seems to be no valid objection to using such class intervals as 40 up to 50, 50 up to 60, and so on. In order to provide comparable results among students in statistics it is suggested here that limits of class intervals be adopted which are divisible by the size of the interval, unless the nature of the data indicates otherwise.

2. Determine the limits of the class intervals. The twelve class intervals necessary to include all the scores in Table I are listed in Table II. The limits of the class intervals are given in column 1 of Table II. For example, the interval of 110.0–119.9 includes all scores of 110 through 119.999 (the 9's running to infinity), but does not include 120. A score of 120 will be included in the top interval of 120.0–129.9. Column 2 is an abbreviated form of column 1. The 110–119 interval in column 2 has the same limits as the interval of 110.0–119.9 in column 1. The .0 and .9 are omitted to facili-

tate the writing of the intervals. The beginning student should write the intervals as indicated in column 1 until he has had considerable experience in expressing class intervals at which time he may express them as they are indicated in column 2. The 110 interval in column 3 has the same limits as the 110–119 and the 110.0–119.9 intervals in columns 2 and 1, respectively. Class intervals are expressed according to column 3 in the majority of published reports of research studies, and it is suggested here that the student use this form only in preparing a manuscript for publication.

TABLE II

A Frequency Distribution of the Scores Listed Individually in Table I

Class Limits	Class Interval		Tally	Frequency (*f*)
(1)	(2)	(3)	(4)	(5)
120.0–129.9	120–129	120	/	1
110.0–119.9	110–119	110		0
100.0–109.9	100–109	100	/	1
90.0– 99.9	90– 99	90	/	1
80.0– 89.9	80– 89	80	~~////~~ ~~////~~	10
70.0– 79.9	70– 79	70	~~////~~ ///	8
60.0– 69.9	60– 69	60	~~////~~ ~~////~~ /	11
50.0– 59.9	50– 59	50	~~////~~ ~~////~~	10
40.0– 49.9	40– 49	40	~~////~~ ///	8
30.0– 39.9	30– 39	30	~~////~~ ~~////~~ ~~////~~ //	17
20.0– 29.9	20– 29	20	////	4
10.0– 19.9	10– 19	10	/	1
				Total (N) = 72

3. Tabulate the scores. This is done by placing a tally (column 4) in the proper compartment; that is, opposite the proper class interval for each score. A system of checking off each score (in the original data) as it is tallied is helpful. After the tallying is completed and checked for total, the number of tally marks for each class interval is indicated in column 5 under the heading of "frequency."

The Mid-Point of a Class Interval

A score of 50 is considered as having a unit of length with 50.5 as the mid-point. (See Fig. 1 of Diagram I in which the limits of this interval are shown to be 50.0 and 51.0.) A graphical representation of the interval 50–54 is given in Fig. 2 of Diagram I. It is readily apparent that the mid-point of this interval is 52.5 on the scale, since 52.5 is the middle of the scaled line representing the interval and since there are 2.5 score units on either side of this point. Note

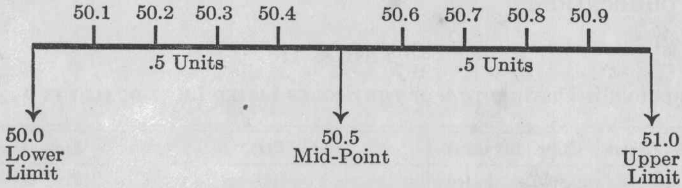

Fig. 1. Illustrative of a Class Interval of 1

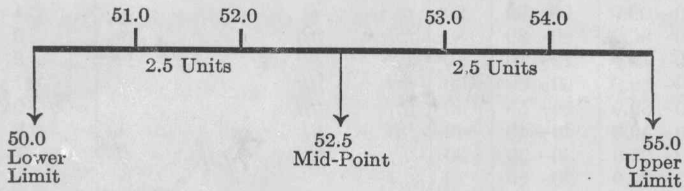

Fig. 2. Illustrative of an Odd Class Interval

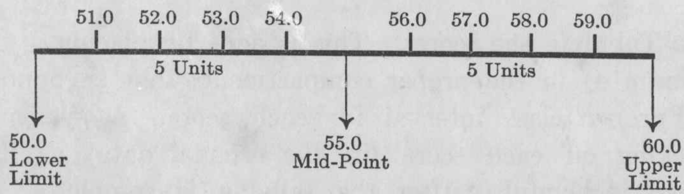

Fig. 3. Illustrative of an Even Class Interval

Diagram I.—Graphical Illustrations of Class Intervals and Mid-Points that the width or size of the class interval is 5, determined as follows: $55.0 - 50.0 = 5$, or, $54 - 50 + 1 = 5$. Also, the mid-point is $50.0 + \frac{55.0 - 50.0}{2} = 52.5$, or, $50.0 + \frac{54.0 - 50.0 + 1}{2} = 52.5$. It should be noted that the mid-point of an odd interval is a fraction (52.5). Figure 3 of Diagram I shows that the mid-point of an even interval is an integer (55.0).

The Mid-Point as Representative of the Scores in the Class Interval

In the frequency distribution it is assumed that the scores within a given class interval are distributed symmetrically throughout the interval, so that the mid-point of the interval may be regarded as the average score for this interval. This assumption is, of course, seldom literally fulfilled. For example, in the interval 20–29 of Table II, two of the four scores (23, 24, 25, 28) fall below the mid-point, one on the mid-point, and one above it; in interval 40–49, three of the eight scores (45, 45, 45, 46, 46, 47, 48, 49) fall on the mid-point and five above it; while in interval 50–59, six of the ten scores (50, 52, 53, 53, 54, 54, 56, 56, 57, 59) fall below the mid-point and four above it. Except in markedly skewed or irregular distributions, however, the assumption that the mid-point is the best representative score is reasonably valid for purposes of subsequent statistical treatment.

Graphs of the Frequency Distribution

The frequency distribution is often represented graphically by means of the frequency polygon, the histogram, and comparative line graphs. The distribution of scores in Table I is represented by a frequency polygon in Diagram II and by a histogram, superimposed on the frequency polygon,