

# **ELECTROMAGNETICS FOR ENGINEERS**

**ARLON T. ADAMS**

**RONALD**

# **ELECTROMAGNETICS FOR ENGINEERS**

**ARLON T. ADAMS**  
SYRACUSE UNIVERSITY

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## RECTANGULAR COORDINATES

$$\nabla\psi = \frac{\partial\psi}{\partial x} \hat{\mathbf{x}} + \frac{\partial\psi}{\partial y} \hat{\mathbf{y}} + \frac{\partial\psi}{\partial z} \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\begin{aligned} \nabla \times \mathbf{F} = & \left[ \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right] \hat{\mathbf{x}} + \left[ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right] \hat{\mathbf{y}} \\ & + \left[ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right] \hat{\mathbf{z}} \end{aligned}$$

## CYLINDRICAL COORDINATES

$$\nabla\psi = \frac{\partial\psi}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial\psi}{\partial\phi} \hat{\boldsymbol{\phi}} + \frac{\partial\psi}{\partial z} \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r} \frac{\partial}{\partial r} (rF_r) + \frac{1}{r} \frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\begin{aligned} \nabla \times \mathbf{F} = & \left[ \frac{1}{r} \frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z} \right] \hat{\mathbf{r}} + \left[ \frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r} \right] \hat{\boldsymbol{\phi}} \\ & + \left[ \frac{1}{r} \frac{\partial}{\partial r} (rF_\phi) - \frac{1}{r} \frac{\partial F_r}{\partial\phi} \right] \hat{\mathbf{z}} \end{aligned}$$

## SPHERICAL COORDINATES

$$\nabla\psi = \frac{\partial\psi}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial\psi}{\partial\theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin\theta} \frac{\partial\psi}{\partial\phi} \hat{\boldsymbol{\phi}}$$

$$\begin{aligned} \nabla \cdot \mathbf{F} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta} (F_\theta \sin\theta) \\ &\quad + \frac{1}{r \sin\theta} \frac{\partial F_\phi}{\partial\phi} \end{aligned}$$

$$\begin{aligned} \nabla \times \mathbf{F} &= \frac{1}{r \sin\theta} \left[ \frac{\partial}{\partial\theta} (F_\phi \sin\theta) - \frac{\partial F_\theta}{\partial\phi} \right] \hat{\mathbf{r}} \\ &\quad + \frac{1}{r} \left[ \frac{1}{\sin\theta} \frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r} (r F_\phi) \right] \hat{\boldsymbol{\theta}} \\ &\quad + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r F_\theta) - \frac{\partial F_r}{\partial\theta} \right] \hat{\boldsymbol{\phi}} \end{aligned}$$

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*To my mother,  
Ruth Whipple*

# Preface

This textbook is intended for the introductory course in electromagnetic fields for students of engineering. The text is divided into five parts, covering vector analysis, electrostatics, magnetostatics, Faraday's Law and material characteristics, and Maxwell's Equations. Wherever possible, the approach is to appeal to the student's physical intuition, and to this end a large number of illustrations is provided. Special features include a chapter on computer solution of electrostatic problems, emphasizing matrix inversion methods, and one on the characteristics of dielectric and magnetic materials. The text contains more than enough material for a one semester course. Numerous problems are given at the end of each chapter, with the more difficult problems indicated by a dagger.

After a survey of the applications of electromagnetic theory, Part I presents the vector concepts of curl, divergence, and gradient in terms of the limiting integral definitions which yield useful physical insights. Physical pictures, such as the divergence meter and the curl meter, are emphasized. Appendix D is a useful adjunct to this chapter, introducing a shorthand notation which leads to the generalized divergence theorem and the generalized Stokes' Law.

In Part II the basic laws of electrostatics are derived by inductive logic rather than by the usual approach of starting with Coulomb's Law. Thus the curl and divergence properties of the electrostatic field, rather than the inverse-square behavior of charges, are emphasized. The discussion of dielectrics emphasizes the drift of bound charges, which results in polarization. Perhaps more emphasis than usual is placed upon the parallel role of the bound, free, and total charge sources. In the discussion of approximate methods the emphasis is on the matrix inversion techniques for electrostatic problems. Examples are given and computer programs for the examples are provided in Appendix E. The problems and examples in this chapter might offer material for term projects.

After introducing the theory of magnetism, Part III further unifies the



concepts of vector analysis by introducing Helmholtz's Theorem, and emphasizes that in electrostatics and magnetostatics we have really been studying the general properties of curlfree and divergenceless vector fields, respectively.

Faraday's experiments are described in abbreviated form, in Part IV, leading up to a statement of Faraday's Law. Some of the effects of motion are discussed in terms of a new electric field  $E'$  measured by a moving observer. Voltage is redefined in terms of  $E'$  and numerous examples are given. A descriptive account of the characteristics of dielectric and magnetic materials is given.

In Part V the subject of Maxwell's equations is approached from several points of view, thereby providing complete justification for the second curl equation.

The author wishes to acknowledge the assistance and advice of a number of persons. The ideas and encouragement of Wilbur R. LePage were especially important in the crucial early stages of the writing. My wife, Judy, provided encouragement and also proofread the entire manuscript. Several colleagues, including Richard McFee, Rajendra Nanavati, Philipp Kornreich, Stephen Kowel, Robert Wallenberg, and Joseph Mautz, reviewed portions of the manuscript, and Joseph Mautz, Daniel Warren, and Andrew Farrar wrote the computer programs for Appendix E. Finally, I am grateful to the secretaries who typed the many versions of the text.

ARLON T. ADAMS

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## VECTOR IDENTITIES

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$\nabla(\psi\phi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\psi\mathbf{A}) = \nabla\psi \cdot \mathbf{A} + \psi(\nabla \cdot \mathbf{A})$$

$$\nabla \times (\psi\mathbf{A}) = \nabla\psi \times \mathbf{A} + \psi(\nabla \times \mathbf{A})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla V) = 0$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

## THE GENERALIZED DIVERGENCE THEOREM AND THE GENERALIZED STOKES' LAW

$$\iiint_v \nabla \cdot \mathbf{F} \, dv = \oint_s \hat{\mathbf{n}} \cdot \mathbf{F} \, ds$$

$$\iint_s (\hat{\mathbf{n}} \times \nabla) \cdot \mathbf{F} \, ds = \oint_c \mathbf{dl} \cdot \mathbf{F}$$

$$\text{Divergence Theorem: } \iiint_v \nabla \cdot \mathbf{F} \, dv = \oint_s \mathbf{F} \cdot \hat{\mathbf{n}} \, ds$$

$$\text{Stokes Law: } \iint_s \nabla \times \mathbf{F} \cdot \hat{\mathbf{n}} \, ds = \oint_c \mathbf{F} \cdot \mathbf{dl}$$

### GAUSS' LAW

$$\nabla \cdot \mathbf{D} = \rho$$

$$\oiint \mathbf{D} \cdot \hat{\mathbf{n}} \, ds = q_{\text{encl}} = \iiint \rho \, dv$$

### AMPERE'S LAW

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{encl}} = \iint \mathbf{J} \cdot \hat{\mathbf{n}} \, ds$$

### FARADAY'S LAW

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$V_{ab} = -\frac{d}{dt} \iint_s \mathbf{B} \cdot \hat{\mathbf{n}} \, ds$$

$$= -\iint_s \frac{\partial \mathbf{B}}{\partial t} \cdot \hat{\mathbf{n}} \, ds + \oint_c \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l}$$

# Contents

## **1 INTRODUCTION 3**

A Survey. Applications of Electromagnetic Theory.

## **PART I VECTOR ANALYSIS**

## **2 VECTOR ALGEBRA 13**

Introduction. Scalar Functions of Position. Vector Functions of Position. Coordinate Systems. Operations in Vector Algebra. Commutativity, Distributivity, and Associativity of Vector Operations.

## **3 VECTOR CALCULUS 25**

The Del ( $\nabla$ ) Operator. Gradient. Divergence. Curl. Further Interpretations of Curl and Divergence. Multiple Products of Vector Calculus.

## **4 DIFFERENTIATION AND INTEGRATION OF VECTORS 43**

Introduction. Differentiation of Vector Functions. Integration of Vector Functions; Line Integrals. Independence of Path. Surface Integrals. Volume Integrals.

## **PART II ELECTROSTATICS**

## **5 THE THEORETICAL BASIS OF ELECTROSTATICS 65**

Introduction. Charge. The Theoretical Basis of Electrostatics. The Electrostatic Potential. Conductors. Flux Concepts. Coulomb's Law.

## **6 GAUSS' LAW, THE POTENTIAL INTEGRAL, AND THE METHOD OF IMAGES** **87**

Applications of Gauss' Law. Laplace's Equation, Poisson's Equation, and the Potential Integral. Image Theory. Linearity and Superposition. Capacity. Electric Fields of Charge Distributions—Field and Source Point Representation. Uniqueness of the Solution of Laplace's or Poisson's Equation.

## **7 DIELECTRIC PHENOMENA** **127**

Introduction. Experimental Observation of Dielectric Effects. Bound Charge and Dipoles. The Polarization Vector—**P**. Postulates for the Electric Field with Dielectrics Present. The Displacement Vector **D**. Boundary Conditions on the Field Vectors **E**, **D**, and **P**. Analogies Between Dielectric and Free Space Problems. Energy Storage in the Electrostatic Field. Forces in the Electrostatic Field.

## **8 BASIC MATRIX INVERSION METHODS** **166**

Introduction. Matrix Inversion Techniques for Systems of Simultaneous Linear Equations. Solving the General Symmetrical Capacitance Problem by Matrix Inversion. The Uses of Symmetry. Further Considerations Regarding Approximate Solutions by Matrix Inversion.

## **9 MATRIX METHODS, CURVILINEAR SQUARES, AND VARIATIONAL METHODS** **186**

Capacitance Problems Involving Dielectrics. Resistance by Matrix Inversion, a Mixed Boundary Value Problem. Asymmetrical and Multiterminal Problems—The Capacitance Matrix. Curvilinear Squares. Variational Methods. Capacitance Coefficients, Capacity, and Even and Odd Excitation Modes.

## PART III MAGNETOSTATICS

### 10 THE THEORETICAL BASIS OF MAGNETOSTATICS 223

Introduction. Sources of the Magnetic Field—Simple Experiments in Magnetostatics. Definition of the Magnetic Induction Field  $\mathbf{B}$ . Current Flow. The Theoretical Basis of Magnetostatics. Flux of the Magnetic Induction Vector  $\mathbf{B}$ . Helmholtz's Theorem and the Magnetic Vector Potential  $\mathbf{A}$ .

### 11 THE POTENTIAL INTEGRAL, INDUCTANCE, AND MAGNETIC FORCES 253

Magnetic Fields of Current Distributions—Field and Source Point Representation. Self and Mutual Inductance. Magnetic Forces and Torques.

### 12 MAGNETIC PHENOMENA 279

Introduction. Experimental Observation of Magnetic Effects. Free and Bound Currents. The Magnetization Vector  $\mathbf{M}$ . The Magnetic Intensity  $\mathbf{H}$ . Relationships for  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{H}$ . Boundary Conditions on the Field Vectors  $\mathbf{B}$ ,  $\mathbf{M}$ , and  $\mathbf{H}$ . Curlfree and Divergenceless Fields. Magnetic Circuits. Relationships Between Bound and Free Sources.

## PART IV FARADAY'S LAW AND MATERIAL CHARACTERISTICS

### 13 FARADAY'S LAW 313

Introduction. A Statement of Faraday's Law. Faraday's Experiments on Magnetic Induction. The Lorentz Force and the Electric Field. The Application of Faraday's Law. Magnetic Energy. Some Limitations on the Application of Faraday's Law.

## 14 DIELECTRIC AND MAGNETIC MATERIALS 348

Introduction. The Effective Electric Field and Dielectric Polarizability. Electronic Polarization. Ionic Polarization (Nonpolar Molecules). Orientation Polarization (Polar Molecules). Ferroelectrics—Permanent Polarization and Hysteresis. Other Types of Dielectrics. Magnetic Materials—Self Field and Magnetic Polarizability. Diamagnetic and Paramagnetic Materials. Ferromagnetic Materials. Antiferromagnetics, Ferrimagnetics, and Other Magnetic Effects. Artificial Dielectric and Magnetic Materials. Temperature Dependence of Material Characteristics—The Curie-Weiss Law. Spontaneous Polarization and Magnetization. Conductors and Conductivity.

## PART V MAXWELL'S EQUATIONS

## 15 BASIC THEORY OF TIME-VARYING FIELDS 381

Introduction. Propagation of Energy. Faraday's Law and Maxwell's Equation. The Wave Equation. Fields in a Dielectric-Loaded Capacitor. Maxwell's Equations and the Boundary Conditions. Plane Waves.

## 16 RADIATION AND SKIN EFFECT 401

Phasors and Fields. Time Harmonic (Phasor) Forms of the Equations of Electromagnetic Theory. Energy Storage, Radiated Energy, and Poynting's Theorem. Vector Potentials and the Wave Equation. Hertzian Dipole Radiation. Skin Depth.

## APPENDIXES

A	SYMBOLS, UNITS AND DIMENSIONS OF BASIC QUANTITIES	421
B	CONSTANTS	422
C	VECTOR CALCULUS IN RECTANGULAR, CYLINDRICAL, AND SPHERICAL COORDINATES	423
D	THE GENERALIZED DIVERGENCE THEOREM AND THE GENERALIZED STOKES' LAW	429
E	COMPUTER PROGRAMS AND SOURCE FUNCTIONS FOR MATRIX INVERSION TECHNIQUES	437

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