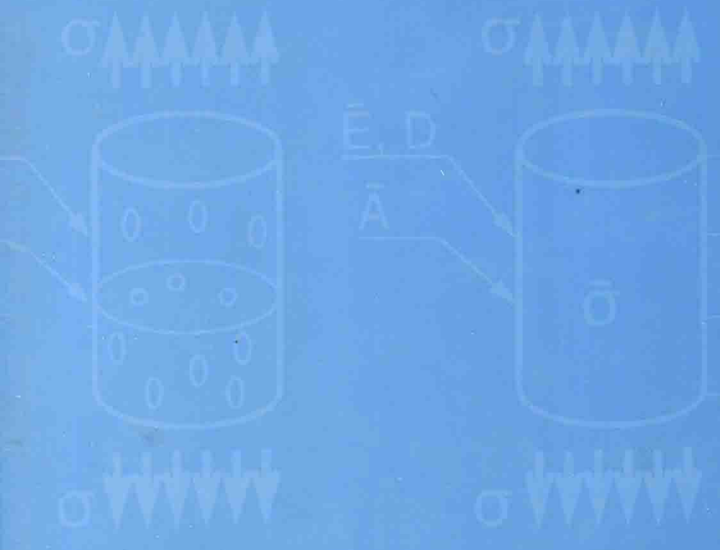
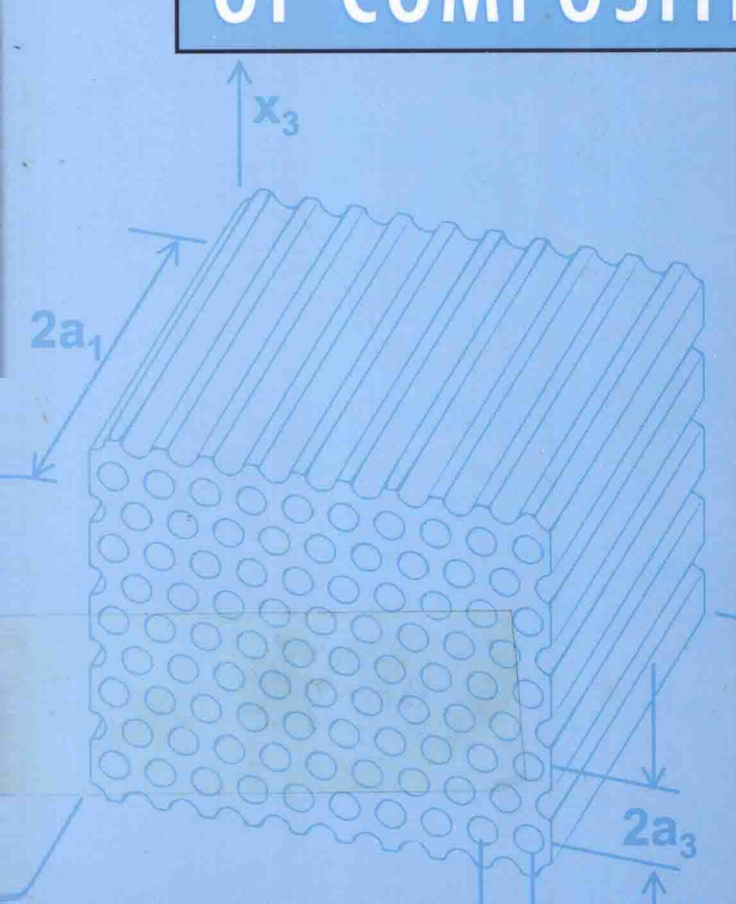


FINITE ELEMENT ANALYSIS OF COMPOSITE MATERIALS



Ever J. Barbero

CRC Press
Taylor & Francis Group

FINITE ELEMENT ANALYSIS OF COMPOSITE MATERIALS

Ever J. Barbero
Department of Mechanical and Aerospace Engineering
West Virginia University
USA

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-4200-5433-0 (Hardcover)

Library of Congress Cataloging-in-Publication Data

Barbero, Ever J.
Finite element analysis of composite materials / Ever J. Barbero.
p. cm.
ISBN-13: 978-1-4200-5433-0 (alk. paper)
ISBN-10: 1-4200-5434-1
1. Composite materials--Mathematical models. 2. Finite element method. I. Title.

TA418.9.C6B368 2007
620.1'18--dc22 2007007536

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

FINITE ELEMENT ANALYSIS OF COMPOSITE MATERIALS

Finite Element Analysis of Composite Materials

Ever J. Barbero

Dedicado a mis padres, Sonia Eulalia y Ever Francisco.

Preface

Finite Element Analysis of Composite Materials deals with the analysis of structures made of composite materials, also called *composites.* The analysis of composites treated in this textbook includes the analysis of the material itself, at the micro-level, and the analysis of structures made of composite materials. This textbook evolved from the class notes of *MAE 646 Advanced Mechanics of Composite Materials* that I teach as a graduate course at West Virginia University. Although this is also a textbook on advanced mechanics of composite materials, the use of the finite element method is essential for the solution of the complex boundary value problems encountered in the advanced analysis of composites, and thus the title of the book.

There are a number of good textbooks on advanced mechanics of composite materials, but none carries the theory to a practical level by actually solving problems, as it is done in this textbook. Some books devoted exclusively to finite element analysis include some examples about modeling composites but fall quite short of dealing with the actual analysis and design issues of composite materials and composite structures. This textbook includes an explanation of the concepts involved in the detailed analysis of composites, a sound explanation of the mechanics needed to translate those concepts into a mathematical representation of the physical reality, and a detailed explanation of the solution of the resulting boundary value problems by using commercial Finite Element Analysis software such as ANSYS™. Furthermore, this textbook includes more than fifty fully developed examples interspersed with the theory, as well as more than seventy-five exercises at the end of chapters, and more than fifty separate pieces of ANSYS code used to explain in detail the solution of example problems. The reader will be able to reproduce the examples and complete the exercises. When a finite element analysis is called for, the reader will be able to do it with commercially or otherwise available software. A Web site is set up with links to download the necessary software unless it is easily available from Finite Element Analysis software vendors. ANSYS and MATLAB™ code is explained in the examples, and the code can be downloaded from the Web site as well. Furthermore, the reader will be able to extend the capabilities of ANSYS by use of ANSYS Parametric Design Language (APDL), user material subroutines, and programmable postprocessing, as demonstrated in the examples included in this textbook.

Chapters 1 through 8 can be covered in a one-semester graduate course. Chapter 2 (Introduction to the Finite Element Method) contains a brief introduction intended for those readers who have not had a formal course or prior knowledge about the finite element method. Chapter 4 (Buckling) is not referenced in the remainder of the textbook and thus it could be omitted in favor of more exhaustive coverage of content in later chapters. Chapters 7 (Viscoelasticity) and 8 (Damage Mechanics) are placed consecutively to emphasize hereditary phenomena. However, Chapter 7 can be skipped if more emphasis on damage and/or delaminations is desired in a one-semester course. A complete continuum damage model with coupled damage-plasticity effects is presented in Chapter 9, immediately following the more fundamental treatment of damage in Chapter 8. Chapter 9 could be omitted for the sake of time, especially if the instructor desires to cover Chapter 10 (Delaminations) as part of a one-semester course.

The *inductive method* is applied as much as possible in this textbook. That is, topics are introduced with examples of increasing complexity, until sufficient physical understanding is reached to introduce the general theory without difficulty. This method will sometimes require that, at earlier stages of the presentation, certain facts, models, and relationships be accepted as fact, until they are completely proven later on. For example, in Chapter 7, viscoelastic models are introduced early to aid the reader in gaining an appreciation for the response of viscoelastic materials. This is done simultaneously with a cursory introduction to the superposition principle and the Laplace transform, which are formally introduced only later in the chapter. For those readers accustomed to the *deductive method*, this may seem odd, but many years of teaching have convinced me that students acquire and retain knowledge more efficiently in this way.

It is assumed that the reader is familiar with basic mechanics of composites as covered in introductory level textbooks such as my previous textbook *Introduction to Composite Material Design*. Furthermore, it is assumed that the reader masters a body of knowledge that is commonly acquired as part of a bachelor of science degree in any of the following disciplines: Aerospace, Mechanical, Civil, or similar. References to books and to other sections in this textbook, as well as footnotes are used to assist the reader in refreshing those concepts and to clarify the notation used. Prior knowledge of continuum mechanics, tensor analysis, and the finite element method would enhance the learning experience but are not necessary for studying with this textbook. The finite element method is used as a tool to solve practical problems. For the most part, ANSYS is used throughout the book. Finite element programming is limited to programming material models, post-processing algorithms, and so on. Basic knowledge of MATLAB is useful but not essential.

Only three software packages are used throughout the book. ANSYS is needed for finite element solution of numerous examples and suggested problems. MATLAB is needed for both symbolic and numerical solution of numerous examples and suggested problems. Additionally, BMI3™, which is available free of charge on the book's Web site, is used in Chapter 4. Several other programs are also mentioned, such as ABAQUS™and LS-DYNA™ but they are not used in the examples. All the code printed in the examples is available on the book's Web site `http://www.mae.wvu.edu/barbero/feacm/`.

Composite materials are now ubiquitous in the marketplace, including extensive applications in aerospace, automotive, civil infrastructure, sporting goods, and so on. Their design is especially challenging because, unlike conventional materials such as metals, the composite material itself is designed concurrently with the composite structure. Preliminary design of composites is based on the assumption of a state of plane stress in the laminate. Furthermore, rough approximations are made about the geometry of the part, as well as the loading and support conditions. In this way, relatively simple analysis methods exist and computations can be carried out simply using algebra. However, preliminary analysis methods have a number of shortcomings that are remedied with advanced mechanics and finite element analysis, as explained in this textbook. Recent advances in commercial finite element analysis packages, with user friendly pre- and post-processing, as well as powerful user-programmable features, have made detailed analysis of composites quite accessible to the designer. This textbook bridges the gap between powerful finite element tools and practical problems in structural analysis of composites. I expect that many graduate students, practicing engineers, and instructors will find this to be a useful and practical textbook on finite element analysis of composite materials based on sound understanding of advanced mechanics of composite materials.

Ever J. Barbero

Acknowledgments

First and foremost, I wish to thank Joan Andreu Mayugo Majo, Domenico Bruno, and Fabrizio Greco for writing Chapters 9 and 10. Chapter 6 is largely based on the work of Raimondo Luciano and Elio Sacco, to whom I am indebted. I wish to thank Tom Damiani and Joan Andreu Mayugo Majo, who taught the course in 2004 and 2006, respectively, both making many corrections and additions to the course notes on which this textbook is based. Acknowledgement is due to those who reviewed the manuscript including Enrique Barbero, Guillermo Creus, Luis Godoy, Paolo Lonetti, Severino Marques, Pizhong Qiao, Timothy Norman, Sonia Sanchez, and Eduardo Sosa. Furthermore, recognition is due to those who helped me compile the solutions manual, including Hermann Alcazar, John Sandro Rivas, and Rajiv Dastane, among others. Also, I wish to thank my colleagues and students for their valuable suggestions and contributions to this textbook. Finally, my gratitude goes to my wife, Ana Maria, and to my children, Margaret and Daniel, who gave up many opportunities to bond with their dad so that I might write this book.

List of Symbols

Symbols Related to Mechanics of Orthotropic Materials

$\boldsymbol{\epsilon}$	Strain tensor
ε_{ij}	Strain components in tensor notation
ϵ_α	Strain components in contracted notation
ϵ_α^e	Elastic strain
ϵ_α^p	Plastic strain
λ	Lame constant
ν	Poisson's ratio
ν_{12}	Inplane Poisson's ratio
ν_{23}, ν_{13}	Interlaminar Poisson's ratios
ν_{xy}	Apparent laminate Poisson ratio x-y
$\boldsymbol{\sigma}$	Stress tensor
σ_{ij}	Stress components in tensor notation
σ_α	Stress components in contracted notation
$[a]$	Transformation matrix for vectors
e_i	Unit vector components in global coordinates
e'_i	Unit vector components in materials coordinates
f_i, f_{ij}	Tsai-Wu coefficients
k	Bulk modulus
l, m, n	Direction cosines
$\widetilde{u}(\varepsilon_{ij})$	Strain energy per unit volume
u_i	Displacement vector components
x_i	Global directions or axes
x'_i	Materials directions or axes
$\mathbf{C}$	Stiffness tensor
C_{ijkl}	Stiffness in index notation
$C_{\alpha,\beta}$	Stiffness in contracted notation
E	Young's modulus
E_1	Longitudinal modulus
E_2	Transverse modulus
E_2	Transverse-thickness modulus
E_x	Apparent laminate modulus in the global x-direction
$G = \mu$	Shear modulus
G_{12}	Inplane shear modulus

G_{23}, G_{13}	Interlaminar shear moduli
G_{xy}	Apparent laminate shear modulus x-y
I_{ij}	Second-order identity tensor
I_{ijkl}	Fourth-order identity tensor
Q'_{ij}	Lamina stiffness components in material coordinates
$[R]$	Reuter matrix
$\mathbf{S}$	Compliance tensor
S_{ijkl}	Compliance in index notation
$S_{\alpha,\beta}$	Compliance in contracted notation
$[T]$	Coordinate transformation matrix for stress
$[\overline{T}]$	Coordinate transformation matrix for strain

Symbols Related to Finite Element Analysis

$\underline{\underline{\partial}}$	Strain-displacement equations in matrix form
$\underline{\epsilon}$	Six-element array of strain components
$\theta_x, \theta_y, \theta_z$	Rotation angles following the right-hand rule (Figure 2.9)
$\underline{\sigma}$	Six-element array of stress components
ϕ_x, ϕ_y	Rotation angles used in plate and shell theory
$\underline{a}$	Nodal displacement array
u_j^e	Unknown parameters in the discretization
$\underline{\underline{B}}$	Strain-displacement matrix
$\underline{\underline{C}}$	Stiffness matrix
$\underline{\underline{K}}$	Assembled global stiffness matrix
$\underline{\underline{K}}^e$	Element stiffness matrix
$\underline{N}$	Interpolation function array
N_j^e	Interpolation functions in the discretization
$\underline{P}^e$	Element force array
$\underline{P}$	Assembled global force array

Symbols Related to Elasticity and Strength of Laminates

γ_{xy}^0	Inplane shear strain
γ_{4u}	Ultimate interlaminar shear strain in the 2-3 plane
γ_{5u}	Ultimate interlaminar shear strain in the 1-3 plane
γ_{6u}	Ultimate inplane shear strain
$\epsilon_x^0, \epsilon_y^0$	Inplane strains
ϵ_{1t}	Ultimate longitudinal tensile strain
ϵ_{2t}	Ultimate transverse tensile strain
ϵ_{3t}	Ultimate transverse-thickness tensile strain
ϵ_{1c}	Ultimate longitudinal compressive strain
ϵ_{2c}	Ultimate transverse compressive strain
ϵ_{3c}	Ultimate transverse-thickness compressive strain
κ_x, κ_y	Bending curvatures

κ_{xy}	Twisting curvature
ϕ_x, ϕ_y	Rotations of the middle surface of the shell (Figure 2.9)
c_4, c_5, c_6	Tsai-Wu coupling coefficients
t_k	Lamina thickness
u_0, v_0, w_0	Displacements of the middle surface of the shell
z	Distance from the middle surface of the shell
A_{ij}	Components of the extensional stiffness matrix $[A]$
B_{ij}	Components of the bending-extension coupling matrix $[B]$
D_{ij}	Components of the bending stiffness matrix $[D]$
$[E_0]$	Extensional stiffness matrix $[A]$, in ANSYS notation
$[E_1]$	Bending-extension matrix $[B]$, in ANSYS notation
$[E_2]$	Bending stiffness matrix $[D]$, in ANSYS notation
F_{1t}	Longitudinal tensile strength
F_{2t}	Transverse tensile strength
F_{3t}	Transverse-thickness tensile strength
F_{1c}	Longitudinal compressive strength
F_{2c}	Transverse compressive strength
F_{3c}	Transverse-thickness compressive strength
F_4	Interlaminar shear strength in the 2-3 plane
F_5	Interlaminar shear strength in the 1-3 plane
F_6	Inplane shear strength
H_{ij}	Components of the interlaminar shear matrix $[H]$
I_F	Failure index
M_x, M_y, M_xy	Moments per unit length (Figure 3.3)
$\widehat{M}_n$	Applied bending moment per unit length
N_x, N_y, N_xy	Inplane forces per unit length (Figure 3.3)
$\widehat{N}_n$	Applied inplane force per unit length, normal to the edge
$\widehat{N}_{ns}$	Applied inplane shear force per unit length, tangential
$(\overline{Q}_{ij})_k$	Lamina stiffness components in global coordinates, layer k
V_x, V_y	Shear forces per unit length (Figure 3.3)

Symbols Related to Buckling

λ, λ_i	Eigenvalues
s	Perturbation parameter
Λ	Load multiplier
$\Lambda^{(cr)}$	Bifurcation multiplier or critical load multiplier
$\Lambda^{(1)}$	Slope of the post-critical path
$\Lambda^{(2)}$	Curvature of the post-critical path
v	Eigenvectors (buckling modes)
$[K]$	Stiffness matrix
$[K_s]$	Stress stiffness matrix
P_{CR}	Critical load

Symbols Related to Free Edge Stresses

$\eta_{xy,x}, \eta_{xy,y}$	Coefficients of mutual influence
$\eta_{x,xy}, \eta_{y,xy}$	Alternate coefficients of mutual influence
F_{yz}	Interlaminar shear force y-z
F_{xz}	Interlaminar shear force x-z
M_z	Interlaminar moment

Symbols Related to Micromechanics

$\overline{\epsilon}_\alpha$	Average engineering strain components
$\overline{\varepsilon}_{ij}$	Average tensor strain components
$\epsilon^0_\alpha, \varepsilon^0_{ij}$	Far-field applied strain components
$\overline{\sigma}_\alpha$	Average stress components
$\mathbf{A}^i$	Strain concentration tensor, i-th phase, contracted notation
$2a_1, 2a_2, 2a_3$	Dimensions of the RVE
A_{ijkl}	Components of the strain concentration tensor
$\mathbf{B}^i$	Stress concentration tensor, i-th phase, contracted notation
B_{ijkl}	Components of the stress concentration tensor
I	6×6 identity matrix
P_{ijkl}	Eshelby tensor
V_f	Fiber volume fraction
V_m	Matrix volume fraction

Symbols Related to Viscoelasticity

$\dot{\varepsilon}$	Stress rate
η	Viscosity
θ	Age or aging time
$\dot{\sigma}$	Stress rate
τ	Time constant of the material or system
Γ	Gamma function
s	Laplace variable
t	Time
$C_{\alpha,\beta}(t)$	Stiffness tensor in the time domain
$C_{\alpha,\beta}(s)$	Stiffness tensor in the Laplace domain
$\widehat{C}_{\alpha,\beta}(s)$	Stiffness tensor in the Carson domain
$D(t)$	Compliance
$D_0, (D_i)_0$	Initial compliance values
$D_c(t)$	Creep component of the total compliance $D(t)$
D', D''	Storage and loss compliances
$E_0, (E_i)_0$	Initial moduli
E_∞	Equilibrium modulus
E, E_0, E_1, E_2	Parameters in the viscoelastic models (Figure 7.1)

$E(t)$	Relaxation
E', E''	Storage and loss moduli
$F[]$	Fourier transform
$(G_{ij})_0$	Initial shear moduli
$H(t - t_0)$	Heaviside step function
$H(\theta)$	Relaxation spectrum
$L[]$	Laplace transform
$L[]^{-1}$	Inverse Laplace transform

Symbols Related to Damage

α_{cr}	Critical misalignment angle at longitudinal compression failure
$\gamma(\delta)$	Damage hardening function
γ_0	Damage threshold
δ_{ij}	Kronecker delta
δ	Damage hardening variable
ε	Effective strain
$\overline{\varepsilon}$	Undamaged strain
ε^p	Plastic strain
$\dot{\gamma}$	Heat dissipation rate per unit volume
$\dot{\gamma}_s$	Internal entropy production rate
$\dot{\lambda}, \dot{\lambda}^d$	Damage multiplier
$\dot{\lambda}^p$	Yield multiplier
σ	Effective stress
$\overline{\sigma}$	Undamaged stress
φ, φ^*	Strain energy density, and complementary SED
χ	Gibbs energy density
ψ	Helmholtz free energy density
Λ	Standard deviation of fiber misalignment
$\mathbf{\Omega} = \Omega_{ij}$	Integrity tensor
d_i	Eigenvalues of the damage tensor
f^d	Damage flow surface
f^p	Yield flow surface
$f(x), F(x)$	Probability density, and its cumulative probability
g^d	Damage surface
g^p	Yield surface
m	Weibull modulus
p	Yield hardening variable
$u(\varepsilon_{ij})$	Internal energy density
A_{ijkl}	Tension-compression damage constitutive tensor
B_{ijkl}	Shear damage constitutive tensor
B_a	Dimensionless number (8.57)
$\overline{C}_{\alpha,\beta}$	Stiffness matrix in the undamaged configuration
$\mathbf{C}^{ed}$	Tangent stiffness tensor

D_{ij}	Damage tensor
D_{1t}^{cr}	Critical damage at longitudinal tensile failure
D_{1c}^{cr}	Critical damage at longitudinal compression failure
D_{2t}^{cr}	Critical damage at transverse tensile failure
$E(D)$	Effective modulus
$\overline{E}$	Undamaged (virgin) modulus
$G_c = 2\gamma_c$	Surface energy
J_{ijkl}	Normal damage constitutive tensor
M_{ijkl}	Damage effect tensor
$R(p)$	Yield hardening function
R_0	Yield threshold
Y_{ij}	Thermodynamic force tensor

Symbols Related to Delaminations

α, β, γ	Mixed mode fracture propagation parameters
ℓ	Delamination length for 2D delaminations
ψ_{xi}, ψ_{yi}	Rotation of normals to the middle surface of the plate
Ω	Volume of the body
Ω_D	Delaminated region
Π_e	Potential energy, elastic
Π^r	Potential energy, total
$\dot{\Gamma}$	Dissipation rate
Λ	Interface strain energy density per unit area
$\partial\Omega$	Boundary of the body
d	One-dimensional damage state variable
k_{xy}, k_z	Displacement continuity parameters
$[A_i], [B_i], [D_i]$	Laminate stiffness sub-matrices
$G(\ell)$	Energy release rate (ERR), total, in 2D
G	Energy release rate (ERR), total, in 3D
G_I, G_{II}, G_{III}	Energy release rate (ERR) of modes I, II, and III
G_c	Critical energy release rate (ERR), total, in 3D
G_I^c	Critical energy release rate mode I
$[H_i]$	Laminate interlaminar shear stiffness matrix
K_I, K_{II}, K_{III}	Stress intensity factors (SIF) of modes I, II, and III
N_i, M_i, T_i	Stress resultants
U	Internal energy
W	Work done by the body on its surroundings

List of Examples