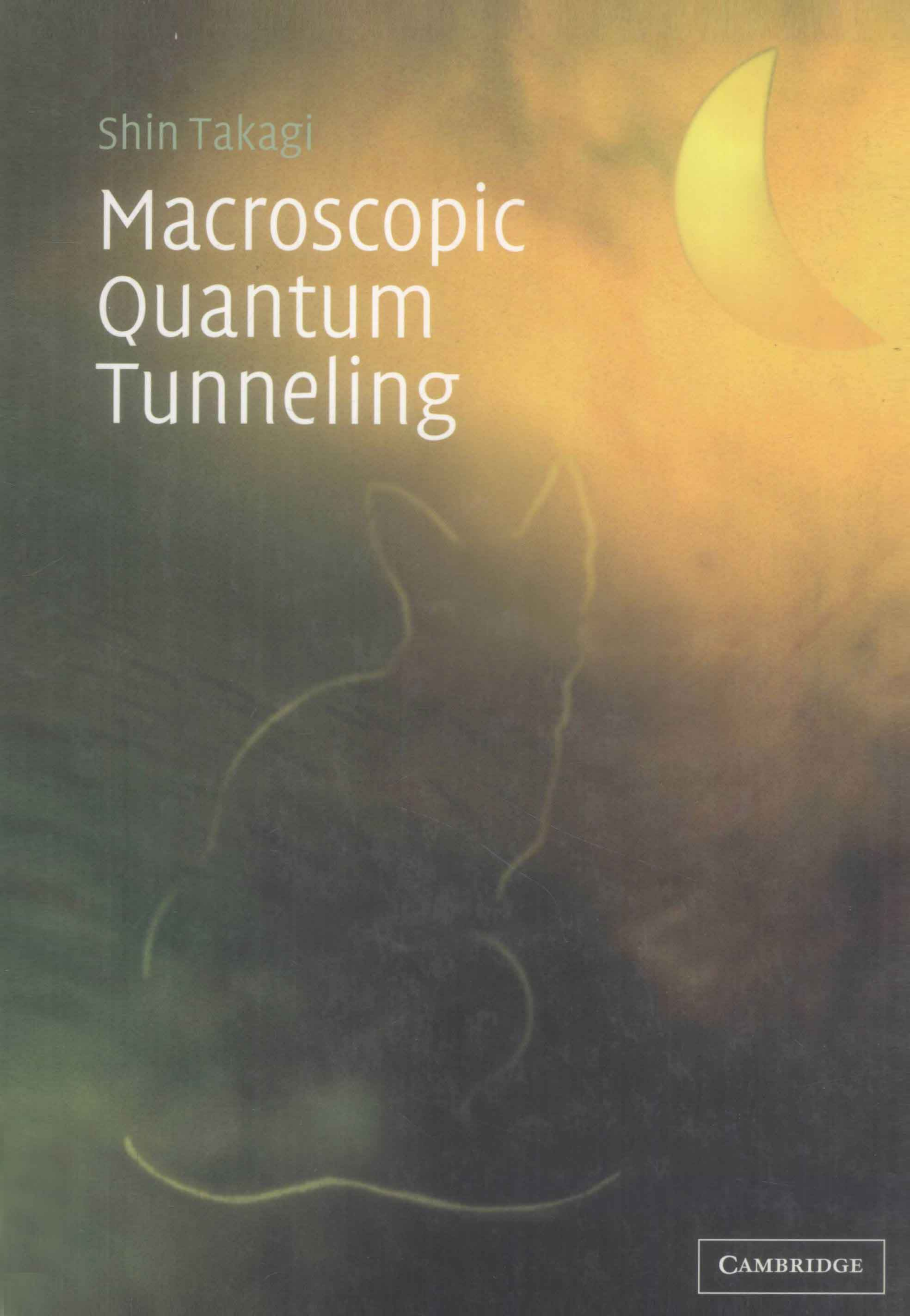
Shin Takagi

# Macroscopic Quantum Tunneling



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# MACROSCOPIC QUANTUM TUNNELING

SHIN TAKAGI
*Fuji Tokoha University,*
*Fuji, Japan*

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**Macroscopic Quantum Tunneling**

Macroscopic quantum phenomena are particularly important when considering the problem of Schrödinger's cat. This book contains a coherent and self-contained account of such phenomena, focusing on the central role played by macroscopic quantum tunneling.

Beginning with an explanation of the nature and significance of the cat problem, Shin Takagi introduces the concept of macroscopic quantum tunneling. He deals with typical examples in detail, elucidating how quantum mechanical coherence may be lost (so-called 'decoherence') or how it may be maintained despite the effects of environment and measurement processes. Recent experimental and theoretical advances are discussed, and the remaining problems described. The final chapter describes an experiment to decide between quantum mechanics and macrorealism in the light of Einstein's moon.

Assuming only a knowledge of elementary quantum mechanics, this book emphasizes conceptual aspects rather than technical details. It provides a firm introduction to the subject for graduate students and researchers.

SHIN TAKAGI obtained his Ph.D. from the University of Tokyo in 1974 and is probably best known for the so-called Leggett–Takagi equation which explains the spin dynamics of superfluid helium 3. He has worked in several fields of theoretical physics including low-temperature physics, quantum field theory in curved space and the problem of time and history in quantum mechanics. Currently Professor of Physical Sciences at Fuji Tokoha University, Shin Takagi has contributed to two other books in Japanese: *Quantum Phenomena in the Macroscopic World* (Shokabo Publishers, 1999) and *The World of the Quantum* (Iwanami Shoten Publishers, 1999).

# Preface

The phrase *tunneling*, which has classical-mechanical overtones, is a way of viewing certain dynamical processes within the paradigm of quantum mechanics. *Macroscopic quantum tunneling*, in particular, offers an indispensable viewpoint from which to ponder the cat.[1] This viewpoint, which proposes to free the cat from the confines of the Gedanken world and put it to an experimental test, has become a realistic one by virtue of recent remarkable advances in experimental technologies.

The readership anticipated for this book consists of those graduate students, both experimental and theoretical, who have just finished their undergraduate courses. Assuming only a knowledge of elementary quantum mechanics, this book surveys *macroscopic quantum tunneling* and its significance. The emphasis is laid on conceptual aspects rather than on technical details. The aim of this book is not so much to elaborate on established and systematized theories as to locate *macroscopic quantum tunneling* in the broad perspective and invite students to one of the most important and fascinating footpaths of physics in the twenty-first century.

### Linguistic policy

(i) The language used is physical mathematics, that is, an intuitive mathematics and reasoning; highbrow theoretical techniques are relegated to appropriate literature.

(ii) In order to distinguish generally accepted statements from the personal opinions of the author, he adopts the following convention:

> A sentence of the type "A is called $\alpha$" is meant to be a generally accepted statement, whereas a sentence of the type "A is to be called $\alpha$" is to be understood to imply

[1] If one regarded quantum mechanics as merely the way to compute transition probabilities, there would be no logical necessity to single out quantum tunnelings, which constitute merely a class of quantum-mechanical transitions. However, it is often the case that the essence of something is recognizable only from a certain viewpoint; a problem may look different not only if the paradigm itself is changed (see e.g. T. S. Kuhn, *The Structure of Scientific Revolutions*, University of Chicago Press, 1962) but also if a viewpoint is changed while retaining the same paradigm.

that "the present author has decided to call A $\alpha$ in the absence of or regardless of common practice".[2]

(iii) For brevity and logical clarity, four kinds of equality symbols are employed; the usual symbol $=$ is to be used in an equation which follows from what precedes it, and $\equiv$ is to rephrase a mathematical symbol with words or another symbol, whereas $:=$ and $=:$ are to be used in definitions.[3] Here are three examples:

(a) The equation

$$\hat{H}_S \equiv \text{Hamiltonian for the \underline{macrosystem}} := \cdots$$

implies that "the symbol $\hat{H}_S$ denotes the Hamiltonian for the macrosystem, and this Hamiltonian is defined as . . . ".

(b) The phrase "by use of $B := S[Q]$" implies "by use of $B$, which is defined by the equation $B = S[Q]$".

(c) The equation $A =: B$ is to be understood to imply "$A$ defines $B$", that is, "$B$ is defined by the equation $B = A$".

(iv) The set consisting of all the real numbers is denoted by **R**, and those of the natural and the complex numbers by **N** and **C**, respectively.[4] For example, the sentence "$x$ is a real number" is sometimes abbreviated as $x \in \mathbf{R}$.

(v) In some cases the absence of appropriate or concise technical terms has required the author to introduce new words, which are underlined. The underlining is to warn the reader that they are not, at least as yet, commonly used nor authorized in academic circles. Here are two examples of such underlined words:

(a) S-Cat is to be used as the abbreviation for Schrödinger's cat.

(b) The sentence "this potential is to be called the bumpy slope" is to be understood as "in the absence of an appropriate authorized word, the present author calls this potential the bumpy slope".

Whenever a strange phrase is encountered, the reader is advised to regard it as a kind of (mathematical) symbol; after all, words are nothing but symbols.

(vi) More than half a century ago, Casimir advocated the use of Broken English by the world scientific community.[5] The present author follows Casimir's spirit to write this book in Beinglish, which, being a revised version of the Broken English, is a common tool of communication among global scientific beings; its grammar, which vaguely resembles that of English, is "customizable" and may be improvised freely.[6]

[2] The sentence of the latter type, therefore, is liable to contain errors and may not necessarily be generally accepted.

[3] Of course, the boundary between rephrasing and defining is somewhat fuzzy.

[4] We follow the modern practice to include 0 in **N**.

[5] H. B. G. Casimir, *Broken English* in *J. Jocular Phys.* **III** (1955) 14 [Reprinted in R. L. Weber, *More Random Walks in Science* (The Institute of Physics, 1982), p.2.]. JJP were irregularly published by the Niels Bohr Institute. The present author is grateful to Professor Chris Pethick for a copy of the collection volume *FAUST and JOURNAL of JOCULAR PHYSICS Volumes I, II and III Reprinted on the Occasion of NIELS BOHR'S CENTENARY October 7, 1985.*

[6] It is unfortunate that spoken Beinglish cannot be reproduced here; it may or may not at all sound like English depending on the speaker.

# Acknowledgements

This book is based on lectures given by the author in several graduate courses in Japan, especially at Tohoku University where he taught a regular course. He is grateful to the staff and students of the following universities for inviting him as a lecturer of short-term intensive courses and making valuable comments: Ochanomizu University, Kyoto University, University of Tsukuba, Osaka City University and Nagoya University. Many thanks are due to the former graduate students of Tohoku University: Norifumi Yamada, Kaoru Hiyama, Taichi Hiraoka, Shigenobu Suzuki, Takashi Nakamura, Yusuke Kanno, Takashi Isozaki and Yoshihiro Yamamoto, from whom the author learned much while acting as their thesis supervisor; many of the results of collaboration with these students have been incorporated into this book without explicitly giving them credit. Valuable comments by yet another former graduate student Kousuke Shizume on the Japanese edition are much appreciated; they were of great help to the author in revising some of the subtle points overlooked therein. It should be mentioned that this book would not have been written if the author had not met Professor Tony Leggett more than 25 years ago. During this quarter of a century the author has learned physics in its essence from Tony, both directly through conversation and collaboration on specific problems and indirectly through reading his writings. Tony also kindly answered many of the author's elementary questions on macroscopic quantum tunneling on the commencement of the writing of the original Japanese edition of this book. For all this, the author cannot thank Tony too much. Finally, thanks are due also to Professors Yosuke Nagaoka and Keiji Kikkawa for providing the author with an opportunity of writing the Japanese edition of this book, to Dr. Nobuaki Miyabe of Iwanami Publishers and Dr. Simon Capelin of Cambridge University Press for their effort in publishing Japanese and English editions, to John B. Laing, a non-physicist colleague of the author, for his generous and time-consuming help of correcting and improving the author's Beinglish, and to poet Inge and relativist Werner for an amusing correspondence about S-Cat, namely the Fitzgerald (not FitzGerald) contracted Schrödinger cat.

# Contents

# 1

# Introduction

We begin by surveying what *macroscopic quantum phenomena* are, and what is the significance of searching for such phenomena, thereby locating *macroscopic quantum tunneling* in the broad perspective of physics in the new century.

## 1.1 The cat and the moon

It should not be necessary to elaborate on a Young-type interference experiment, which has by now been realized not only with electrons or neutrons but also with atoms such as He, Ne and Na. In a typical experiment, a particle of a given kinetic energy is sent through a double slit to a planar array of particle counters. What happens is that one and only one of the counters fires and is marked by a bright spot. As many particles of the same kind and the same kinetic energy as the first particle are sent one by one successively, bright spots accumulate and eventually emerge as an almost smooth interference pattern. This impressive emergence of the pattern may best be appreciated by watching a movie that records such an experiment in real time. In view of recent remarkable advances in experimental technology, one cannot but be curious about the prospect in the not-unforeseeable future: can the Young-type experiment be realized with an even bigger object, and how big an object can one deal with? Here is a dialogue between Weizsäcker[1] and Glauber[2] at a meeting on quantum mechanics in the early 1990s.

W: However far the technology should advance, one would not be able to see an interference pattern with tennis balls.
G: It might be possible with soccer balls, though.[3]

[1] C. F. von Weizsäcker is a theoretician known for his contribution to nuclear physics, etc. He is a brother of the former president of Germany.
[2] R. J. Glauber is a theoretician known for his contribution to quantum optics, etc.
[3] The molecule $C_{60}$ consisting of 60 carbon atoms is often called a soccer ball because of its shape. **Note added in the English edition**: In 1999, only 2 years after the Japanese edition was published, an interference pattern was observed successfully with $C_{60}$ (see Ref. [10] in the bibliography).

The fundamental equation of quantum mechanics[4] is the Schrödinger equation[5] which describes the time evolution of a given system. Its most important property is linearity. Let $|\Psi(t)\rangle$ be the state of the system at time $t$. (Hereafter a *state* is to be understood as a *quantum state* unless otherwise mentioned.) The Schrödinger equation may be written generally in the following (integrated) form:

$$|\Psi(t)\rangle = \hat{U}(t)|\Psi(0)\rangle. \tag{1.1.1}$$

The symbol $\hat{U}(t)$ denotes a unitary operator determined by the Hamiltonian $\hat{H}$ of the system. For the present purpose, it is sufficient to regard $\hat{U}(t)$ as a sort of a linear black box; given a state at time 0, (i) the state at an arbitrary time $t$ is determined uniquely by the above equation, and (ii) the following equality holds for arbitrary $|\Psi_1\rangle$, $|\Psi_2\rangle$ and $t$:

$$\hat{U}(t)(|\Psi_1\rangle + |\Psi_2\rangle) = \hat{U}(t)|\Psi_1\rangle + \hat{U}(t)|\Psi_2\rangle. \tag{1.1.2}$$

This relationship embodies the above-mentioned linearity, which is supported by interference effects demonstrated in various experiments, especially ones of the Young-type.

In the microscopic world,[6] a variety of superpositions (namely, linear combinations) of states have been confirmed experimentally. One of the familiar examples is a superposition of spin-up and spin-down states. By virtue of the linearity (1.1.2), a superposition in the microscopic world can in principle be magnified to one in the macroscopic world. A mechanism of such a magnification is Schrödinger's linear theater, of which a simplified version would run as follows. On the stage is a box containing a cat. The play is so designed that a radioactive nucleus is thrown into the box and is swallowed by the cat at time 0, and that there are two possible states $|\phi_\pm\rangle$ for the nucleus at time 0 such that the nucleus has not yet decayed if it is in $|\phi_+\rangle$, but has decayed already if in $|\phi_-\rangle$. The cat will remain intact if the nucleus is in $|\phi_+\rangle$ at time 0, but will eventually die due to radiation hazard if the nucleus is in $|\phi_-\rangle$ at time 0. Let $|\psi\rangle$ be the state of the cat at time 0, and $|\Psi_\pm\rangle$ be the state describing the cat either remaining intact (subscript +) or being dead (subscript −), at some time $T$, after swallowing the nucleus which was in $|\phi_\pm\rangle$. Note that $|\Psi_\pm\rangle$ are states of the entire system composed of the cat and the nucleus:

$$|\Psi_\pm\rangle := \hat{U}(T)|\psi, \phi_\pm\rangle, \quad |\psi, \phi_\pm\rangle \equiv |\psi\rangle|\phi_\pm\rangle. \tag{1.1.3}$$

The curtain is to be closed at the time $T$. So much for the setting of the stage. Now, immediately before the curtain is opened at time 0, the nucleus is to be prepared

[4] Quantum mechanics here includes quantum field theory as well.

[5] Some of the readers might associate the Schrödinger equation with the equation "$\hat{H}|\Psi\rangle = E|\Psi\rangle$". The latter, however, being a special case of the former, is appropriate only if "$|\Psi\rangle$ is a stationary state".

[6] The word "world" here is meant to represent vaguely the whole collection of various physical phenomena.

in neither of the two states $|\phi_\pm\rangle$, but in the superposition $|\phi_+\rangle + |\phi_-\rangle$.[7] Hence, the initial state of the entire system is of the following form:

$$|\Psi(0)\rangle = |\psi\rangle(|\phi_+\rangle + |\phi_-\rangle) = |\psi, \phi_+\rangle + |\psi, \phi_-\rangle. \qquad (1.1.4)$$

Combining this with (1.1.1), (1.1.2) and (1.1.3), one finds the state at the time $T$:

$$|\Psi(T)\rangle = \hat{U}(T)(|\psi, \phi_+\rangle + |\psi, \phi_-\rangle) = |\Psi_+\rangle + |\Psi_-\rangle. \qquad (1.1.5)$$

This equation shows that the superposition in the microscopic world $(|\phi_+\rangle + |\phi_-\rangle)$ can be magnified to that in the macroscopic world $(|\Psi_+\rangle + |\Psi_-\rangle)$:

MMM: Magnification {microscopic $\longrightarrow$ macroscopic}

Given the state $|\Psi(T)\rangle$, which is a superposition of $|\Psi_+\rangle$ and $|\Psi_-\rangle$, the cat can neither be said to be alive ($|\Psi_+\rangle$) nor dead ($|\Psi_-\rangle$); it may only be said to be in the state of $|\Psi_+\rangle$ AND $|\Psi_-\rangle$.

In view of the fact that superposition of distinct states (e.g. $|\phi_\pm\rangle$ in the above example) in the microscopic world has been confirmed experimentally, the appearance of *macroscopic superposition* (or, equivalently, *macroscopic linear combination*) such as (1.1.5) cannot be avoided so long as linearity of the Schrödinger equation is taken for granted. (Here, "macroscopic superposition" is meant to imply "superposition of *macroscopically distinct* states".) However, this sort of strange state is incompatible with the *macrorealism*,[8] according to which the cat, exposed to the radiation, must necessarily be either in the state $|\Psi_+\rangle$ or in $|\Psi_-\rangle$ ($|\Psi_+\rangle$ OR $|\Psi_-\rangle$); a cat in the state $|\Psi_+\rangle$ AND $|\Psi_-\rangle$ is totally incomprehensible.[9]

During one of his walks, Einstein is said to have asked his colleague[10] "Do you really believe that the moon is there only when you look at it?" What lies at the core of the discussion is the problem of the transition[11] from AND of quantum mechanics to OR of macrorealism,

TAO: Transition {AND $\longrightarrow$ OR}.

[7] A radioactive nucleus evolves into a state of this type even if it was originally in $|\phi_+\rangle$.

[8] Loosely speaking, a naive everyday-life realism. See Chapter 9 for details.

[9] According to Schrödinger as translated into English, it is 'ridiculous'; he ridiculed it by creating his linear theater but without forgetting to mention '... the living and the dead cat (pardon the expression)...'. Would Schrödinger have said 'Scat!' to the S-Cat (≡ Schrödinger's cat)? Note that the audience are not allowed to enter the theater before the closing time $T$! They are invited to examine the cat only somewhat later. Then, some of them will find the cat alive and the others will find it dead.

[10] A. Pais, *Rev. Mod. Phys.* **51** (1979), 863–914, Section X. Although this question may not have addressed specifically to the problem concerning the macroscopic world, it is undoubtedly an eloquent representative of the macrorealism.

[11] Also called Collapse, Objectification or REalisation (put together as CORE).

If a measuring apparatus replaces the cat, this problem reduces to the "problem of measurement in quantum mechanics", which has been debated since the birth of quantum mechanics.

It should be noted that the state (1.1.5) is not a simple product of a state of the nucleus and a state of the cat but a sum (linear combination) of such products. In this state, the nucleus and the cat cannot be separated; rather they are as it were inseparably entangled. In general such a state is called an *entangled state* (see Chapter 4 for details).

## 1.2 Leggett program

Let us take a look at the "quantum measurement problem" or the "*S-Cat* (Schrödinger's cat) paradox" from a laboratory-rooted point of view. This paradox presupposes the universal validity of quantum mechanics even in the macroscopic world. This premise, however, lacks in experimental evidence. If it were not valid, the paradox would either disappear or change its character. If, on the other hand, the premise is valid and a macroscopic superposition is realized, one should expect *QIMDS*,[12] namely, *quantum interference of macroscopically distinct states*. The question then is this: how macroscopic can an object be for a laboratory experiment to be able to detect QIMDS, thereby confirming a macroscopic superposition with the object? Note here a traditional opinion against QIMDS:

> Even if quantum mechanics was valid in the macroscopic world, it would be impossible in practice to detect QIMDS.

The argument runs as follows:[13]

> A macroscopic system has a large number of degrees of freedom. Accordingly, QIMDS must result as a sum of a large number of interference effects. Even if each of the effects separately produces such a clear-cut interference pattern as that in the Young-type experiment, the net result of summing these patterns would be the disappearance of any interference effect, because in general they are slightly out of phase with each other; that is, the peaks in one of the patterns are slightly displaced compared to those in another. In the example of Schrödinger's linear theater, the entire system in fact consists of the nucleus, the cat and the whole environment surrounding them, although the environment was disregarded for brevity in the preceding section. Thus, the number of degrees of freedom is infinite in effect, and interference will completely disappear.

[12] An acronym invented by A. J. Leggett.

[13] This argument is often followed by the statement "Therefore, AND is synonymous with OR for all practical purposes". As emphasized in the ensuing paragraph, however, this sort of *for all practical purposes argument* does not resolve the TAO problem.

The disappearance of interference just argued is called the *decoherence* due to the *environment*.[14] This argument will certainly apply to a majority of situations including a real cat. However, since there is no definite boundary between the microscopic and the macroscopic worlds, possibilities should remain for QIMDS to be detectable with a fairly macroscopic object, so long as quantum mechanics is valid. Even if the number of degrees of freedom is formally infinite, it might not be impossible to reduce the number of those which are harmful to QIMDS; an appropriate control (e.g. cooling to a sufficiently low temperature) over the environment could achieve the desired reduction.

These considerations led Leggett to propose the following program around 1980 (in what follows, "QM ≡ quantum mechanics" and "MR ≡ macrorealism"):

- (0) Search both experimentally and theoretically for a macroscopic system which is expected, provided that QM remains valid, to show evidence of QIMDS under an appropriately controlled environment.
- (1-0) If the experimental result can be interpreted, on the basis of QM, to show evidence of QIMDS, proceed to the step (2) below.
- (1-1) If the experimental result unambiguously denies QIMDS against the quantum-mechanical prediction, then QM may be concluded to be invalid for a system as macroscopic as the one in question. Proceed to modify QM in the light of the negative result.
- (2) Scrutinize, without invoking QM, whether or not the experimental result is compatible with MR.
- (2-0) If it is, the experiment in question can not decide between QM and MR. Go back to the step (0) and refine the experiment.
- (2-1) If it is not, one may conclude that MR is not valid but QM remains to be valid for a system as macroscopic as the one in question. Go back to the step (0) to continue the search for still more macroscopic candidates of QIMDS.

A comment is in order on the step (2). However uncomfortable one might feel with macroscopic superposition predicted by QM, it is illegitimate to reject QM on the basis of one's subjective feeling. A way to quantify this discomfort is to adopt MR, on the basis of which one may derive certain inequalities (Leggett–Garg inequalities[15]) to be satisfied by some measurable quantities (time-correlation functions). Furthermore, it can be shown that there are circumstances where QM

[14] The environment here includes many internal degrees of freedom of the macroscopic system in question (e.g. the cat) apart from those which are reserved to distinguish the macroscopically distinct states (e.g. to distinguish whether the cat is alive or dead). Of course, interference between microscopically different states can also be affected and often washed out by environment. The thesis of the above traditional argument is that the decoherence is inevitable and fatal in the case of QIMDS.

[15] They correspond and have the same mathematical structure as Bell's inequalities which are appropriate in the Einstein–Podolsky–Rosen problem, namely, testing QM against the local realism.

violates these inequalities. A sufficiently skilful experiment should be able to reveal this discrepancy, thereby deciding between QM and MR. See Chapter 9 for details.

Hidden in this program is the expectation that quantum mechanics will cease to be valid for a sufficiently macroscopic system.[16] The program itself, however, is independent both of this expectation and of a belief that QM is absolutely valid; it is a down-to-earth research program to enlarge the range of applicability of QM step-by-step from the microscopic world to the more macroscopic one. This program is to be called the *Leggett program*.[17]

## 1.3 What is meant by "macroscopic"?

### *1.3.1 Intuitive consideration*

We have frequently used the word *macroscopic*. In order to avoid confusion, it is necessary to agree on its meaning as it is used in this book. As a starting point let us consider a Young-type experiment, where a pair of distinct states is involved; they represent a particle passing through either the upper or the lower slit. If the distance between the two slits is macroscopic (say 0.1 mm), one might be inclined to regard the pair of states as macroscopically distinct from each other, even if the system in question is a microscopic particle such as an electron or a neutron. What is macroscopic here, however, is a mere distance; the number of particles involved is only one. By contrast, the word "macroscopic" in this book refers to those situations where a large number of particles are involved, or to be more precise, the number $N$ of the *dynamical degrees of freedom* is large.

The phrase "dynamical degrees of freedom" (hereafter to be abbreviated as degrees of freedom) should also be used with caution. Imagine a tennis ball passing through a wall without being squeezed. There can be no objection to calling this phenomenon a macroscopic tunneling; if the ball is regarded as a collection of atoms, the number of degrees of freedom involved in this phenomenon is comparable to the number of atoms. However, the same phenomenon can also be described by a single degree of freedom, namely, the center of mass. These two descriptions are related to each other by a transformation of variables and are mathematically equivalent. On the basis of this example, it could be argued that the number of degrees of freedom involved, which is not invariant under transformations of variables, cannot quantify the word "macroscopic"; a physical conclusion should not depend on the choice

[16] See A. J. Leggett, *The Problems of Physics*, Oxford University Press (1987), Chapter 5, Skeletons in the cupboard.

[17] The program announced by Felix Klein (1849–1925) on the occasion of his appointment to a professorship at the University of Erlangen, well known as Erlangen Programm, was so insightful that it played a long-lasting leading role in the synthesis of geometry. The Leggett program, which is still under development, will be regarded by the late twenty-first century physicists to have played a role in physics comparable to the Klein program in twentieth century mathematics.

of a mathematical description. This objection, which might look reasonable at first sight, may be disposed of as follows. Our intuition which regards the above phenomenon as a macroscopic tunneling does not rely on the number of *collective degrees of freedom* such as the center of mass but on the number of *microscopic degrees of freedom* such as the positions of constituent atoms. Collective degrees of freedom are the elite degrees of freedom which are singled out by rearranging the microscopic ones, all of which our intuition leads us to treat on an equal footing. In accordance to our intuition, we adopt the democratic way of counting the number of degrees of freedom, which in general is of the same order as the number of particles composing the system in question. Thus, the above objection is irrelevant. Of course, the number of constituent particles depends on what we count as fundamental particles; $N$ neutrons may be counted as $3N$ quarks, for instance. However, the difference between $N$ and $3N$ is irrelevant as well; the word "macroscopic" may be quantified only by orders of magnitude.

### *1.3.2 S-Cattiness*

Until the 1970s, the phrase macroscopic quantum phenomena represented collectively superfluidity and superconductivity. For example, the phenomenon of liquid He creeping up along the wall of a glass and flowing out of it is both undoubtedly macroscopic and explicable only in terms of quantum mechanics. In this kind of phenomena, however, microscopic interference at the level of one (or two) particles is enhanced by virtue of cooperation of many particles (or, many pairs of particles), resulting in an effect of macroscopic scale; QIMDS is not involved. Today they are often called *macroscopic quantum phenomena of the first kind* and are distinguished from *macroscopic quantum phenomena of the second kind* in which QIMDS is involved.

Let us elaborate on the difference between the two kinds. Consider, as a typical example of the first kind, superfluid $^4$He flowing out of a glass. The wavefunction representing its state is conceptually of the form:[18]

$$\prod_{k=1}^{N}\{\psi(\mathbf{r}_k - \mathbf{d}/2) + \psi(\mathbf{r}_k + \mathbf{d}/2)\}, \qquad (1.3.1)$$

where $\mathbf{r}_k$ denotes the position of the $k$-th of the $N$ atoms composing the liquid $^4$He, $\mathbf{d}/2$ that of the center of the glass, and $-\mathbf{d}/2$ a position outside the glass (Fig. 1.1). Equation (1.3.1) implies that each of the atoms is in a state of superposition $\psi(\mathbf{r} - \mathbf{d}/2) + \psi(\mathbf{r} + \mathbf{d}/2)$ and that the state of the entire liquid is their product.

[18] It should be noted that the following expression is merely schematic. Bose–Einstein condensation by itself is not enough to give rise to superfluidity; interaction among atoms is necessary for a state roughly of the following form to be kept stable.