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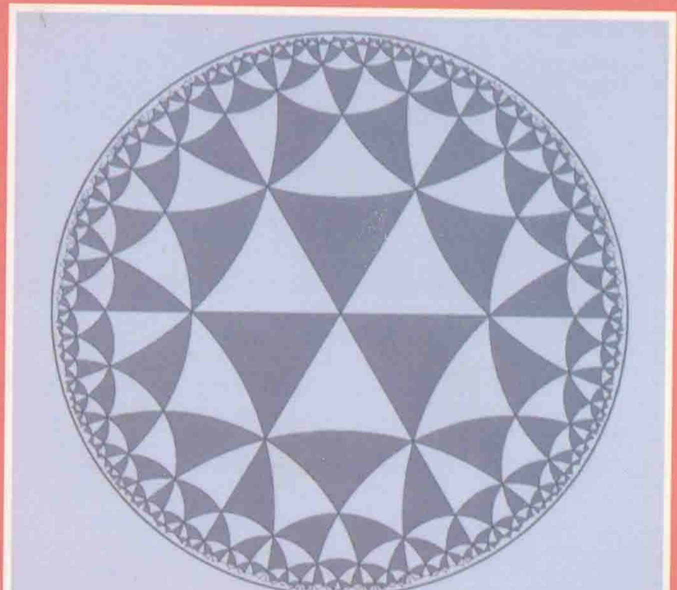
COMPLEX VARIABLES

Introduction and Applications

SECOND EDITION

复变量 第2版

MARK J. ABLOWITZ
ATHANASSIOS S. FOKAS



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Complex Variables
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The study of complex variables is important for students in engineering and the physical sciences and is a central subject in mathematics. In addition to being mathematically elegant, complex variables provide a powerful tool for solving problems that are either very difficult or virtually impossible to solve in any other way.

Part I of this text provides an introduction to the subject, including analytic functions, integration, series, and residue calculus. It also includes transform methods, ordinary differential equations in the complex plane, numerical methods, and more. Part II contains conformal mappings, asymptotic expansions, and the study of Riemann–Hilbert problems. The authors also provide an extensive array of applications, illustrative examples, and homework exercises.

This new edition has been improved throughout and is ideal for use in introductory undergraduate and graduate level courses in complex variables.

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Preface

The study of complex variables is beautiful from a purely mathematical point of view and provides a powerful tool for solving a wide array of problems arising in applications. It is perhaps surprising that to explain real phenomena, mathematicians, scientists, and engineers often resort to the "complex plane." In fact, using complex variables one can solve many problems that are either very difficult or virtually impossible to solve by other means. The text provides a broad treatment of both the fundamentals and the applications of this subject.

This text can be used in an introductory one- or two-semester undergraduate course. Alternatively, it can be used in a beginning graduate level course and as a reference. Indeed, Part I provides an introduction to the study of complex variables. It also contains a number of applications, which include evaluation of integrals, methods of solution to certain ordinary and partial differential equations, and the study of ideal fluid flow. In addition, Part I develops a suitable foundation for the more advanced material in Part II. Part II contains the study of conformal mappings, asymptotic evaluation of integrals, the so-called Riemann–Hilbert and DBAR problems, and many of their applications. In fact, applications are discussed throughout the book. Our point of view is that students are motivated and enjoy learning the material when they can relate it to applications.

To aid the instructor, we have denoted with an asterisk certain sections that are more advanced. These sections can be read independently or can be skipped. We also note that each of the chapters in Part II can be read independently. Every effort has been made to make this book self-contained. Thus advanced students using this text will have the basic material at their disposal without dependence on other references.

We realize that many of the topics presented in this book are not usually covered in complex variables texts. This includes the study of ordinary differential equations in the complex plane, the solution of linear partial differential equations by integral transforms, asymptotic evaluation of integrals, and Riemann–Hilbert problems. Actually some of these topics, when studied at all, are only included in advanced graduate level courses. However, we believe that these topics arise so frequently in applications that early exposure is vital. It is fortunate that it is indeed possible to present this material in such a way that it can be understood with only the foundation presented in the introductory chapters of this book.

We are indebted to our families, who have endured all too many hours of our absence. We are thankful to B. Fast and C. Smith for an outstanding job of word processing the manuscript and to B. Fast, who has so capably used mathematical software to verify many formulae and produce figures.

Several colleagues helped us with the preparation of this book. B. Herbst made many suggestions and was instrumental in the development of the computational section. C. Schober, L. Luo, and L. Glasser worked with us on many of the exercises. J. Meiss and C. Schober taught from early versions of the manuscript and made valuable suggestions.

David Benney encouraged us to write this book and we extend our deep appreciation to him. We would like to take this opportunity to thank those agencies who have, over the years, consistently supported our research efforts. Actually, this research led us to several of the applications presented in this book. We thank the Air Force Office of Scientific Research, the National Science Foundation, and the Office of Naval Research. In particular we thank Arje Nachman, Program Director, Air Force Office of Scientific Research (AFOSR), for his continual support.

Since the first edition appeared we are pleased with the many positive and useful comments made to us by colleagues and students. All necessary changes, small additions, and modifications have been made in this second edition. Additional information can be found from www.cup.org/titles/catalogue.

Contents

Sections denoted with an asterisk (*) can be either omitted or read independently.

Part I

Fundamentals and Techniques of Complex Function Theory

The first portion of this text aims to introduce the reader to the basic notions and methods in complex analysis. The standard properties of real numbers and the calculus of real variables are assumed. When necessary, a rigorous axiomatic development will be sacrificed in place of a logical development based upon suitable assumptions. This will allow us to concentrate more on examples and applications that our experience has demonstrated to be useful for the student first introduced to the subject. However, the important theorems are stated and proved.

1

Complex Numbers and Elementary Functions

This chapter introduces complex numbers, elementary complex functions, and their basic properties. It will be seen that complex numbers have a simple two-dimensional character that submits to a straightforward geometric description. While many results of real variable calculus carry over, some very important novel and useful notions appear in the calculus of complex functions. Applications to differential equations are briefly discussed in this chapter.

1.1 Complex Numbers and Their Properties

In this text we use Euler's notation for the imaginary unit number:

$$i^2 = -1 \tag{1.1.1}$$

A complex number is an expression of the form

$$z = x + iy \tag{1.1.2}$$

Here x is the real part of z, $\text{Re}(z)$; and y is the imaginary part of z, $\text{Im}(z)$. If $y = 0$, we say that z is real; and if $x = 0$, we say that z is pure imaginary. We often denote z, an element of the complex numbers as $z \in \mathbb{C}$; where x, an element of the real numbers is denoted by $x \in \mathbb{R}$. Geometrically, we represent Eq. (1.1.2) in a two-dimensional coordinate system called the **complex plane** (see Figure 1.1.1).

The real numbers lie on the horizontal axis and pure imaginary numbers on the vertical axis. The analogy with two-dimensional vectors is immediate. A complex number $z = x + iy$ can be interpreted as a two-dimensional vector (x, y).

It is useful to introduce another representation of complex numbers, namely polar coordinates (r, θ):

$$x = r\cos\theta \qquad y = r\sin\theta \qquad (r \geq 0) \tag{1.1.3}$$

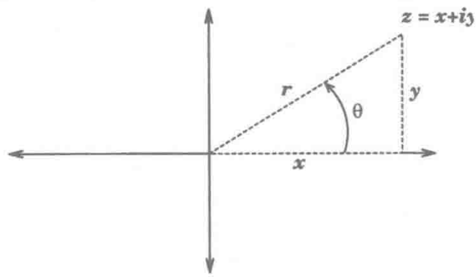

Fig. 1.1.1. The complex plane ("z plane")

Hence the complex number z can be written in the alternative polar form:

$$z = x + iy = r(\cos\theta + i\sin\theta) \tag{1.1.4}$$

The radius r is denoted by

$$r = \sqrt{x^2 + y^2} \equiv |z| \tag{1.1.5a}$$

(note: $\equiv$ denotes equivalence) and naturally gives us a notion of the **absolute value** of z, denoted by $|z|$, that is, it is the length of the vector associated with z. The value $|z|$ is often referred to as the **modulus** of z. The angle θ is called the argument of z and is denoted by $\arg z$. When $z \neq 0$, the values of θ can be found from Eq. (1.1.3) via standard trigonometry:

$$\tan\theta = y/x \tag{1.1.5b}$$

where the quadrant in which x, y lie is understood as given. We note that $\theta \equiv \arg z$ is **multivalued** because $\tan\theta$ is a periodic function of θ with period π. Given $z = x + iy$, $z \neq 0$ we identify θ to have one value in the interval $\theta_0 \leq \theta < \theta_0 + 2\pi$, where θ_0 is an arbitrary number; others differ by integer multiples of 2π. We shall take $\theta_0 = 0$. For example, if $z = -1 + i$, then $|z| = r = \sqrt{2}$ and $\theta = \frac{3\pi}{4} + 2n\pi$, $n = 0, \pm1, \pm2, \ldots$. The previous remarks apply equally well if we use the polar representation about a point $z_0 \neq 0$. This just means that we translate the origin from $z = 0$ to $z = z_0$.

At this point it is convenient to introduce a special exponential function. The polar exponential is defined by

$$\cos\theta + i\sin\theta = e^{i\theta} \tag{1.1.6}$$

Hence Eq. (1.1.4) implies that z can be written in the form

$$z = re^{i\theta} \tag{1.1.4$'$}$$

This exponential function has all of the standard properties we are familiar with in elementary calculus and is a special case of the complex exponential