GENERAL TOPOLOGY

ÁKOS CSÁSZÁR

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Translated by
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PREFACE TO THE ENGLISH EDITION

The literature of general topology is enriched each year by several hundreds of papers containing new scientific results and besides these, by some new books or new editions of older books. Some of them deal with a specific domain of this discipline and therefore need certain preliminaries of topology, but the majority treat the subject starting from basic principles.

This phenomenon is explained by the fact that one of the characteristic features of modern mathematics is the penetration of topological methods into many chapters of mathematics, first of all into analysis and geometry. Hence it is desirable that students interested in general topology can embark on this subject in a rather early phase of their studies. From a purely logical point of view it would be possible to present general topology in a form disregarding any previous knowledge in mathematics, by beginning, after the enumeration of the background from set theory, with the basic definitions and axioms. However, I am convinced that a treatment of this kind is an abuse of the possibilities of the axiomatic method which can produce a formal understanding but cannot furnish a real insight into the importance of the concepts. Indeed, there is no doubt that the abstract theories characterizing modern mathematics are important only because their general concepts include a series of interesting special cases; their general statements, applied to concrete examples, imply interesting conclusions, and I am firmly convinced that the presentation of such a theory is useful only if it includes the necessary preparation for the procedure of abstraction leading to the definitions, the explanation of the special cases contained in them, and the applications resulting from the theorems. However, in order to reach the concepts of general topology in a natural way, starting from concrete notions, and to illustrate the definitions by non-trivial examples, one needs some background in analysis; the larger this background, the richer is the illustrative material available in presenting the theory, but of course then the possibility of getting acquainted with topology is transferred to a later phase of studies in mathematics.

After considering all this, I decided to postulate for the reader a knowledge corresponding to the material of a first year undergraduate course in analysis. Instead of presuming "nothing else than ability in abstract thinking", I assume, to the extent indicated above, acquaintance with elementary analysis. On the other hand, background from analysis, algebra, or set theory, not familiar to a student having finished the first year at university, is not used without a detailed explanation at the corresponding place in the text.

This limitation of mathematical background implies that I do not use the concept of the cardinality of a set – except the distinction of finite, countable, and uncountable sets – as well as the theory of infinite cardinal and ordinal numbers. The Kuratowski–Zorn lemma and the Zermelo theorem, stated without proof, give however the possibility of presenting every essential concept and result of general topology, except, of course, the concepts based on the notion of cardinality (e.g. the weight of a topological space) and some more intricate counter-examples. An expert reader can judge whether this avoidance of deeper methods of set theory essentially truncates the material or not.

According to the introductory character of the book the purpose of the exercises at the end of each section is to give the reader the possibility of controlling the understanding of the material by enlightening the content of a definition or a theorem by means of special cases and counter-examples. I do not have the illusion (in contrast to the authors of many similar books) that the reader will be able to discover alone each further result of general topology; accordingly every exercise (except those which are straightforward) is divided into smaller units and often the essential thoughts necessary to the solution are given in brackets.

Another aid to the beginner is the clear presentation of the logical structure of longer proofs. This is done by means of the symbols $\Rightarrow$ and $\Leftrightarrow$; the latter is expressed in the text by the usual abbreviation "iff". The symbol ▌ shows the end of a proof; if it is put at the end of a statement, then the latter is obvious in view of the corresponding definitions or the preceding statements.

According to the character of the book, references to the literature are omitted. Instead there is a list of textbooks and monographs on general topology, rather large without being complete. It does not contain works in languages other than English, French, or German; new editions are mentioned only if their language differs from that of the first one, or if they were essentially enlarged. The works contained in this list give a rich possibility of getting deeper knowledge in general topology; a part of them contains a large list of papers on the subject.

The introductory character of the book was emphasized above several times. In apparent contradiction to this, I should like to point out that I tried to present to the reader the up-to-date features of general topology and to give a survey on some fairly recent theories. In particular, I present some types of topological structures other than topologies in some extent and I point out their role in the study of various questions of general topology.

The first Hungarian edition of this book (Bevezetés az általános topológiába) appeared in 1970. The English edition was enlarged by the treatment of some new subjects, especially in Chapters 4, 8 and 9, and many new exercises were added.

It is my duty to express my sincerest gratitude to my colleagues M. Bognár, S. Gacsályi, J. Gerlits, P. Hamburger, I. Juhász, for their aid in preparing the manuscript. Some proofs and a series of exercises are based on their ideas.

Á. Császár

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