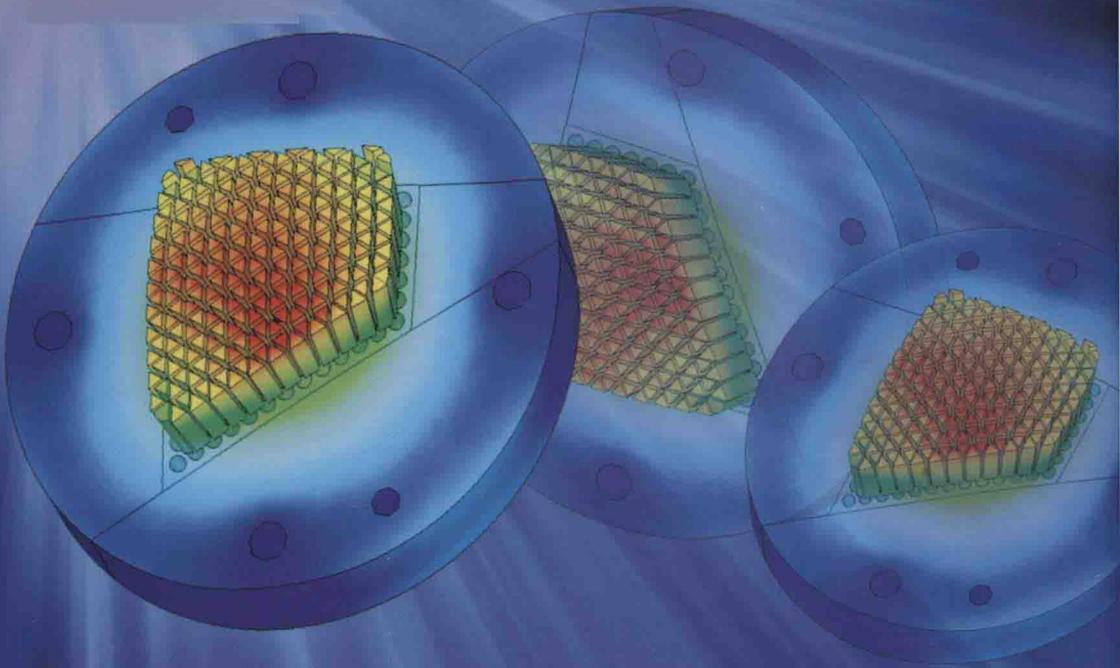


Second Edition



MODELING AND SIMULATION

Pratiksha Saxena



Alpha Science

Modeling and Simulation

Second Edition

Pratiksha Saxena



Alpha Science International Ltd.
Oxford, U.K.

Modeling and Simulation

Second Edition

398 pgs. | 113 figs. | 17 tbls.

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Second Edition 2014

ALPHA SCIENCE INTERNATIONAL LTD.

7200 The Quorum, Oxford Business Park North

Garsington Road, Oxford OX4 2JZ, U.K.

www.alphasci.com

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ISBN 978-1-84265-930-4

Printed in India

Dedicated to
My Dear Husband
'Dr. Mohit Saxena'

Preface to the Second Edition

This book introduces you to model of a system and various simulation techniques. Designed for engineering and computer students as well as operation research specialists and system analysts, it describes different simulation techniques for operation research related concepts, like models for queuing system, inventory system and PERT networks. Followed by system dynamic analysis and different types of simulation languages.

The simulation and modeling technology has major applications in virtual prototyping. It brings about a revolution in complex applications, involving parts from several application domains, and many more useful facilities. This book is useful as an introductory course on modeling and simulation. It is easily accessible to people, who do not previously have a background in modeling and simulation. The book contains many examples of models in different application domains, as well as examples combining several domains.

Chapter 1 gives introduction to system, model and simulation. It introduces the nature of physical and natural systems to facilitate a deeper understanding of real-world systems. It also describes different activities and models.

Chapter 2 addresses the issues of why systems need to be simulated, what to simulate, and how to simulate. This chapter describes the essential nature of simulation, the role of modeling, and the fundamental limits of modeling and simulation. It also addresses applications of simulation in different areas. It presents different kinds of simulation like real time simulation, hybrid simulation and Monte Carlo simulation techniques with examples.

Chapter 3 addresses random variables and features of probability distributions. This chapter describes basic definitions, probability, different kind of approaches to calculate probability and characteristics of probability distributions.

Chapter 4 describes a number of special probability distributions as Normal, Binormal, Poisson and many other distributions. It also describes their characteristics and applications.

Chapter 5 details the basic techniques in continuous simulation which is further organized for different kind of continuous models. It combines good examples of continuous model with full description of system and its models. This chapter traces the evolution of simulation techniques, analyzes the principal causes underlying the developments, and presents a comparative analysis of the techniques.

Chapter 6 presents techniques for discrete simulation models. This chapter recognizes models with the properties of discrete system simulation and also compares different techniques used for this purpose.

Chapter 7 introduces concept of time series. It discusses various components of time series, methods to calculate trend line and their application. With the help of trend line, decisions can be forecasted on the basis of past data.

Chapter 8 discusses statistical inference, with the help of which decisions for whole of the population is taken on the basis of sample data. Technique for test of hypothesis is described in this chapter. It includes detailed t-test and z-test.

Chapter 9 details chi-square test which is a non-parametric test. It describes methodology of chi-square test and its applications.

Chapter 10 details the principles underlying the execution of queuing system. Firstly, it gives basic principle for queuing theory and then it describes methods to simulate it.

Chapter 11 details the inventory theory and different types of inventory models. It also describes different terms and types associated to inventory models and defines simulation of inventory models with the help of flow charts and algorithms.

Chapter 12 details CPM and PERT networks. It describes components, techniques and applications of both kind of networks. It also computes simulation of these networks.

Chapter 13 describes system dynamics and diagram with the help of casual, reinforcing and balancing loops. It also gives different kinds of exponential models and analyzes the interdependence between the choice of input stimuli and performance data.

Chapter 14 presents different simulation languages used for different purposes.

Chapter 15 discusses verification and validation techniques for simulation model. This chapter provides classification of techniques and area of application.

Chapter 16 describes few real world applications of simulation models.

Introduction

Simulation is the process of working on a model to develop level of understanding for interaction of system components. A model represents simplified version of system at some particular point in time or space under specified conditions according to the objective of study. Model is a mimic of system and simulation represents certain characteristics and behavior of abstract system. While taking system in consideration, due to a large number of components and their interactions, complexity comes into the picture. Dynamic complexity is also associated with systems due to cause and effect separated by time and space. Simulation process takes only those components and features of system into account which is important for objective of study. The procedures which may take long time with system converts into minute's process with the use of simulation. Simulation process is used to compress time and space in an effective manner to give better understanding of system.

Study of simulation was started in 20th century and day by day area of application is spreading. Simulation can be classified in three categories in which computer simulation has worldwide applications. First category is of physical simulation in which real objects are substituted with physical objects and then simulation runs are applied on these physical models. Second category includes interaction of human being with physical objects and is called interactive simulation. Driving and flight simulators are examples of interactive simulators where these models are used to give training for different purposes. These models respond to the action taken by human beings. A computer simulation run includes model or hypothetical part on computer to study the system. In the modern era of technology, computer simulation is applicable in a number of areas including physics, chemistry, economics, social science, engineering and networking. It is very useful as it reduces time and money involved in the process. Computer simulation is a program that simulate model of system or it may be in the form

of network of computers. This network of computers or computer program can convert the simulation run into hours which may be ongoing to a number of days with actual system.

In the past 60 years, simulation theory and application has achieved good results. Application of simulation model is increasing day by day. For instance, a number of models are investigated and solved with different features in different fields. A variety of simulation models are in use for manufacturing, telecommunication and service industry.

Simulation and modeling is an effective and powerful approach to capture and analyze complex systems. Most of the decisions are based on computer generated data derived from simulation. Simulation is best method to improve efficiency and to reduce cost. Simulation is widely used to analyze possible strategies for improvement of system performance. Simulation effectively analyzed and supported model can save money on acquisition and live testing to verify results. It provides alternative system configurations, therefore it is easy to choose best system features with the help of simulation models.

A system is represented by model and simulation is process of execution of model. Every model has entities, attributes and state which is represented by a set of input variables. For a system there may be a number of models depending on the objective of study. Only those features are included in a particular model of the system which is required for study. There are different kind of computer simulations, each having unique features and objective. Computer simulation was firstly used during Manhattan Project in World War II to model the process of nuclear detonation. In 1960s simulation was mainly used as a tool for education, training and system analysis. But revolution in computer hardware and software advances the simulation applications.

On the basis of attributes computer models can be classified as:

Discrete event simulation: In this type of simulation events are managed in time. The simulator maintains a queue of events which is sorted by simulation time; it reads the queue and triggers new events as each event is processed. Each event changes the state of system. Queuing model is based on this type of simulation. State variables change only at a countable number of points in time. Four main mechanisms have been defined to carry discrete even simulation; event-based, activity-based, process-based and three-phase approaches. Simulation languages defined for this purpose are Python, Simpy, Facsimile and JSL.

Continuous event simulation: It is concerned with models in which state variables change continuously with time. Continuous simulation uses differential equations to deal with state and output of the simulation runs. Main application is for electrical circuits, flight simulators and management simulation games. ACSL, MyM and Simgua are simulation languages used for continuous simulation.

Static and dynamic simulation: For dynamic simulation models state variables change over time where as static models is defined at a single point of

time. Monte Carlo simulation and optimization models fall in static simulation models where as discrete event and agent based models are dynamic simulation model.

Deterministic and stochastic model: When behavior of model is entirely predictable, it is said to be deterministic model and stochastic model is one whose behavior can not be entirely predictable.

Verification and validation techniques: After constructing suitable model for a system, verification and validation techniques are applied. The validation and verification of a simulation is acknowledged as an integral part of a simulation project. These techniques are very useful to check how accurate are these simulations and to what extent do they produce dependable results?

Life cycle validation and verification techniques are extremely important for the success of a simulation study. Correct model is essential for correct results. In the development of a system, it is very important to determine if the system meets specifications and if its outputs are correct. This process of verification, validation and planning starts early in the development life cycle. Development of any system is not complete without rigorous testing and verification. This planning is necessary from the beginning of the development life cycle. There are many different verification and validation techniques which are applicable at different stages of the development life cycle. The result of these techniques is formed as documentation which is used to support certification.

Various statistical methods are used for simulation of a system, few of which are defined here.

1. ANOVA

ANOVA is a statistical method to compare means of several groups which generalize the two sample t-test to more than two groups. Relation between t and ANOVA is given as $F = t^2$. Several types of ANOVA are defined according to number of treatments.

ONE WAY ANOVA is used to test the difference of mean for more than two individual groups. TWO WAY ANOVA is used when subjects undergo with different treatments repeatedly. FACTORIAL ANOVA deals with study of more than two treatment variables. It includes analysis of 2^k factorial designs. Mixed design ANOVA and Multivariate analysis of variance are other types of ANOVA techniques.

For one way ANOVA technique general hypothesis is considered as,

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 \dots \dots \dots = \mu_n$$

that is, the arithmetic mean of populations from which n samples are randomly drawn are equal to one another and

$H_1 : \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \dots \neq \mu_n$

Specimen table of this technique is defined as,

| Source of variation | Sum of square | Degree of freedom | Mean square | Variance ratio <i>F</i> |
|---------------------|---------------|-------------------|-------------|-------------------------|
| Between Samples | SSC | | MSC | MSC/MSE |
| Within Samples | SSE | | MSE | |
| Total | SST | $n - 1$ | | |

Where,

SST = Total sum of squares of variations

SSC = Sum of squares between samples

SSE = Sum of squares within samples

MSC = Mean sum of squares between samples

MSE = Mean sum of squares with in samples

This technique is extended for two way and factorial ANOVA analysis.

Least square method

This statistical method is commonly used for regression, curve fitting and estimation for trend of outcomes. In this process equations for approximating curve are fitted with raw data. It defines best fit curve having minimum deviation from all data points. If data is given for independent variable x and dependent variable y as,

$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n)$$

for the curve $y = f(x)$, and deviations from each data point is defined as,

$$d_1 = y_1 - f(x_1) \quad d_2 = y_2 - f(x_2) \quad \dots \quad d_n = y_n - f(x_n)$$

Then least square method follows the property of minimum deviation from all data points as,

$$\sum_{i=1}^n d_i^2 = \text{minimum}$$

Different normal equations are produced for different type of curves.

Poisson process

Poisson process is very useful for simulation of queuing models. In general, Poisson process can be categorized as,

- (a) Pure birth process which is based on arrival intensity
- (b) Process of individual increments
- (c) The most random process with given intensity

The number of arrivals at a finite interval of time is given by Poisson distribution,

$$P(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$$

Sequence of discrete event is generally modeled as a Poisson process with essential assumption of system.

Random number generation

Random number generator is an algorithm which generates a sequence of numbers, approximating properties of random numbers. It uses states to generate the numbers and widely used for simulation. These algorithms are classified as linear feedback shift register, Fibonacci generators, Lagged Fibonacci generators and linear congruential generators.

Optimization

Simulation techniques are used very well for most of the optimization techniques. For example simulation of inventory models, queuing models and different kind of programming models has world wide applications. Simulation models are also used to optimize complex stochastic systems. Gradient-based search methods, Stochastic approximation methods, Response search method, Heuristic search method and Programming methods are important techniques of optimization which are mainly used for simulation of system. A number of softwares have been developed to achieve this.

Application of simulation

Simulation is a powerful tool to develop effective model applicable in different fields. There are different kind of computer simulation, each has unique features and objective. Computer simulation was firstly used during Manhattan Project in World War II to model the process of nuclear detonation. In 1960s simulation was mainly used as a tool for education, training and system analysis. But revolution in computer hardware and software advances the simulation applications. Some of the applications are discussed here.

Manufacturing Industry: In recent years, there are lots of improvement in manufacturing industry by using simulation techniques and simulation softwares. It is a significant tool for improving manufacturing operations, lowering cost, improving productivity and quality, and reducing time to market new products. New technologies and standards are used to create most useful simulation models for manufacturing systems. Cadbury Adams plant uses **Visual18** simulation software to reorganize its plant layout. **Wilde FEA** is another powerful and cost-effective simulation model to work on manufacturing problems. It is used

to improve forming and machining processes and solving production problems through software and consulting services.

DYNAFORM is used to tool surface modeling, draw bead force prediction, binder wrap analysis, draw die simulation, trimming operations, spring back simulation, and multiple stage tooling simulation.

Supply chain Management: Simulation plays a key role for complete Business Process Management solution. Business process discovery, design phase, analysis and control designing use simulation as a powerful tool. In this era of globalization, supply chain management become important as well as complex in terms of efficiency, operational scope, competition and reliability. In this sense simulation model makes it easy by developing related softwares. **SIMFLEX** is a supply-chain simulation software package which is used to measure dynamic performance metrics of a proposed design. **LLamasoft** is supply chain modeling and analytics software. It is used for strategic planning and provides faster modeling of enterprise operations using multiple analysis techniques and deep operations research algorithms with rich graphical output and visualization tools.

Application in Airforce-Frasca: PC-9 FTD is an Aircraft Specific Simulation for the PC-9. It has Four-monitor Instructor Station with Graphical Instructor Station (GIST) and full instrument repeating and TruVision textured visual system. **Bell 412 Full Flight Simulator** is a full flight simulator for Frasca Bell 412 with high-end simulation capabilities. Main features are Three Channel Visual system, Six Axis Motion Platform, Digital DSP Based Sound Simulation, Weapon Simulation with Scoring and Doppler Navigation Unit.

Application in Driving: Driving Simulators are used to train drivers, education courses of driving in education institutions, business and games and entertainment. It is also being used for research purposes in the area of human factors and medical research. **BR Series of Professional Driving Simulators** is designed for home and school use and **BR series of desktop driving simulators** are designed for professional use in driver training, testing, rehabilitation, and research. A wide variety of driving scenarios is built-in to each simulator to collect and record a variety of driver performance measurements. **GEOQUAKE** is a 2D driving simulator which was released in August 2007 and afterwards different version came with add-on features. Version 1.61 was released in August 2009. **TUTOR** is a simulator for professional training of bus and truck drivers. It is developed by Lander Simulation and Training Solutions in 2004 in INTA, Spain.

There are a large number of applications in the area of simulation and simulation software, such as space simulation, simulation in hospital industry, simulation in supply chain management, inventory management and other areas. Finance, insurance and transportation are also other important areas. Simulation become very effective technique and applicable in industries, research and education.

Contents

| | |
|--|-----------------|
| <i>Preface to the Second Edition</i> | <i>vii</i> |
| <i>Introduction</i> | <i>xvii</i> |
| 1. SYSTEM MODELS | 1.1—1.13 |
| 1.1 System, Components and Model..... | 1.1 |
| 1.2 System Environment | 1.3 |
| 1.3 Type of Activities | 1.5 |
| 1.4 Continuous and Discrete System..... | 1.5 |
| 1.5 System Modeling..... | 1.7 |
| 1.6 Types of Model..... | 1.8 |
| <i>Exercise</i> | <i>1.11</i> |
| <i>References</i> | <i>1.12</i> |
| 2. SYSTEM SIMULATION | 2.1—2.20 |
| 2.1 Introduction | 2.1 |
| 2.2 The Power of Simulation..... | 2.2 |
| 2.3 Technique of Simulation..... | 2.3 |
| 2.4 Why to Simulate..... | 2.5 |
| 2.5 When to Use Simulation..... | 2.6 |
| 2.6 Advantage of Simulation..... | 2.7 |
| 2.6.1 Simulation in business..... | 2.7 |
| 2.6.2 Simulation in science and engineering..... | 2.8 |
| 2.6.3 Simulation in soft sciences..... | 2.8 |
| 2.7 Basic Nature of Simulation | 2.8 |
| 2.8 Comparison of Analytical and Simulation Method | 2.9 |
| 2.9 Real-Time Simulation | 2.10 |
| 2.10 Hybrid Simulation..... | 2.11 |
| 2.11 Monte Carlo Simulation | 2.13 |
| 2.11.1 Integration | 2.16 |
| 2.11.2 Optimization..... | 2.16 |

| | |
|---|-----------------|
| 2.11.3 Inverse problems | 2.16 |
| 2.12 Simulation of Pure Pursuit Problem | 2.17 |
| 2.13 Simulation Study | 2.19 |
| <i>Exercise</i> | 2.20 |
| <i>References</i> | 2.20 |
| 3. RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS..... | 3.1—3.30 |
| 3.1 Introduction | 3.1 |
| 3.2 Basic Definitions | 3.2 |
| 3.2.1 Random experiment | 3.2 |
| 3.2.2 Outcome | 3.2 |
| 3.2.3 Sample space | 3.2 |
| 3.2.4 Event | 3.2 |
| 3.2.5 Exhaustive events | 3.2 |
| 3.2.6 Favourable events | 3.3 |
| 3.2.7 Mutually exclusive event | 3.3 |
| 3.2.8 Equally likely events | 3.3 |
| 3.2.9 Independent events | 3.3 |
| 3.2.10 Dependent events | 3.3 |
| 3.2.11 All inclusive events | 3.3 |
| 3.2.12 Complementary events | 3.3 |
| 3.3 Probability | 3.3 |
| 3.3.1 Classical probability | 3.4 |
| 3.3.2 Statistical or empirical probability | 3.4 |
| 3.3.3 Axiomatic probability | 3.5 |
| 3.3.4 Subjective probability | 3.6 |
| 3.4 Important Theorems on Probability | 3.6 |
| 3.5 General Notations | 3.8 |
| 3.6 Conditional Probability | 3.9 |
| 3.7 Bay's Theorem | 3.9 |
| 3.8 Random Function (Variable) | 3.17 |
| 3.9 Theorem Results for Random Variables | 3.18 |
| 3.10 Distribution function | 3.18 |
| 3.11 Discrete random variable and distribution function | 3.19 |
| 3.12 Continuous random variable and distribution function | 3.21 |
| 3.13 Important measures | 3.22 |
| <i>Exercise</i> | 3.28 |
| 4. SPECIAL PROBABILITY DISTRIBUTION | 4.1—4.50 |
| 4.1 Introduction | 4.1 |
| 4.2 Important Definition | 4.1 |
| 4.2.1 Mathematical expectation | 4.1 |
| 4.2.2 Expected value of function of a random variable | 4.2 |

| | | |
|--------|---|------|
| 4.2.3 | Moment about origin..... | 4.2 |
| 4.2.4 | Mean..... | 4.2 |
| 4.2.5 | Variance..... | 4.2 |
| 4.2.6 | Covariance | 4.2 |
| 4.2.7 | Variance of a linear combination of random variables | 4.3 |
| 4.2.8 | Moment generating function | 4.3 |
| 4.2.9 | Cumulants | 4.3 |
| 4.2.10 | Characteristic function | 4.3 |
| 4.2.11 | Mean deviation..... | 4.4 |
| 4.2.12 | Coefficient of kurtosis | 4.4 |
| 4.2.13 | Coefficient of skewness..... | 4.4 |
| 4.3 | Binomial Distribution..... | 4.4 |
| 4.3.1 | Moments of binomial distribution..... | 4.5 |
| 4.3.2 | Coefficient of skewness..... | 4.6 |
| 4.3.3 | Coefficient of kurtosis | 4.6 |
| 4.3.4 | Moment generating function of binomial distribution | 4.6 |
| 4.3.5 | Moment generating function about mean..... | 4.6 |
| 4.3.6 | Characteristic function of binomial distribution..... | 4.7 |
| 4.3.7 | Cumulants of the binomial distribution..... | 4.7 |
| 4.3.8 | Additive property | 4.8 |
| 4.4 | Poisson Distribution | 4.11 |
| 4.4.1 | Moments of poisson distribution..... | 4.13 |
| 4.4.2 | Coefficient of skewness..... | 4.15 |
| 4.4.3 | Coefficient of kurtosis | 4.15 |
| 4.4.4 | Moment generating function | 4.15 |
| 4.4.5 | Characteristic function | 4.15 |
| 4.4.6 | Cumulants | 4.16 |
| 4.4.7 | Additive property | 4.16 |
| 4.5 | Normal Distribution | 4.19 |
| 4.5.1 | Mode of normal distribution | 4.23 |
| 4.5.2 | Mean of normal distribution..... | 4.24 |
| 4.5.3 | Moment generating function of normal distribution | 4.25 |
| 4.5.4 | Moments of normal distribution function..... | 4.25 |
| 4.5.5 | Cumulant general function of normal distribution | 4.27 |
| 4.5.6 | Coefficient of skewness for normal distribution | 4.27 |
| 4.5.7 | Coefficient of kurtosis for normal distribution..... | 4.28 |
| 4.5.8 | Bernstein's theorem..... | 4.28 |
| 4.5.9 | A linear combination of independent normal variates..... | 4.28 |
| 4.6 | Characteristics of the Normal Distribution and Graph of Normal Distribution | 4.29 |
| 4.7 | Relation between Poisson, Binomial and Normal Distribution..... | 4.31 |

| | | |
|-----------|---|-----------------|
| 4.8 | Importance of Normal Distribution..... | 4.31 |
| 4.8 | Error Function..... | 4.32 |
| 4.9 | Area Under the Normal Curve | 4.33 |
| 4.10 | Working Procedure..... | 4.35 |
| 4.11 | Negative Binomial Distribution | 4.40 |
| 4.12 | Geometric Distribution..... | 4.41 |
| 4.13 | Hyper-Geometric Function | 4.41 |
| 4.14 | Multinomial Distribution | 4.42 |
| 4.15 | Uniform (or Rectangular) Distribution | 4.42 |
| 4.16 | Gamma Distribution..... | 4.43 |
| 4.17 | Beta Distribution of First Kind | 4.44 |
| 4.18 | Beta Distribution of Second Kind..... | 4.44 |
| | <i>Exercise</i> | 4.48 |
| 5. | SIMULATION OF CONTINUOUS SYSTEMS | 5.1—5.14 |
| 5.1 | Introduction | 5.1 |
| 5.2 | Mathematical Modeling Tool | 5.2 |
| 5.3 | Analog Versus Digital Simulation | 5.6 |
| 5.4 | Simulation of A Water Reservoir..... | 5.7 |
| 5.5 | Simulation of Servo System | 5.10 |
| 5.6 | Simulation of An Autopilot | 5.12 |
| | <i>Exercise</i> | 5.13 |
| | <i>References</i> | 5.13 |
| 6. | SIMULATION OF DISCRETE SYSTEMS..... | 6.1—6.21 |
| 6.1 | Discrete System..... | 6.1 |
| 6.2 | Simulation of Discrete System | 6.2 |
| 6.3 | Discrete Event Simulators | 6.4 |
| 6.4 | Component of Discrete Event Simulation | 6.4 |
| 6.5 | How Discrete Event Simulation Works | 6.5 |
| 6.6 | Overview of Discrete System..... | 6.6 |
| 6.7 | Fixed Time Step Versus Event to Event Model | 6.9 |
| 6.8 | Distributed Lag Models..... | 6.10 |
| 6.9 | Cobweb Model | 6.11 |
| 6.10 | Random Number Generation..... | 6.12 |
| | <i>Exercise</i> | 6.18 |
| | <i>References</i> | 6.20 |
| 7. | TIME SERIES..... | 7.1—7.19 |
| 7.1 | Introduction | 7.1 |
| 7.2 | Components of Time Series..... | 7.2 |
| 7.2.1 | Secular trend or trend component | 7.2 |
| 7.2.2 | Seasonal component..... | 7.3 |