Ten Chapters in Turbulence

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TEN CHAPTERS IN TURBULENCE

Turbulence is ubiquitous in science, technology and daily life and yet, despite years of research, our understanding of its fundamental nature is tentative and incomplete. More generally, the tools required for a deep understanding of strongly interacting many-body systems remain underdeveloped.

Inspired by a research programme held at the Newton Institute in Cambridge, this book contains reviews by leading experts that summarize our current understanding of the nature of turbulence from theoretical, experimental, observational and computational points of view. The articles cover a wide range of topics, including the scaling and organized motion in wall turbulence; small scale structure; dynamics and statistics of homogeneous turbulence, turbulent transport and mixing; and effects of rotation, stratification and magnetohydrodynamics, as well as superfluid turbulence.

The book will be useful to researchers and graduate students interested in the fundamental nature of turbulence at high Reynolds numbers.

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Preface

Vorticity fields that are not overly damped develop extremely complex spatial structures exhibiting a wide range of scales. These structures wax and wane in coherence; some are intense and most of them weak; and they interact nonlinearly. Their evolution is strongly influenced by the presence of boundaries, shear, rotation, stratification and magnetic fields. We label the multitude of phenomena associated with these fields as *turbulence*[1] and the challenge of predicting the statistical behaviour of such flows has engaged some of the finest minds in twentieth century science.

The progress has been famously slow. This slowness is in part because of the bewildering variety of turbulent flows, from the ideal laboratory creations on a small scale to heterogeneous flows on the dazzling scale of cosmos. Philip Saffman (*Structure and Mechanisms of Turbulence II*, Lecture Notes in Physics **76**, Springer, 1978, p. 273) commented: "... we should not altogether neglect the possibility that there is no such thing as 'turbulence'. That is to say, it is not meaningful to talk about the properties of a turbulent flow independently of the physical situation in which it arises. In searching for a theory of turbulence, perhaps we are looking for a chimera ... Perhaps there is no 'real turbulence problem', but a large number of turbulent flows and our problem is the self-imposed and possibly impossible task of fitting many phenomena into the Procrustean bed of a universal turbulence theory."[2]

More than forty years have passed since Steven Orszag (*J. Fluid Mech.* **41**, 363, 1970) made the whimsical comment, as if with an air of resignation: "It must be admitted that the principal result of fifty years of turbulence research is the recognition of the profound difficulties of the subject". The hope he expressed that "This is not meant to imply that a fully satisfactory theory is beyond hope", appearing almost as an afterthought in his paper, is still unrealized but progress has been made. In recent years one of the driving forces behind this progress has been the ever increasing power of computer simulations.[3] These simulations, in conjunction with ever more ambitious laboratory and field experiments, have helped us understand the

[1] We will not use the term here in its more general connotation of complex behaviour in an array of many-dimensional systems.

[2] Saffman was merely pointing out that he was open to this multiplicity, not declaring that research in turbulence is tantamount to taxonomy.

[3] The progress in numerical simulations of turbulence is a tribute to Orszag's untiring efforts. Sadly, he passed away during the preparation of this book.

role of organised motion in near-wall turbulence, and the structure and dynamics of small scales in statistically homogeneous turbulence and other turbulent flows. Together, experiments and simulations have enhanced our understanding of turbulent mixing and dispersion, allowing us to probe the validity and refinement of many classical scaling predictions, and have successfully complemented each other. In strongly stratified turbulence, for example, we have learnt that we are not free to prescribe the vertical Froude number; rather, nature dictates that it is of order unity. This understanding has been crucial to developing a self-consistent scaling theory of stratified turbulence. Similarly, in rapidly rotating turbulence, simulations have supplemented laboratory experiments and there is now a vigorous debate as to what precise role is played by inertial waves in forming the long-lived columnar vortices evident in both simulations and experiments. Yet another area in which simulations and experiments have admirably supplemented each other is turbulence in superfluids, which displays some of the same macroscopic phenomena as classical turbulence even though the microscopic physics in the two instances is quite different.

This book was conceived during an Isaac Newton programme on turbulence held in Cambridge in the Fall of 2008, and in particular after the workshop on *Inertial-Range Dynamics and Mixing* organised by the editors. The chapters, which take the form of reviews, are written by leading experts in the field and should appeal to specialists and non-specialists alike. They cover topics from small-scale turbulence in velocity and passive scalar fields to organized motion in wall flows, from dispersion and mixing to quantum turbulence as well as rotating, stratified and magnetic flows. They reflect the breadth, the nature and the features of turbulence already mentioned. The chapters progress from the those dealing with more general issues, such as homogeneous turbulence and passive scalars through to wall flows, ending with chapters dealing with stratification, rapidly rotating flows, the effect of electromagnetic fields and quantum turbulence. Each is intended to be a comprehensive account of lasting value that will help to open up new lines of enquiry. Yet, the title of the book conforms to the pragmatic style of the articles, rather than to a grand vision which promises more than it delivers.

The editors wish to thank the director and staff of the Isaac Newton Institute for their constant support during the 2008 turbulence programme, and Peter Bartello, David Dritschel and Rich Kerswell who co-organised the programme with great enthusiasm. They wish to thank CUP for the professionalism in preparing the book, and all the authors for their hard work and patience.

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Contents

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Small-Scale Statistics and Structure of Turbulence – in the Light of High Resolution Direct Numerical Simulation

Yukio Kaneda and Koji Morishita

1.1 Introduction

Fully developed turbulence is a phenomenon involving huge numbers of degrees of dynamical freedom. The motions of a turbulent fluid are sensitive to small differences in flow conditions, so though the latter are seemingly identical they may give rise to large differences in the motions.[1] It is difficult to predict them in full detail.

This difficulty is similar, in a sense, to the one we face in treating systems consisting of an Avogadro number of molecules, in which it is impossible to predict the motions of them all. It is known, however, that certain relations, such as the ideal gas laws, between a few number of variables such as pressure, volume, and temperature are insensitive to differences in the motions, shapes, collision processes, etc. of the molecules.

Given this, it is natural to ask whether there is any such relation in turbulence. In this regard, we recall that fluid motion is determined by flow conditions, such as boundary conditions and forcing. It is unlikely that the motion would be insensitive to the difference in these conditions, especially at large scales. It is also tempting, however, to assume that, in the statistics at sufficiently small scales in fully developed turbulence at sufficiently high Reynolds number, and away from the flow boundaries, there exist certain kinds of relation which are universal in the sense that they are insensitive to the detail of large-scale flow conditions. In fact, this idea underlies Kolmogorov's theory (Kolmogorov, 1941a, hereafter referred as K41), and has been at the heart of many modern studies of turbulence. Hereafter, universality in this sense is referred to as universality in the sense of K41.

Although most of the energy in turbulence resides at large scales, most of

[a] This work was undertaken while both authors were at Nagoya University.

[1] This does not prevent satisfactory averages being measured, at least those belonging to small scales.

the degrees of dynamical freedom resides in the small scales. In Fourier space, for example, most of the Fourier modes are in the high-wavenumber range. Hence properly understanding the nature of turbulence at small scales is interesting, not only from the theoretical, but also from the practical point of view, because such an understanding can be expected to be useful for developing models of turbulence to properly reduce the degrees of freedom to be treated.

This chapter will review studies of the nature of turbulence at small scales. Of course, more intensive studies have been performed on this interesting subject than we can cover here; in addition, we cannot review all of the issues related to each study that we do cover. We present a review of a few topics in the light of recent progress in high resolution direct numerical simulation (DNS) of turbulence. An analysis is also made on elongated local eddy structure and statistics. An emphasis is placed upon the Reynolds number dependence of the statistics and on the difference between active and non-active regions in turbulence.

1.2 Background supporting the idea of universality

1.2.1 Kolmogorov's 4/5 law

The existence of universality in the sense of K41 has not yet been proven rigorously, but there is evidence supporting it. Among this is Kolmogorov's 4/5 law (Kolmogorov, 1941c), which is derived as a consequence of the Navier–Stokes (NS) equation governing fluid motion.

Let $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ be an incompressible turbulent velocity field obeying the Navier–Stokes equation,

$$\frac{\partial}{\partial t}\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f}, \tag{1.1}$$

and the incompressibility condition,

$$\nabla\cdot\mathbf{u} = \mathbf{0}, \tag{1.2}$$

where ν is the kinematic viscosity, p the pressure, $\mathbf{f}$ the external force per unit mass, and ρ the fluid density.

For homogeneous and isotropic (HI) turbulence, the NS equation with the incompressibility condition (1.2) yields the Kármán–Howarth equation (Kármán and Howarth, 1938)

$$B_3^L(r) = -\frac{4}{5}\langle\epsilon\rangle r + 6\nu\frac{\partial B_2^L(r)}{\partial r} + F(r) - \frac{3}{r^4}\int_0^r \frac{\partial B_2^L(\tilde{r})}{\partial t}\tilde{r}^4 d\tilde{r}, \tag{1.3}$$

where $\langle\epsilon\rangle$ is the average of the rate of energy dissipation ϵ per unit mass, and $B_n^L(r)$ is the nth order structure function of the longitudinal velocity difference δu^L defined as

$$B_n^L(r) \equiv \left\langle \left[\delta u^L(r)\right]^n \right\rangle, \quad \delta u^L(r) \equiv [\mathbf{u}(\mathbf{x}+r\mathbf{e}) - \mathbf{u}(\mathbf{x})] \cdot \mathbf{e}, \tag{1.4}$$

in which $\mathbf{e}$ is an arbitrary unit vector. In (1.3), F is expressed in terms of the correlation $g(r) \equiv \langle[\mathbf{u}(\mathbf{r}+\mathbf{x}) - \mathbf{u}(\mathbf{x})] \cdot [\mathbf{f}(\mathbf{r}+\mathbf{x}) - \mathbf{f}(\mathbf{x})]\rangle$. It is shown by simple algebra that

$$F(r) = \frac{6}{r^4}\int_0^r \tilde{r}G(\tilde{r})d\tilde{r}, \quad G(r) = \int_0^r \tilde{r}^2 g(\tilde{r})d\tilde{r}.$$

If the forcing $\mathbf{f}$ is confined only to large scales, say $\sim L$ (where the symbol $\sim$ denotes an equality up to a coefficient of order unity), and the viscosity ν is very small, then it is plausible to assume that in (1.3),

(i) the forcing term $F(r)$ is negligible at $r \ll L$,
(ii) the viscosity term works only at small scales, say $\sim \eta$, so that it is negligible at $r \gg \eta$, and
(iii) the statistics is almost stationary at small scales, so that the last term is negligible at $r \ll L$.

Under these assumptions, (1.3) yields the 4/5 law,

$$B_3^L(r) = -\frac{4}{5}\langle\epsilon\rangle r, \tag{1.5}$$

for $L \gg r \gg \eta$.

Note that the 4/5 law (1.5) applies not only to the stationary but also to the freely-decaying case, as long as one may assume (iii), in addition to (i) and (ii), where L is to be understood appropriately, e.g., as the characteristic length scale of the energy containing eddies.

The relation (1.5) asserts that $B_3^L(r)$ is specified only by $\langle\epsilon\rangle$ and r. It holds independently of the shapes, internal structures, deformations, positions, alignments, interactions, collision and reconnection processes, etc. of small-scale eddies, however the term 'eddies' may be defined, and also of the forcing and boundary conditions outside the range $r \ll L$, as long as (i), (ii) and (iii) hold. (This doesn't mean that $B_3^L(r)$ is independent of these factors and conditions, as they may still affect $\langle\epsilon\rangle$: rather, the relation (1.5) means that their influence, if any, is only through $\langle\epsilon\rangle$.)

The relation (1.5) holds independently of these factors, just as the ideal gas laws hold independently of the shapes, internal structures, interactions, collision processes etc. of the molecules comprising the gas, and independently of the shape of the container of the gas. The relation is in this sense

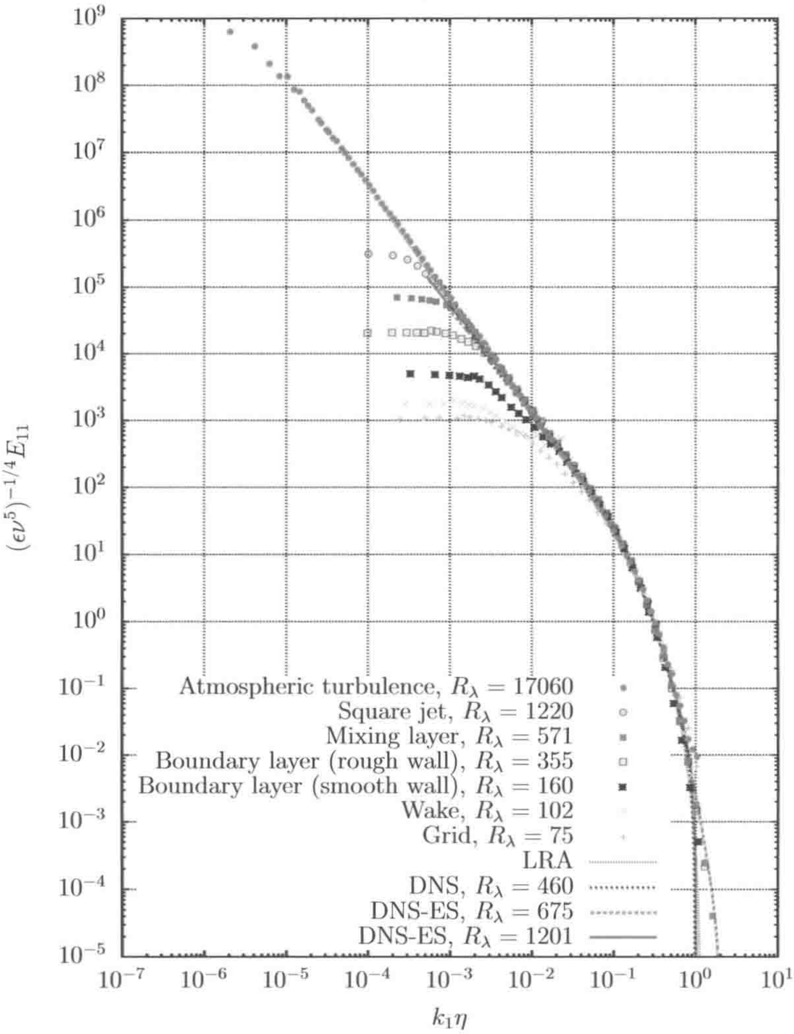

Figure 1.1 Normalized longitudinal one-dimensional energy spectrum. The data except those by DNS-ES are re-plotted from Tsuji (2009).

universal, and supports the idea of existence of universality in the sense of K41.

1.2.2 Energy spectrum

More support for the existence of universality in the sense of K41 is given by the second-order two-point velocity correlations, or, equivalently, the velocity correlation spectra observed in experiments and DNS. If the second-order moments of $\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$ are universal in a certain sense at small scale,