

STRUCTURAL DAMPING

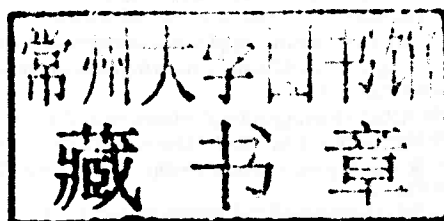
**Applications in
Seismic Response Modification**

**Zach Liang
George C. Lee
Gary F. Dargush
Jianwei Song**

STRUCTURAL DAMPING

Applications in Seismic Response Modification

Zach Liang
George C. Lee
Gary F. Dargush
Jianwei Song



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

MATLAB® is a trademark of The MathWorks, Inc. and is used with permission. The MathWorks does not warrant the accuracy of the text or exercises in this book. This book's use or discussion of MATLAB® software or related products does not constitute endorsement or sponsorship by The MathWorks of a particular pedagogical approach or particular use of the MATLAB® software.

CRC Press

Taylor & Francis Group

6000 Broken Sound Parkway NW, Suite 300

Boca Raton, FL 33487-2742

© 2012 by Taylor & Francis Group, LLC

CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

Printed in the United States of America on acid-free paper

Version Date: 20111012

International Standard Book Number: 978-1-4398-1582-3 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Structural damping : applications in seismic response modification / Zach Liang ... [et al.].

p. cm. -- (Advances in earthquake engineering)

"A CRC title."

Includes bibliographical references and index.

ISBN 978-1-4398-1582-3 (hardcover : alk. paper)

1. Earthquake resistant design. 2. Damping (Mechanics) I. Liang, Zach.

TA658.44.S77 2012

624.1'762--dc23

2011036557

Visit the Taylor & Francis Web site at

<http://www.taylorandfrancis.com>

and the CRC Press Web site at

<http://www.crcpress.com>

STRUCTURAL DAMPING

**Applications in
Seismic Response Modification**

Preface

Today, earthquake engineering research is devoting a major effort toward establishing seismic performance requirements associated with large inelastic deformations of the structure. At the same time, structural response modification systems (certainly, passive energy dissipation and seismic isolation systems) have been widely used. It is reasonable to expect that the next research emphasis in performance-based engineering is to integrate both frontiers for more optimal seismic performance of structures. As far as safety, performance, and cost for both components of inelastic deformation and structural response control are concerned, structural damping is a core knowledge area. In this book, recent advances in structural damping are presented and their applications to the design of passive structural response modification devices are given to complement the current supplemental damping design practice for high-level damping. Integration with seismic performance requirement is not addressed in this book.

Aseismic design using supplemental dampers has steadily gained popularity in the earthquake engineering profession over the past several decades. Many practical applications of various dampers can be found worldwide and, in the United States, damper design has been included in building codes. To date, damping design is primarily based on the concepts of the energy equation, effective proportional damping, and simplified linear single-degree-of-freedom (SDOF) response spectrum. These concepts, along with their associated underlying assumptions, support the idea that installing supplemental dampers in structures will dissipate energy. Nonlinear damping is presented by an effective damping ratio through linearization schemes, and the damping coefficients of structures are assumed to be classic damping matrices in order to establish a procedure for damping design. In addition, it is assumed that statistical procedures that use earthquake records can be carried out through proportional scaling of their amplitudes.

These assumptions have enabled us to develop a design procedure for supplemental dampers. However, it is not well understood that some of these assumptions work well only when the amount of damping in structures is within a specific low level. The first main objective of this book is to provide a theoretical foundation on the role of damping in the dynamic response of structures, especially when the level of damping is high or when nonlinearities become important design issues. The second main objective is to provide response spectra-based design principles and guidelines for practical applications of damping devices to reduce earthquake-induced structural vibration.

Generally speaking, structural responses under seismic excitations are dynamic processes. There are three resisting force components to counter the earthquake load, one of which is the damping force. While damping technology has been developed and advanced in a range of mechanical and aerospace engineering applications over many years, it has become a popular approach in structural engineering only in the later part of the twentieth century.

While the development and application of energy dissipation devices in structural engineering continue to expand, there are a number of fundamental issues related to the dynamic behavior of structures with supplemental damping as a system that require further study. Limitations and impacts of using energy dissipation devices need to be clarified and established. This book intends to fill the knowledge gap by helping earthquake engineers to better understand the dynamic behavior of structures and to more effectively use the design codes for dampers.

The key elements in this book are summarized as follows.

A straightforward concept often advocated in damping design is that “more energy dissipated by the added dampers will result in less vibration energy remaining in the structure, and thus the structural response is reduced.” This is not always true. A higher level of damping may not effectively reduce the responses of a given structure. In some cases, high-level damping may even magnify the responses, because the level of structural response depends not only on local energy

dissipation, but also on energy input and its redistribution. Thus, minimizing the conservative energy of the vibrating system in structures with supplemental damping is a more appropriate general guiding principle.

Several other basic issues in structural damping are carefully reviewed. These include the maximum energy dissipation principle under preset damping force and allowed structural displacement; the damping adaptability of devices that can operate in a large dynamic range of earthquake loads; the viscoelastic behavior of any damping elements that take the supporting stiffness as well as installation practice into consideration; nonproportional damping that needs to be minimized as much as possible in design practice; the limitation of using damping force that provides a practical engineering limit beyond which adding more damping provides diminishing gains; and the problem of damping and stiffness nonlinearity that cannot be accurately approximated by today's design approaches. In addition, a design principle based on energy distribution is discussed, which may be useful for generally damped multi-degree-of-freedom (MDOF) systems.

The characteristics of nonlinear damping and nonlinear structures are complex issues to address in damping design. In this book, nonlinearity is considered in three cases. The first case involves a linear structure with small nonlinear damping, the second case is for a linear structure with larger nonlinear damping, and the third case applies when both the structure and the damping are nonlinear. In the first case, because the damping force is rather small, almost any type of linearization can be used without causing any significant design discrepancies. In the second case, care must be taken to choose proper linearization; employing nonlinear design spectra can often be a reasonable approach. In the third case, linearization methods, though adopted by many building codes, can provide highly inaccurate results. Methods such as the equal displacement approach (using R-factors), equal energy approach, and pushover approach all have their limitations. Since the nonlinear design spectra approach requires too many response spectra, not only for specific damping and stiffness, but also for specific levels of ground excitations, it is not useful in practice. Thus, nonlinear time history analysis must be used. This latter method, though always workable for the first two cases, can yield an unacceptable computation burden, making it unattractive for use in day-to-day practical damping design. For this reason, time history analysis is not emphasized in all discussions.

Two types of design approaches are provided in this book. The first is the design response spectra approach. Specifically, the design is modified by a simplified factor, the damping ratios of the first several vibration modes of the structure. For readers familiar with the design response spectra method for an SDOF linear system, this approach provides a good design when damping is small and for structural responses in elastic ranges. The modified approach addresses cases where damping is large and nonlinear. Examples of how to modify the response spectra design are also provided.

The second type of damper design is based on time history analysis, for which several issues are important. These issues include how to select and scale earthquake records to be consistent with the response spectra, accuracy of modeling of dampers as well as the structure-damper system both as elastic and inelastic systems, and interpretation of response time histories and peak values.

There are many issues related to the role of large damping in the design of earthquake protective systems. Some involve fundamental theories, while others focus on practical details, such as device installations. In this book, focus is first given to the fundamental issues. Detailed technical descriptions and step-by-step design procedures are developed based on the basic principles. These fundamental issues are limited to areas within the scope of structural dynamics principles, although attention is also given to related topics of damper selection, damper specifications, and damper installation.

The arrangement of this book is as follows. Part I provides a foundation for generally damped MDOF systems by emphasizing damping force, energy dissipation, and structural impedances, which are important in structural dynamics and damping control. In Chapter 1, the necessary background of linear SDOF systems, including the concepts of natural frequency and damping ratio, is introduced. Free and forced harmonic vibrations are discussed, and the concepts of damping force and energy dissipation are systemically explained. The effect of damping on free and harmonic

vibration is reviewed. Effective damping, as systemically described by Timoshenko, is the fundamental formula in currently used damper design. This concept is analyzed and an alternative force-based approach for linearization of damping is also provided.

In Chapter 2, arbitrary excitations for SDOF systems, including periodic, transient, and random excitations, are considered and the earthquake responses of structures are examined. Parallel with the description of these three types of arbitrary excitations and the corresponding responses of SDOF systems, mathematical tools are introduced. In particular, Fourier series and Fourier/Laplace transforms, which are basic approaches to represent vibration signals, are reviewed. The integral transforms allow a different idea to be considered; that is, the view in the frequency domain is seen as a modal vibration model, whereas the direct response of the system provides the response model in the time domain. Furthermore, to account for random vibrations, the concepts of correlation analysis and spectral analysis are introduced. These necessary mathematical tools are used in the rest of the book. In the last section of this chapter, earthquake response is discussed with a primary focus on the response spectrum. Again, the effect of damping is emphasized.

In Chapter 3, linear MDOF systems with proportional damping are introduced. First, the undamped system is examined and the fundamental approach of eigen-decomposition is considered. The concept of the Rayleigh quotient is introduced as the foundation of modal analysis. Then, proportional damping is discussed, followed by modal analysis and system decoupling. For practical applications, modal participation, modal truncation, and modal parameter estimation, as well as several forms of proportional damping matrices, are presented.

Nonproportionally damped and overdamped systems are discussed in Chapter 4. Although most structures are more or less generally damped, only the equations that are needed for practical use are presented. Theoretical developments on generally damped MDOF systems are not covered here in significant depth, but are available elsewhere (e.g., Liang et al. 2007). Instead, explanations for engineering application of the theoretical principles and design examples are provided.

Part II introduces some design principles and guidelines for damping control. The focus is on using damping force more accurately and effectively in the design of structures with supplemental damping. In Chapter 5, the basic principles of damper design and damping devices are given. The first group of principles are associated with various dampers. These include generic modeling of damping force for dampers and damping parameters of structures with added dampers; conventionally used Timoshenko effective damping ratio based on energy and an alternative approach for the effective damping ratio based on damping force; maximum energy dissipation per device per cycle, which leads to the rectangular law that provides a method for optimal damping design and the upper limit of damping vibration control; damping adaptability, which provides another basic rule for damper selection; damping ratio affected by the physical parameters of the total system and the effectiveness of the structural parameters in affecting the damping ratio; similarity and difference of response spectrum and dynamic stiffness, which provide an alternative rule other than response spectrum as a design criteria; and the relationship between damping and stiffness, which is an often overlooked issue in practical damper design.

Chapter 6 is a continuation of the discussion on design principles, but the focus is on the nonlinearity and damping irregularity of the total system of structures with supplemental damping. The pros and cons of currently used simplified damper design procedures are presented and discussed. In the simplified design procedure, the designers do not have to obtain the exact mode shapes for modes higher than the fundamental one, nor do they have to calculate the exact first mode shape. However, for systems with larger damping, special considerations must be given. These considerations result in modifications to the currently used simplified approach based on the design spectra approach. These suggested modifications are presented in the NEHRP (2009) provisions and are discussed in Part III.

In general, Part III provides more detailed design procedures based on the classification of specific damping devices. In this book, dampers are classified based on their linearity and rate dependency to facilitate the subsequent development of design guidelines in a logical manner, rather than

on their displacement dependency and velocity dependency. Chapter 7 deals with linear damping and linearized nonlinear damping. When the supplemental damping is not sufficiently large, this approach can greatly reduce the computational burden, yet provide reasonable accuracy. The simplified design approach of current codes is introduced. While the design logic is virtually identical among these codes, some improvements are suggested to enhance the design procedure. The first part of Chapter 7 presents an approach for using SDOF systems, which is directly related to the design spectrum. This serves as the basis for the entire simplified design process, as well as provides an initial estimation of whether supplemental damping should be used. The multiple-story-single-period (MSSP) structures and multiple modes are then introduced as the main platform for developing design guidelines. Chapter 7 also presents more general linear damping.

When the added damping is sufficiently large, to avoid the errors introduced by the linearization process, nonlinear damping needs to be considered. This is examined in Chapter 8. One typical nonlinear response is the parallelogram-form force–displacement relationship, referred to as the bilinear damper. To estimate the structural responses, specific bilinear response spectra are used. To further obtain the response vectors of nonlinear MDOF systems, separation of the displacement and acceleration is performed and the combination of the first several effective “modes” is rendered. In Chapter 8, another important type of damper, the sublinear damper, is discussed and expressed by the sublinear response spectra. Unlike bilinear damping, in which all the effective “modes” of interest can be treated as bilinear, sublinear damping rarely contributes accurate information for higher “modes.” Therefore, an alternative approach of an equivalent linear MDOF system, which likely will be generally damped, is used. Detailed design steps on mode shape normalization, general damping indices, and response computation, as well as selecting damper specifications, are discussed. For sublinear systems, an iterative design procedure for a nonlinearly damped structure is also suggested. This includes the identification of the model, initial design, and response estimation. Note that the spectra-based estimation proposed in this chapter provides simplified calculations with considerably less computational burden. It should be used together with time history analysis for design safety, efficiency of damper use, and cost-effective optimizations.

The materials covered in Part III are incomplete. Much remain to be fully developed. Because most design professionals are familiar with the current design codes (e.g., NEHRP 2009), an approach that follows the NEHRP provisions with added “notations” and “recommendations” is followed in presenting the materials in Chapters 7 and 8. It is hoped that this information will be useful as a supplement to the existing NEHRP provisions. It is obvious that many research and development challenges remain to be faced by the earthquake engineering research community and codification professional groups. This book will hopefully also help clarify some of these future research needs.

The materials presented in this book were gradually developed by the authors during the past 20 years in conjunction with their research activities sponsored by the National Science Foundation and the Federal Highway Administration through the National Center for Earthquake Engineering Research and subsequently, the Multidisciplinary Center for Earthquake Engineering Research. The authors would like to acknowledge these funding agencies for the opportunity to work on structural damping–related subjects, and the Samuel P. Capen Endowment fund of the University at Buffalo, State University of New York, for partial financial support. They would like to express their appreciation to professors Joseph Penzein of the University at California, Berkeley, and Masanobu Shinozuka of the University of California, Irvine, for helpful technical discussions and to many of their colleagues at the University at Buffalo, in particular, professors Michael Constantinou, Andre Filiatrault, Andrei Reinhorn, T. T. Soong, and Andrew Whittaker, for their inspiring discussions and pioneering research efforts in damping design and related areas that greatly benefited this writing effort. In addition, the authors are indebted to the following individuals for their invaluable assistance in technical editing and formatting of the manuscript: Jane Stoye Welch, Shuchuan Zhang, Nasi Zhang, Yihui Zhou, Hao Xue, Dezhang Sun, and Chao Huang. Last but not the least, the authors express their sincere appreciation and affection to their wives, Yiwei, Grace, Andrea, and Li, for

their patience, encouragement, and love, as the authors devoted countless evenings and weekends while this book was being written.

Zach Liang, George C. Lee, Gary F. Dargush, and Jianwei Song

*University at Buffalo
State University of New York*

MATLAB® is a registered trademark of The MathWorks, Inc. For product information, please contact:

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098 USA
Tel: 508 647 7000
Fax: 508-647-7001
E-mail: info@mathworks.com
Web: www.mathworks.com

Series Editor



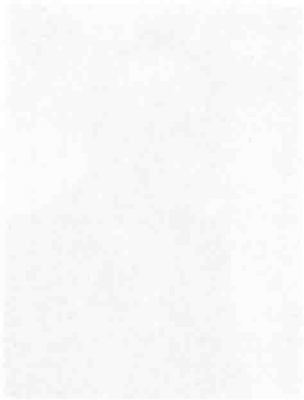
Dr. Franklin Cheng received a BS degree (1960) from the National Cheng-Kung University, Taiwan, and an MS degree (1962) from the University of Illinois at Urbana-Champaign. He gained industrial experience with C. F. Murphy and Sargent & Lundy in Chicago, Illinois. Dr. Cheng then received a PhD degree (1966) in civil engineering from the University of Wisconsin–Madison. Dr. Cheng joined the University of Missouri, Rolla (now named Missouri University of Science and Technology) as assistant professor in 1966 and then associate professor and professor in 1969 and 1974, respectively. In 1987, the Board of Curators of the University appointed him curators' professor, the highest professorial position in the system comprising four campuses. He has been Curators' Professor Emeritus of Civil Engineering since 2000. In 2007, the American Society of Civil Engineers recognized Dr. Cheng's accomplishments

by electing him to honorary membership, which is now renamed as distinguished membership. Honorary membership is the highest award the Society may confer, second only to the title of ASCE President. Honorary members on this prestigious and highly selective list are those who have attained acknowledged eminence in a branch of engineering or its related arts and sciences. Until 2007, there have been 565 individuals who were elected to this distinguished grade of membership since 1853. For the year of 2007, only 10 honorary members were selected from more than 14,000 members.

Dr. Cheng was honored for his significant contributions to earthquake structural engineering, optimization, nonlinear analysis, and smart structural control; and for his distinguished leadership and service in the international engineering community; as well as for being a well-respected educator, consultant, author, editor, and member of numerous professional committees and delegations. His cutting edge research helped recognize the vital importance of the possibilities of automatic computing in the future of civil engineering. He was one of the pioneers in allying computing expertise to the design of large and complex structures against dynamic loads. His research expanded over the years to include the important topics of structural optimization and design of smart structures. In fact, he is one of the foremost experts in the world on the application of structural dynamics and optimization to the design of structures. Due to the high caliber and breadth of his research expertise, Dr. Cheng has been regularly invited to serve on the review panels for the National Science Foundation (NSF), hence setting the direction of future structural research. In addition, he has been instrumental in helping the NSF develop collaborative research programs with Europe, China, Taiwan, Japan, and South Korea. Major industrial corporations and government agencies have sought Dr. Cheng's consultancy. He has consulted with Martin Marietta Energy Systems, Inc., Los Alamos National Laboratory, Kjaime Corporation, Martin & Huang International, Inc., and others.

Dr. Cheng received four honorary professorships from China and chaired 7 of his 24 NSF delegations to various countries for research cooperation. He is the author of more than 280 publications, including 5 textbooks: *Matrix Analysis of Structural Dynamics – Applications and Earthquake Engineering*, *Dynamic Structural Analysis*, *Smart Structures – Innovative Systems for Seismic Response Control*, *Structure Optimization – Dynamic and Seismic Applications*, and *Seismic Design Aids for Nonlinear Pushover Analysis of Reinforced Concrete and Steel Bridges*. Dr. Cheng has been recipient of numerous honors and awards including Chi Epsilon, MSM–UMR Alumni

Merit for Outstanding Accomplishments, Faculty Excellence Award, Halliburton Excellence Award, and recognitions in 21 biographical publications such as *Who's Who in Engineering*, and *Who's Who in the World*. He has twice been the recipient of the ASCE State-of-the-Art Award in 1998 and 2004.



Series Preface

The major goal of earthquake engineering is to limit seismic-induced structural damages at reasonable cost. To achieve this goal, one of the most effective approaches to reduce structural response is to utilize supplemental dampers, provided such a damping system is properly designed. In this sense, understanding structural damping is critically important for successful seismic design. This book *Structural Damping: Applications in Seismic Response Modification* is a valuable addition to the Advances in Earthquake Engineering Series for it serves the goal of optimal response control.

The book was written under the direction of Dr. George C. Lee, SUNY distinguished professor, who had served as chair of the Department of Civil Engineering, dean of the School of Engineering and Applied Science, and director of the Multidisciplinary Center for Earthquake Engineering Research. He and his coauthors have integrated the essential materials developed by his research team in the past 20 years. The key features of this book may be briefly summarized as follows:

1. The book provides an integrated systematic presentation of the dynamic response of structures with nonproportional or nonlinear damping, as well as with overdamped vibration modes. This theoretical base is important for understanding the dynamic behavior of structures with large damping, prior to the development of aseismic design using damping devices.
2. This volume is a pioneering work to provide comprehensive design principles of structural damping, including design procedures and guidelines for aseismic design of structures with enhanced damping.
3. Based on a comprehensive formulation, limitations of current design practice for large damping are clearly illustrated, and improvements to handle enhanced damping are given.
4. This volume offers a discussion of the safety issues of structures with enhanced damping, based on theoretical formulation and practical design consideration.

This book is useful not only in the practicing engineering community but also to researchers and educators, because numerous research and development challenges remain to be pursued.

Authors

Dr. Zach Liang is a research professor in the Department of Mechanical and Aerospace Engineering, University at Buffalo. He has an MS degree in Mechanical Engineering, Tianjin University, China, 1982 and a PhD in Mechanical and Aerospace Engineering, University at Buffalo, 1987. His research interests include structural dynamics, vibration and control, damping of systems and devices, reliability design and random process, earthquake engineering, testing and measurement, and nondestructive evaluation.

Dr. George C. Lee is a SUNY distinguished professor in the Department of Civil, Structural, and Environmental Engineering, University at Buffalo. Previously, he served as director of the Multidisciplinary Center for Earthquake Engineering Research (MCEER). He received a BS degree from the National Taiwan University, and MS and PhD degrees from Lehigh University. His academic and research interests include steel structures, earthquake engineering, bridge engineering, and multihazard bridge design of principles.

Dr. Gary F. Dargush is a professor and the department chairman of Mechanical and Aerospace Engineering, University at Buffalo. He has a PhD in Civil Engineering, University at Buffalo, 1987. His research interests include computational mechanics, boundary element methods, finite element methods, earthquake engineering, structural dynamics, geomechanics, thermomechanics, and computational fluid dynamics.

Dr. Jianwei Song is a senior research scientist in the Department of Civil and Environmental Engineering, University at Buffalo. He received a PhD in Mechanical Engineering, Tianjin University, China, 1989. His research interests include structural dynamics and control, vibration modal analysis and signal processing, advanced experimental system development, earthquake engineering, and nondestructive test/evaluation.

Contents

Series Preface.....	xv
Preface.....	xvii
Series Editor.....	xxiii
Authors.....	xxv

PART I *Vibration Systems*

Chapter 1	Free and Harmonic Vibration of Single-Degree-of-Freedom Systems.....	3
1.1	Model of Linear SDOF Vibration Systems	3
1.1.1	Equation of Motion and Basic Dynamic Parameters.....	3
1.1.1.1	Equilibrium of Vibration Forces.....	3
1.1.1.2	Basic Parameters of the Physical Model.....	5
1.1.1.3	Characteristic Equation and Modal Model.....	7
1.1.2	Homogeneous Solution, Free-Decay Vibration, and the Response Model	11
1.1.3	Forced Vibration with Harmonic Excitation	16
1.1.3.1	Steady-State Response	16
1.1.3.2	Method of Complex Response for Steady-State Displacement	18
1.1.3.3	Response of Harmonic Excitation with Zero Initial Condition	19
1.1.3.4	Responses with Nonzero Initial Conditions	23
1.1.4	Ground Excitation	24
1.1.4.1	Governing Equation.....	24
1.1.4.2	Responses of Harmonic Ground Excitations.....	26
1.2	Dynamic Magnification.....	29
1.2.1	Dynamic Magnification Factor, General Excitation	30
1.2.1.1	Dynamic Magnification Factor of Displacement.....	30
1.2.1.2	Dynamic Magnification Factor of Acceleration	32
1.2.1.3	Peak Values of Dynamic Magnification Factors	33
1.2.1.4	Dynamic Magnification Factors Reaching Unity	35
1.2.1.5	Half-Power Points and Resonant Region.....	36
1.2.1.6	Response Reduction due to Increase of Damping	38
1.2.2	Dynamic Magnification Factor, Ground Excitation.....	40
1.2.2.1	Dynamic Magnification Factor of Relative Displacement	41
1.2.2.2	Dynamic Magnification Factor of Absolute Acceleration.....	44
1.3	Energy Dissipation and Effective Damping	50
1.3.1	Energy Dissipated per Cycle	50
1.3.1.1	Linear Viscous Damping.....	50
1.3.1.2	Energy Dissipated by Damping Force.....	51
1.3.1.3	Damping Coefficient and Damping Ratio of Linear Viscous Damping.....	52
1.3.1.4	Linear System	53

1.3.2	Damping and Seismic Force	54
1.3.2.1	Parametric Equation	54
1.3.3	Effective Damping	58
1.3.3.1	Effective Damping Coefficient	59
1.3.3.2	Effective Damping Ratio	60
1.3.3.3	General Forced Vibration, Energy Dissipation	61
1.3.3.4	Alternative Form of Damping Ratio.....	62
1.4	Summary	63
	References	63
Chapter 2	Linear Single-Degree-of-Freedom Systems with Arbitrary Excitations	65
2.1	Periodic Excitations	65
2.1.1	Periodic Signals	65
2.1.1.1	Periodic Functions	65
2.1.2	Fourier Series	68
2.1.2.1	Fourier Coefficients	68
2.1.3	Discrete Fourier Transform.....	72
2.1.3.1	Discretization of Signals.....	72
2.1.3.2	Discrete Fourier Series	74
2.1.4	General Damping Force	76
2.1.4.1	Linearity of Fourier Series	78
2.1.4.2	Steady-State Structural Force	78
2.1.4.3	Dry Friction Damping	81
2.1.4.4	General Nonlinear Viscous Damping	85
2.1.5	Response to Periodic Excitations	89
2.1.5.1	General Response	89
2.1.5.2	The n^{th} Steady-State Response.....	90
2.1.5.3	Transient Response	90
2.2	Transient Excitations	91
2.2.1	Transient Signals	91
2.2.2	Fourier Transform	91
2.2.2.1	Important Features of the Fourier Transform: A Summary	93
2.2.3	Laplace Transform	94
2.2.3.1	Important Features of the Laplace Transform: A Summary	95
2.2.4	Impulse Response.....	96
2.2.5	General Force and the Duhamel Integral.....	98
2.2.5.1	Convolution Integral	98
2.2.6	Transfer Function of Unit Impulse Response.....	100
2.2.7	Integral Transform of Convolution.....	102
2.3	Random Excitations.....	103
2.3.1	Random Variables	103
2.3.1.1	Mean Value, Mathematic Expectation.....	105
2.3.1.2	Variance and Mean Square Value	107
2.3.1.3	Standard Deviation and Root Mean Square Value	108
2.3.2	Random Process	109
2.3.2.1	Random Time Histories.....	109
2.3.2.2	Statistical Averaging.....	109

2.3.3	Correlation Functions and Power Spectral Density Functions.....	112
2.3.3.1	Correlation Analysis	112
2.3.3.2	Power Spectral Density Function	115
2.3.4	Correlation between Forcing Function and Impulse Response Function.....	124
2.3.5	Basic Approach to Dealing with Random Vibrations.....	128
2.4	Earthquake Responses of SDOF Linear Systems	128
2.4.1	Response Spectrum	128
2.4.2	Design Spectra	131
2.4.3	Control Factor for Damper Design: The Base Shear	137
2.4.3.1	Spectral Accelerations and Displacement	141
2.5	Summary	141
	References	141

Chapter 3	Linear Proportionally Damped Multi-Degree-of-Freedom Systems	143
3.1	Undamped MDOF Systems.....	143
3.1.1	Eigen-Parameters of Linear Undamped Systems	143
3.1.1.1	Governing Equations	143
3.1.1.2	Modal Response of Free Vibrations	145
3.1.1.3	General Eigen-Parameters	146
3.1.2	Brief Discussion of Vectors and Matrices	148
3.1.2.1	Vector.....	148
3.1.2.2	Vector Norm	149
3.1.2.3	Orthonormal Vectors	150
3.1.2.4	Unit Vector.....	150
3.1.2.5	Vector Space	152
3.1.2.6	Linear Independence	152
3.1.3	Symmetric Matrix and Rayleigh Quotient.....	154
3.1.3.1	Eigen-Parameters of Symmetric Matrices.....	154
3.1.3.2	Rayleigh Quotient	156
3.2	Proportionally Damped MDOF Systems	159
3.2.1	Modal Analysis and Decoupling Procedure	159
3.2.1.1	Governing Equation of Damped MDOF Systems.....	159
3.2.1.2	Decoupling by Means of Rayleigh Quotient	160
3.2.1.3	Decoupling by Means of Linear Transformation	162
3.2.2	Free-Decay Vibration	164
3.3	Modal Participation and Truncation.....	169
3.3.1	Modal Participation Factor.....	169
3.3.2	Modal Contribution Indicator	170
3.3.2.1	Theory of the Indicator.....	170
3.3.2.2	Realization of Indicators.....	172
3.3.3	Response Computation of Truncated Modal Superposition	173
3.3.3.1	Computation Procedure	173
3.3.4	Peak Responses	181
3.4	Base Shear and Lateral Force.....	182
3.5	Natural Frequency and Mode Shape Estimation.....	185
3.5.1	Natural Frequency	185
3.5.2	Mode Shape Estimation	188
3.6	Coefficient Matrix for Proportional Damping.....	190
3.6.1	Rayleigh Damping.....	190

3.6.2	Caughey Damping.....	191
3.6.3	Modification of Caughey Damping.....	192
3.6.4	Overdamped Modes	193
3.6.5	Alternative Expression of Proportional Damping.....	194
3.6.5.1	Generalized Symmetric Damping Matrix.....	195
3.7	Summary	196
	References	196
Chapter 4	Multi-Degree-of-Freedom Systems with General Damping.....	197
4.1	State Equations and Conventional Treatment.....	197
4.1.1	State Matrix and Eigen-Decomposition	197
4.1.1.1	State Equations	197
4.1.2	Accompanist Matrix of Mode Shapes.....	204
4.1.3	Linear Independency and Orthogonality Conditions.....	207
4.1.4	Approach of Complex Damping.....	212
4.1.5	Solutions in $2n$ Modal Space	218
4.1.5.1	Mode Decoupling	218
4.1.5.2	Alternative Computation Method	219
4.2	Damper Design for Nonproportionally Damped Systems	225
4.2.1	Issues with General Damper Design.....	225
4.2.2	Essence of the Solution of Nonproportionally Damped System	226
4.2.3	Modal Truncations for Nonproportionally Damped Systems	230
4.3	Overdamped Subsystems.....	230
4.3.1	Concept of Overdamped System	231
4.3.2	Design Response Spectra for Overdamped Subsystem.....	236
4.3.2.1	Spectral Value	236
4.3.2.2	Overdamping Constant	237
4.4	Responses of Generally Damped Systems and the Design Spectra.....	240
4.4.1	Approach in $2n$ - and n -Space, Design Codes.....	241
4.4.2	Modal Solution in n -Dimensional Space.....	244
4.4.3	Modal Truncations for a Generally Damped System.....	248
4.5	Modal Participation and Modal Criteria	249
4.5.1	Criteria on Complex Mode.....	249
4.5.1.1	Modal Energy Index	249
4.5.1.2	Complex Modal Factor	251
4.5.2	Modal Participation Factors	253
4.5.2.1	Nonproportionally Damped Modes	254
4.5.2.2	Proportionally Damped Modes	254
4.5.2.3	Overdamped Subsystems.....	256
4.5.3	Modal Contribution Indicators	256
4.5.3.1	Modal Mass Ratio.....	256
4.5.3.2	Static Modal Energy Ratio	260
4.5.4	Modal Reconstruction of Generally Damped System	262
4.5.4.1	Modal Reconstruction for Damper Design.....	264
4.5.4.2	Damper Design without Stiffness Matrix.....	268
4.6	Summary	275
	References	275