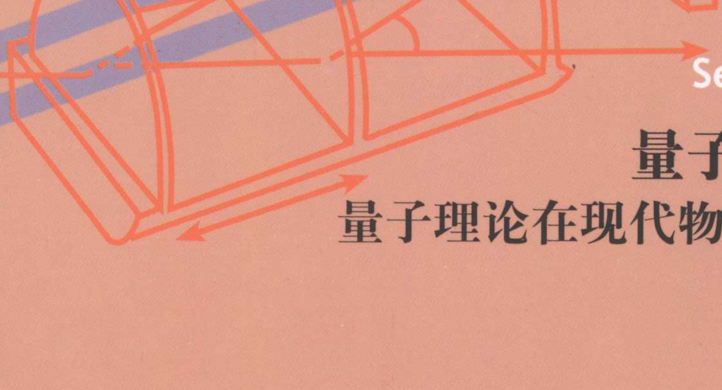
Jean-Louis Basdevant
Jean Dalibard

THE QUANTUM MECHANICS SOLVER

HOW TO APPLY QUANTUM THEORY TO MODERN PHYSICS

Second Edition



量子力学题解

量子理论在现代物理中的应用

第2版

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With 59 Figures, Numerous Problems and Solutions

Springer

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The Quantum Mechanics Solver

ÉCOLE POLYTECHNIQUE

The Ecole Polytechnique, one of France's top academic institutions, has a longstanding tradition of producing exceptional scientific textbooks for its students. The original lecture notes, the *Cours de l'Ecole Polytechnique*, which were written by Cauchy and Jordan in the nineteenth century, are considered to be landmarks in the development of mathematics.

The present series of textbooks is remarkable in that the texts incorporate the most recent scientific advances in courses designed to provide undergraduate students with the foundations of a scientific discipline. An outstanding level of quality is achieved in each of the seven scientific fields taught at the *Ecole*: pure and applied mathematics, mechanics, physics, chemistry, biology, and economics. The uniform level of excellence is the result of the unique selection of academic staff there which includes, in addition to the best researchers in its own renowned laboratories, a large number of world-famous scientists, appointed as part-time professors or associate professors, who work in the most advanced research centers France has in each field.

Another distinctive characteristic of these courses is their overall consistency; each course makes appropriate use of relevant concepts introduced in the other textbooks. This is because each student at the Ecole Polytechnique has to acquire basic knowledge in the seven scientific fields taught there, so a substantial link between departments is necessary. The distribution of these courses used to be restricted to the 900 students at the Ecole. Some years ago we were very successful in making these courses available to a larger French-reading audience. We now build on this success by making these textbooks also available in English.

Preface to the Second Edition

Quantum mechanics is an endless source of new questions and fascinating observations. Examples can be found in fundamental physics and in applied physics, in mathematical questions as well as in the currently popular debates on the interpretation of quantum mechanics and its philosophical implications.

Teaching quantum mechanics relies mostly on theoretical courses, which are illustrated by simple exercises often of a mathematical character. Reducing quantum physics to this type of problem is somewhat frustrating since very few, if any, experimental quantities are available to compare the results with. For a long time, however, from the 1950s to the 1970s, the only alternative to these basic exercises seemed to be restricted to questions originating from atomic and nuclear physics, which were transformed into exactly soluble problems and related to known higher transcendental functions.

In the past ten or twenty years, things have changed radically. The development of high technologies is a good example. The one-dimensional square-well potential used to be a rather academic exercise for beginners. The emergence of quantum dots and quantum wells in semiconductor technologies has changed things radically. Optronics and the associated developments in infrared semiconductor and laser technologies have considerably elevated the social rank of the square-well model. As a consequence, more and more emphasis is given to the physical aspects of the phenomena rather than to analytical or computational considerations.

Many fundamental questions raised since the very beginnings of quantum theory have received experimental answers in recent years. A good example is the neutron interference experiments of the 1980s, which gave experimental answers to 50 year old questions related to the measurability of the phase of the wave function. Perhaps the most fundamental example is the experimental proof of the violation of Bell's inequality, and the properties of entangled states, which have been established in decisive experiments since the late 1970s. More recently, the experiments carried out to quantitatively verify decoherence effects and "Schrödinger-cat" situations have raised considerable interest with respect to the foundations and the interpretation of quantum mechanics.

This book consists of a series of problems concerning present-day experimental or theoretical questions on quantum mechanics. All of these problems are based on actual physical examples, even if sometimes the mathematical structure of the models under consideration is simplified intentionally in order to get hold of the physics more rapidly.

The problems have all been given to our students in the École Polytechnique and in the École Normale Supérieure in the past 15 years or so. A special feature of the École Polytechnique comes from a tradition which has been kept for more than two centuries, and which explains why it is necessary to devise original problems each year. The exams have a double purpose. On one hand, they are a means to test the knowledge and ability of the students. On the other hand, however, they are also taken into account as part of the entrance examinations to public office jobs in engineering, administrative and military careers. Therefore, the traditional character of stiff competitive examinations and strict meritocracy forbids us to make use of problems which can be found in the existing literature. We must therefore seek them among the forefront of present research. This work, which we have done in collaboration with many colleagues, turned out to be an amazing source of discussions between us. We all actually learned very many things, by putting together our knowledge in our respective fields of interest.

Compared to the first version of this book, which was published by Springer-Verlag in 2000, we have made several modifications. First of all, we have included new themes, such as the progress in measuring neutrino oscillations, quantum boxes, the quantum thermometer etc. Secondly, it has appeared useful to include, at the beginning, a brief summary on the basics of quantum mechanics and the formalism we use. Finally, we have grouped the problems under three main themes. The first (Part A) deals with Elementary Particles, Nuclei and Atoms, the second (Part B) with Quantum Entanglement and Measurement, and the third (Part C) with Complex Systems.

We are indebted to many colleagues who either gave us driving ideas, or wrote first drafts of some of the problems presented here. We want to pay a tribute to the memory of Gilbert Grynberg, who wrote the first versions of "The hydrogen atom in crossed fields", "Hidden variables and Bell's inequalities" and "Spectroscopic measurement on a neutron beam". We are particularly grateful to François Jacquet, André Rougé and Jim Rich for illuminating discussions on "Neutrino oscillations". Finally we thank Philippe Grangier, who actually conceived many problems among which the "Schrödinger's cat", the "Ideal quantum measurement" and the "Quantum thermometer", Gérald Bastard for "Quantum boxes", Jean-Noël Chazalviel for "Hyperfine structure in electron spin resonance", Thierry Jolicoeur for "Magnetic excitons", Bernard Equer for "Probing matter with positive muons", Vincent Gillet for "Energy loss of ions in matter", and Yvan Castin, Jean-Michel Courty and Do-

minique Delande for "Quantum reflection of atoms on a surface" and "Quantum motion in a periodic potential".

Palaiseau, April 2005

Jean-Louis Basdevant
Jean Dalibard

Contents

Part II Quantum Entanglement and Measurement

Summary of Quantum Mechanics

In the following pages we remind the basic definitions, notations and results of quantum mechanics.

1 Principles

Hilbert Space

The first step in treating a quantum physical problem consists in identifying the appropriate Hilbert space to describe the system. A Hilbert space is a complex vector space, with a Hermitian scalar product. The vectors of the space are called kets and are noted $|\psi\rangle$. The scalar product of the ket $|\psi_1\rangle$ and the ket $|\psi_2\rangle$ is noted $\langle\psi_2|\psi_1\rangle$. It is linear in $|\psi_1\rangle$ and antilinear in $|\psi_2\rangle$ and one has:

$$\langle\psi_1|\psi_2\rangle = (\langle\psi_2|\psi_1\rangle)^* \ .$$

Definition of the State of a System; Pure Case

The state of a physical system is completely defined at any time t by a vector of the Hilbert space, normalized to 1, noted $|\psi(t)\rangle$. Owing to the superposition principle, if $|\psi_1\rangle$ and $|\psi_2\rangle$ are two possible states of a given physical system, any linear combination

$$|\psi\rangle \propto c_1|\psi_1\rangle + c_2|\psi_2\rangle \ ,$$

where c_1 and c_2 are complex numbers, is a possible state of the system. These coefficients must be chosen such that $\langle\psi|\psi\rangle = 1$.

Measurement

To a given physical quantity A one associates a self-adjoint (or Hermitian) operator $\hat{A}$ acting in the Hilbert space. In a measurement of the quantity A, the only possible results are the eigenvalues a_α of $\hat{A}$.

Consider a system in a state $|\psi\rangle$. The probability $\mathcal{P}(a_\alpha)$ to find the result a_α in a measurement of A is

$$\mathcal{P}(a_\alpha) = \left\| \hat{P}_\alpha |\psi\rangle \right\|^2 ,$$

where $\hat{P}_\alpha$ is the projector on the eigensubspace $\mathcal{E}_\alpha$ associated to the eigenvalue a_α.

After a measurement of $\hat{A}$ which has given the result a_α, the state of the system is proportional to $\hat{P}_\alpha|\psi\rangle$ (wave packet projection or reduction).

A single measurement gives information on the state of the system after the measurement has been performed. The information acquired on the state before the measurement is very "poor", i.e. if the measurement gave the result a_α, one can only infer that the state $|\psi\rangle$ was not in the subspace orthogonal to $\mathcal{E}_\alpha$.

In order to acquire accurate information on the state before measurement, one must use N independent systems, all of which are prepared in the same state $|\psi\rangle$ (with $N \gg 1$) . If we perform N_1 measurements of $\hat{A}_1$ (eigenvalues $\{a_{1,\alpha}\}$), N_2 measurements of $\hat{A}_2$ (eigenvalues $\{a_{2,\alpha}\}$), and so on (with $\sum_{i=1}^{p} N_i = N$), we can determine the probability distribution of the $a_{i,\alpha}$, and therefore the $\| \hat{P}_{i,\alpha}|\psi\rangle \|^2$. If the p operators $\hat{A}_i$ are well chosen, this determines unambiguously the initial state $|\psi\rangle$.

Evolution

When the system is not being measured, the evolution of its state vector is given by the Schrödinger equation

$$\mathrm{i}\hbar \frac{d}{dt}|\psi\rangle = \hat{H}(t)\, |\psi(t)\rangle ,$$

where the hermitian operator $\hat{H}(t)$ is the Hamiltonian, or energy observable, of the system at time t.

If we consider an isolated system, whose Hamiltonian is time-independent, the energy eigenstates of the Hamiltonian $|\phi_n\rangle$ are the solution of the time independent Schrödinger equation:

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle .$$

They form an orthogonal basis of the Hilbert space. This basis is particularly useful. If we decompose the initial state $|\psi(0)\rangle$ on this basis, we can immediately write its expression at any time as: