Fritz Riehle

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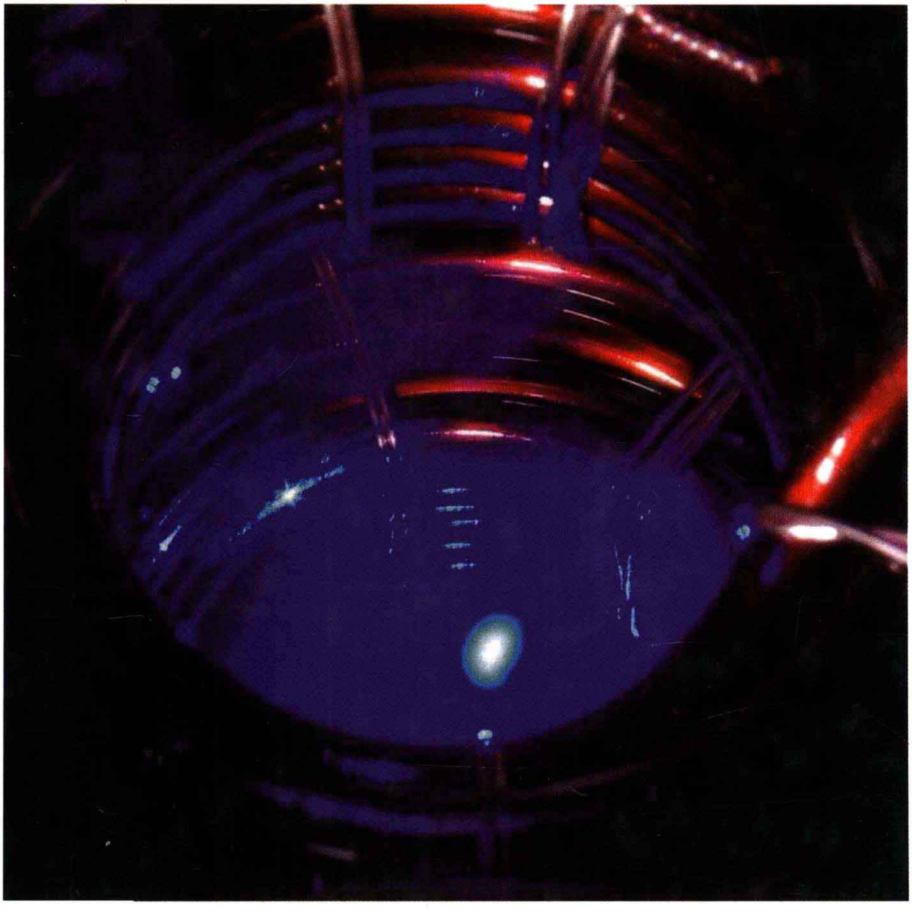
# Frequency Standards

Basics and Applications



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## Basics and Applications

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Dr. Fritz Riehle
Physikalisch-Technische Bundesanstalt, Braunschweig
Germany
e-mail: fritz.riehle@ptb.de



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Fritz Riehle

# Frequency Standards

Basics and Applications

**To the memory of my parents**

**and to Hildegard and Ruth**

# Preface

The contributions of accurate time and frequency measurements to global trade, traffic and most sub-fields of technology and science, can hardly be overestimated. The availability of stable sources with accurately known frequencies is prerequisite to the operation of world-wide digital data networks and to accurate satellite positioning, to name only two examples. Accurate frequency measurements currently give the strongest bounds on the validity of fundamental theories. Frequency standards are intimately connected with developments in all of these and many other fields as they allow one to build the most accurate clocks and to combine the measurements, taken at different times and in different locations, into a common system.

The rapid development in these fields produces new knowledge and insight with breath-taking speed. This book is devoted to the basics and applications of frequency standards. Most of the material relevant to frequency standards is scattered in excellent books, review articles, or in scientific journals for use in the fields of electrical engineering, physics, metrology, astronomy, or others. In most cases such a treatise focusses on the specific applications, needs, and notations of the particular sub-field and often it is written for specialists. The present book is meant to serve a broader community of readers. It addresses both graduate students and practising engineers or physicists interested in a general and introductory actual view of a rapidly evolving field. The volume evolved from courses for graduate students given by the author at the universities of Hannover and Konstanz. In particular, the monograph aims to serve several purposes.

First, the book reviews the basic concepts of frequency standards from the microwave to the optical regime in a unified picture to be applied to the different areas. It includes selected topics from mechanics, atomic and solid state physics, optics, and methods of servo control. If possible, the topics which are commonly regarded as complicated, e.g., the principles and consequences of the theory of relativity, start with a simple physical description. The subject is then developed to the required level for an adequate understanding within the scope of this book.

Second, the realisation of commonly used components like oscillators or macroscopic and atomic frequency references, is discussed. Emphasis is laid not only on the understanding of basic principles and their applications but also on practical examples. Some of the subjects treated here may be of interest primarily to the more specialised reader. In these cases, for the sake of conciseness, the reader is supplied with an evaluated list of references addressing the subject in necessary detail.

Third, the book should provide the reader with a sufficiently detailed description of the most important frequency standards such as, e.g., the rubidium clock, the hydrogen maser, the caesium atomic clock, ion traps or frequency-stabilised lasers. The criteria for the "impor-

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tance" of a frequency standard include their previous, current, and future impact on science and technology. Apart from record-breaking primary clocks our interest also focusses on tiny, cheap, and easy-to-handle standards as well as on systems that utilise synchronised clocks, e.g., in Global Navigation Satellite Systems.

Fourth, the book presents various applications of frequency standards in contemporary high-technology areas, at the forefront of basic research, in metrology, or for the quest for most accurate clocks. Even though it is possible only to a limited extent to predict future technical evolution on larger time scales, some likely developments will be outlined. The principal limits set by fundamental principles will be explored to enable the reader to understand the concepts now discussed and to reach or circumvent these limitations. Finally, apart from the aspect of providing a reference for students, engineers, and researchers the book is also meant to allow the reader to have intellectual fun and enjoyment on this guided walk through physics and technology.

Chapter 1 reviews the basic glossary and gives a brief history of the development of clocks. Chapters 2 and 3 deal with the characterisation of ideal and real oscillators. In Chapter 4 the properties of macroscopic and in Chapter 5 that of microscopic, i.e., atomic and molecular frequency references, are investigated. The most important methods for preparation and interrogation of the latter are given in Chapter 6. Particular examples of frequency standards from the microwave to the optical domain are treated in Chapters 7 to 10, emphasising their peculiarities and different working areas together with their main applications. Chapter 11 addresses selected principles and methods of measuring optical frequencies relevant for the most evolved current and future frequency standards. The measurement of time as a particular application of frequency standards is treated in Chapter 12. The remainder of the book is devoted to special applications and to the basic limits.

I would like to thank all colleagues for continuous help with useful discussions and for supporting me with all kinds of information and figures. I am thankful to the team of Wiley–VCH for their patience and help and to Hildegard for her permanent encouragement and for helping me with the figures and references. I am particularly grateful to A. Bauch, T. Binnewies, C. Degenhardt, J. Helmcke, P. Hetzel, H. Knöckel, E. Peik, D. Piester, J. Stenger, U. Sterr, Ch. Tamm, H. Telle, S. Weyers, and R. Wynands for careful reading parts of the manuscript. These colleagues are, however, not responsible for any deficiencies or the fact that particular topics in this book may require more patience and labour as adequate in order to be understood. Furthermore, as in any frequency standard, feedback is necessary and highly welcome to eliminate errors or to suggest better approaches for the benefit of future readers.

Fritz Riehle
(fritz.riehle@ptb.de)
Braunschweig
June 2004

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# 1 Introduction

## 1.1 Features of Frequency Standards and Clocks

Of all measurement quantities, frequency represents the one that can be determined with by far the highest degree of accuracy. The progress in frequency measurements achieved in the past allowed one to perform measurements of other physical and technical quantities with unprecedented precision, whenever they could be traced back to a frequency measurement. It is now possible to measure frequencies that are accurate to better than 1 part in $10^{15}$. In order to compare and link the results to those that are obtained in different fields, at different locations, or at different times, a common base for the frequency measurements is necessary. Frequency standards are devices which are capable of producing stable and well known frequencies with a given accuracy and, hence, provide the necessary references over the huge range of frequencies (Fig. 1.1) of interest for science and technology. Frequency standards link the different areas by using a common unit, the hertz. As an example, consider two identical clocks whose

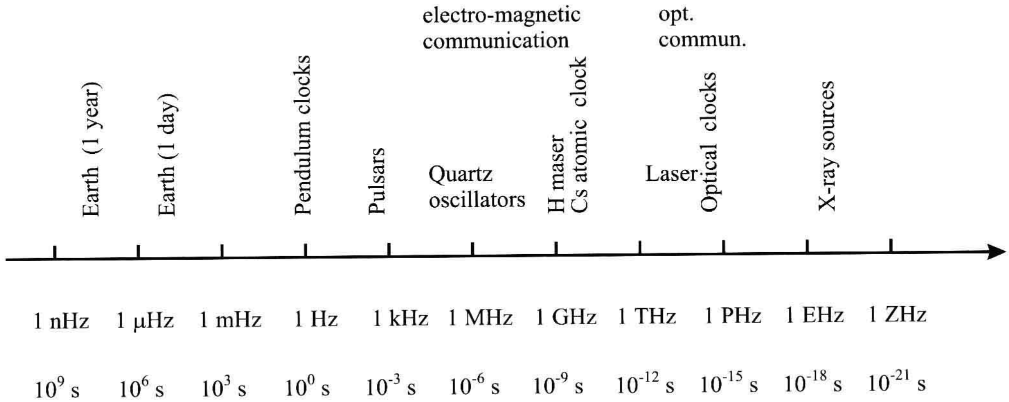


**Figure 1.1:** Frequency and corresponding time scale with clocks and relevant technical areas.

relative frequencies differ by $1 \times 10^{-15}$. Their readings would disagree by one second only after thirty million years. Apart from the important application to realise accurate clocks and time scales, frequency standards offer a wide range of applications due to the fact that numerous physical quantities can be determined very accurately from measurements of related frequencies. A prominent example of this is the measurement of the quantity *length*. Large distances are readily measured to a very high degree of accuracy by measurement of the time interval that a pulse of electromagnetic waves takes to traverse this distance. Radar guns used

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by the police represent another example where the quantity of interest, i.e. the speed of a vehicle is determined by a time or frequency measurement. Other quantities like magnetic fields or electric voltages can be related directly to a frequency measurement using the field-dependent precession frequency of protons or using the Josephson effect, allowing for exceptionally high accuracies for the measurement of these quantities.

The progress in understanding and handling the results and inter-relationships of celestial mechanics, mechanics, solid-state physics and electronics, atomic physics, and optics has allowed one to master steadily increasing frequencies (Fig. 1.1) with correspondingly higher accuracy (Fig. 1.2). This evolution can be traced from the mechanical clocks (of resonant

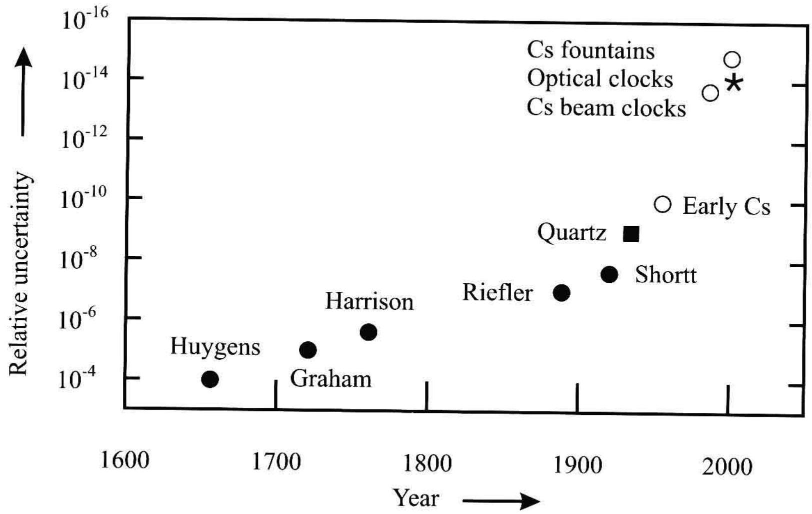


**Figure 1.2:** Relative uncertainty of different clocks. Mechanical pendulum clocks (full circles); quartz clock (full square); Cs atomic clocks (open circles); optical clocks (asterisk). For more details see Section 1.2.

frequencies $\nu_0 \approx 10^0$ Hz) via the quartz and radio transmitter technology ($10^3$ Hz $\leq \nu_0 \leq 10^8$ Hz), the microwave atomic clocks ($10^8$ Hz $\leq \nu_0 \lesssim 10^{10}$ Hz) to today's first optical clocks based on lasers ($\nu_0 \lesssim 10^{15}$ Hz). In parallel, present-day manufacturing technology with the development of smaller, more reliable, more powerful, and at the same time much cheaper electronic components, has extended the applications of frequency technology. The increasing use of quartz and radio controlled clocks, satellite based navigation for ships, aircraft and cars as well as the implementation of high-speed data networks would not have been possible without the parallel development of the corresponding oscillators, frequency standards, and synchronisation techniques.

Frequency standards are often characterised as active or passive devices. A "passive" frequency standard comprises a device or a material of particular sensitivity to a single frequency or a group of well defined frequencies (Fig. 1.3). Such a frequency reference may be based on macroscopic resonant devices like resonators (Section 4) or on microscopic quantum systems (Section 5) like an ensemble of atoms in an absorber cell. When interrogated by a suitable oscillator, the frequency dependence of the frequency reference may result in an absorption

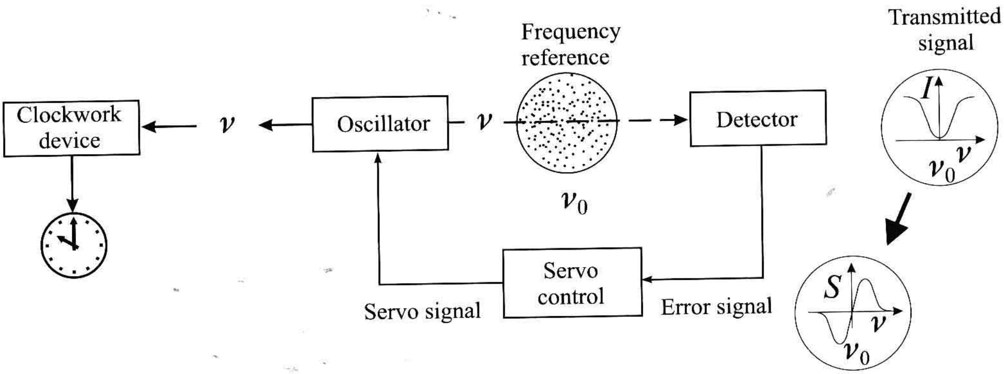


**Figure 1.3:** Schematics of frequency standard and clock.

line with a minimum of the transmission at the resonance frequency $\nu_0$. From a symmetric absorption signal $I$ an anti-symmetric error signal $S$ may be derived that can be used in the servo-control system to generate a servo signal. The servo signal acting on the servo input of the oscillator is supposed to tune the frequency $\nu$ of the oscillator as close as possible to the frequency $\nu_0$ of the reference. With a closed servo loop the frequency $\nu$ of the oscillator is "stabilised" or "locked" close to the reference frequency $\nu_0$ and the device can be used as a frequency standard provided that $\nu$ is adequately known and stable.

In contrast to the passive standard an "active" standard is understood as a device where, e.g., an ensemble of excited atomic oscillators directly produces a signal with a given frequency determined by the properties of the atoms. The signal is highly coherent if a fraction of the emitted radiation is used to stimulate the emission of other excited atoms. Examples of active frequency standards include the active hydrogen maser (Section 8.1) or a gas laser like the He-Ne laser (Section 9.1).

A frequency standard can be used as a clock (Fig. 1.3) if the frequency is suitably divided in a clockwork device and displayed. As an example consider the case of a wrist watch where a quartz resonator (Section 4.1) defines the frequency of the oscillator at 32 768 Hz = $2^{15}$ Hz that is used with a divider to generate the pulses for a stepping motor that drives the second hand of the watch.

The specific requirements in different areas lead to a variety of different devices that are utilised as frequency standards. Despite the various different realisations of frequency standards for these different applications, two requirements are indispensable for any one of these devices. First, the frequency generated by the device has to be stable in time. The frequency, however, that is produced by a real device will in general vary to some extent. The variation may depend, e.g., on fluctuations of the ambient temperature, humidity, pressure, or on the operational conditions. We value a "good" standard by its capability to produce a stable frequency with only small variations.

A stable frequency source on its own, however, does not yet represent a frequency standard. It is furthermore necessary that the frequency $\nu$ is known in terms of absolute units. In the internationally adopted system of units (Systéme International: SI) the frequency is measured in units of Hertz representing the number of cycles in one second (1 Hz = 1/s). If the frequency of a particular stable device has been measured by comparing it to the frequency of another source that can be traced back to the frequency of a primary standard [1] used to *realise* the SI unit, our stable device then – and only then – represents a frequency standard.

After having fulfilled these two prerequisites, the device can be used to calibrate other stable oscillators as further secondary standards.

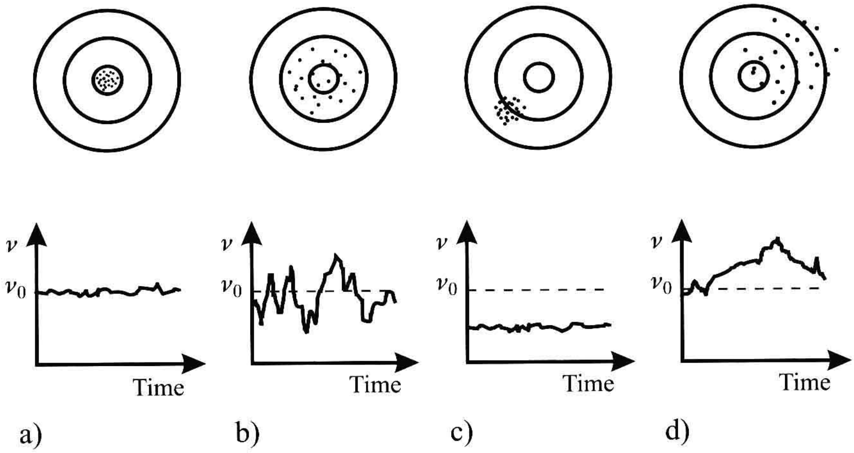


**Figure 1.4:** Bullet holes on a target (upper row) show four different patterns that are precise and accurate (a), not precise but accurate (b), precise but not accurate (c), not precise and not accurate (d). Correspondingly a frequency source (lower row) shows a frequency output that is stable and accurate (a), not stable but accurate (b), stable but not accurate (c), and not stable and not accurate (d).

There are certain terms like stability, precision, and accuracy that are often used to characterise the quality of a frequency standard. Some of those are nicely visualised in a picture used by Vig [2] who compared the temporal output of an oscillator with a marksman's sequence of bullet holes on a target (Fig. 1.4). The first figure from the left shows the results of a highly skilled marksman having a good gun at his disposal. All holes are positioned accurately in the centre with high precision from shot to shot. In a frequency source the sequence of firing bullets is replaced by consecutive measurements of the frequency $\nu$, where the deviation of the frequency from the centre frequency $\nu_0$ corresponds to the distance of each bullet hole from the centre of the target.[2] Such a stable and accurate frequency source may be used as a frequency standard. In the second picture of Fig. 1.4 the marks are scattered with lower precision but enclosing the centre accurately. The corresponding frequency source would suffer from reduced temporal stability but the mean frequency averaged over a longer period would be accurate. In the third picture all bullet holes are precisely located at a position off the centre. The corresponding frequency source would have a frequency offset from the desired

[1] A primary frequency standard is a frequency standard whose frequency correponds to the adopted definition of the second , with its specified accuracy achieved without external calibration of the device [1].

[2] The distances of bullet holes in the lower half plane are counted negative.