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ROBUST CONTROL DESIGN 2000

*A Proceedings volume from the 3rd IFAC Symposium
Prague, Czech Republic, 21 - 23 June 2000*

Edited by

V. KUČERA and M. ŠEBEK

Volume 2



PERGAMON

ROBUST CONTROL DESIGN 2000 (ROCOND 2000)

*A Proceedings volume from the 3rd IFAC Symposium,
Prague, Czech Republic, 21 – 23 June 2000*

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Foreword

The 3rd IFAC ROCOND Symposium on Robust Control Design took place in Hotel Renaissance, Prague, Czech Republic during June 21 – 23, 2000.

The ROCOND symposia have developed from an IFAC conference on Control System Design, which was held in Zürich, Switzerland in 1991. The 1st IFAC ROCOND Symposium was organized in Rio de Janeiro, Brazil in 1994. The series then continued by the 2nd IFAC ROCOND Symposium in Budapest, Hungary in 1997.

Following the tradition, the aim of ROCOND 2000 was to bring together the robust control community to discuss the trends in the field and to present new methods and applications.

The technical program included 21 sessions on robust control and related topics in identification and signal processing. The methods presented in these sessions included linear matrix inequalities, polynomial techniques, sliding modes, optimal control, fuzzy and adaptive control. Attention was paid to linear as well as nonlinear systems.

The highlights of the technical program were two plenary lectures by world famous experts in the field: Robust Control and Filtering Design for Discrete-Time Systems, by J. Geromel (Universidade Estadual de Campinas, Brazil) and H₂-optimization: Theory and Application to Robust Control Design, by H. Kwakernaak (Twente University, Netherlands).

Very well attended was the invited application session on Parameter-Space Tools for Robust Control, organized by J. Ackermann (German Aerospace Research Establishment, Oberpfaffenhofen). Part of the Symposium was an Europoly Workshop, which included 3 sessions of high quality papers on the theory and applications of polynomial design methods in control and signal processing.

It is my pleasure to congratulate U. Shaked (Tel Aviv University, Israel), the chairman of the International Program Committee and M. Šebek (Czech Technical University in Prague, Czech Republic), the chairman of the National Organizing Committee for their achievements in preparing and running the Symposium. I hope that the robust control community enjoyed this event and already look forward to the 4th IFAC ROCOND Symposium to be held in Milano, Italy in the year 2003.

Vladimír Kučera
Editor

CONTENTS

VOLUME 1

POLYNOMIAL METHODS, SINGULAR PERTURBATIONS

A Polynomial Approach to l_1 Optimal Control Problems A. CASAVOLA, D. FAMULARO	1
Necessary and Sufficient Conditions for Robust Positivity of Polynomic Functions via Sign Decomposition C. ELIZONDO-GONZALEZ	7
An LMI Condition for Robust Stability of Polynomial Matrix Polytopes D. HENRION, D. ARZELIER, D. PEAUCELLE, M. ŠEBEK	13
Descriptor- and Non-Descriptor Controllers in H_∞ Control of Descriptor Systems A. REHM, F. ALLGÖWER	19
Stabilization and Robust Stability of Singularly Perturbed Systems I. HODAKA, M. SUZUKI	25
A Linear Matrix Inequality Approach to the Peak-to-Peak Guaranteed Cost Filtering Design R.M. PALHARES, P.L.D. PERES, J.A. RAMÍREZ	29

FILTERING

H_∞ Filtering of Continuous-Time Linear Systems with Multiplicative Noise E. GERSHON, U. SHAKED, I. YAESH, D.J.N. LIMEBEER	35
Disturbance Attenuation in Luenberger Function Observer Designs – A Parametric Approach G.R. DUAN, G.P. LIU, S. THOMPSON	41
H_∞ Bounds for Wiener Filtering and State-Feedback Control with Actuator Disturbances P. BOLZERN, P. COLANERI, G. DE NICOLAO, U. SHAKED	47
Unbiased Robust H_∞ Filtering by Means of LMI Optimisation S. BITTANTI, F.A. CUZZOLA	53
An H_∞ Technique for the Reconstruction of the Primary Current in Industrial Transformers S. BITTANTI, F.A. CUZZOLA, G. CORBETTA	59
Robust H_∞ Filter Design via Parameter-Dependent Lyapunov Functions C.E. de SOUZA, M. FU, A. TROFINO	65

POSTER PAPERS I

Design of Stable Observer Based Controllers for Robust Pole Assignment G.P. LIU, G.R. DUAN, S. DALEY	71
The Robust and Reliable Servo System P.M.G. FERREIRA	77
Robust Model Reference Control for Multivariable Linear Systems: A Parametric Approach G.R. DUAN, W.Q. LIU, G.P. LIU	83
H_∞ and Robust Control of Hybrid Stochastic Systems with Jumps of State Vector P.V. PAKSHIN	89

The Calculation of Stability Radius with D Stability Region and Non-Linear Coefficients Y. WANG, K.J. HUNT	95
Evaluation of Robust Performance for Interval Systems Based on Characteristic Roots Area Y. OKUYAMA, F. TAKEMORI	101
Polyhedral Regions of Robust Local Stability for Linear Systems with Saturating Controls B.E.A. MILANI, A.D. COELHO, W.O. ASSIS	107
On a Coefficientwise Stability Margin Analysis for Perturbed Polynomials J. BONDIA, J. PICÓ	113

LMI TECHNIQUES

Robust Analysis and Synthesis of SISO Linear Systems Subject to Simultaneous Perturbations Z. DUAN, L. HUANG, L. WANG	119
Synthesis of a Static Anti-Windup Compensator via Linear Matrix Inequalities M. SAEKI, N. WADA	125
Synthesis of a Static Anti-Windup Compensator for Systems with Magnitude and Rate Limited Actuators N. WADA, M. SAEKI	131
Robust Servo System Design by Mixed H_2/H_∞ Control Application to Three-Inertia Benchmark Problem Y-W. CHOE	137
Flexible Arm Multiobjective Control via Youla Parameterization and LMI Optimization B. CLEMENT, G. DUC	143
Robust Redundancy Determination and Evaluation of the Dual Variables of Linear Programming Problems in the Presence of Uncertainty I. IOSLOVICH, P.-O. GUTMAN	149

SLIDING MODES

Sliding-Mode Estimators for a Class of Nonlinear Systems A. ALESSANDRI	155
Multi-Model Switching Control of a Class of Nonlinear Systems Using Sliding Modes M.L. CORRADINI, T. LEO, G. ORLANDO	161
Adaptive Output Tracking Backstepping Sliding Mode Control of Nonlinear Systems A.J. KOSHKOEI, A.S.I. ZINOBER	167
Maple Design of Dynamical Sliding Adaptive Backstepping Controllers J.C. SCARRATT, R.E. MILLS, A.S.I. ZINOBER	173
Robust Regulation via Sliding Modes of a Rotary Inverted Pendulum L.E. RAMOS, J. RUIZ-LEÓN, S. ČELIKOVSKÝ	179

POSTER PAPERS II

The Definition of Column Reduced Λ -Generalized Polynomial Matrices J. RUIZ-LEÓN, A. CASTELLANOS	185
H_2 Analysis and Control of Parameter Dependent Systems via LMIS and Parameter Dependent Lyapunov Functions J. de OLIVEIRA, C.E. de SOUZA, A. TROFINO	191
Improved Classification Algorithm for the Counter Propagation Network L. KOVÁCS, G. TERSTYÁNSZKY	197
Simulation Results of a Stabilization of an Inverted Pendulum by Filtered Dynamic Output Feedback R. POTHIN, S. ČELIKOVSKÝ, C.H. MOOG	201

Time-Domain Robust Stability Test Under Plant and Controller Interval Uncertainty P. CUGUERO, S. TORNIL, T. ESCOBET, J. SALUDES, V. PUIG	207
Robust, Multi-Objective Control for Real Uncertainties via Parameter-Dependent Lyapunov Functions D. ARZELIER, D. PEAUCELLE	213
Synthesis of a Robust Controller and Fault Detection Filter for an Unstable System A. CARDOSO, A. DOURADO	219
A Constrained Global Optimization Algorithm with Application to PI Robust Synthesis D. FAMULARO, P. PUGLIESE, Ya.D. SERGEYEV	225
Active Suspension System by H_2 and H_∞ : A Weights Choice Procedure for Vehicle Performance E. ABDELLAHI, D. MEHDI, M. M'SAAD	231

SYSTEMS WITH DELAY

Robust Sampled-Data H_∞ Control of Systems with State Delays E. FRIDMAN, U. SHAKED	237
Dynamical Output Feedback Controller for Uncertain Continuous-Time Time-Delay Systems Containing Saturating Actuator D. MEHDI, Q.-L. HAN	243
Robust H_∞ Filtering for Linear Continuous-Time Uncertain Systems with Multiple Delays: An LMI Approach R.M. PALHARES, P.L.D. PERES, C.E. de SOUZA	249
Predictor-Based Solution to the H^∞ Control of Dead-Time Systems L. MIRKIN	255
Stabilization of Linear Discrete Time Delay Systems with Additive Disturbance and Saturating Actuators S. TARBOURIECH, G. GARCIA, P.L.D. PERES, I. QUEINNEC	261

IDENTIFICATION

Identification and Control of Non-Stationary Time Delayed Systems P. DRÉANO, R. LAURENT	267
Towards a Generalized Robust Analysis by Fictitious Identification C. MANCEAUX-CUMER	273
Robust Servo Control Design Using Identified Models P. GÁSPÁR, I. SZÁSZI	279
Two-Stage and Projection Methods for Closed-Loop Identification Using Generalized Orthonormal Basis Functions P. GÁSPÁR, Z. SZABÓ, J. BOKOR	285
Controller Order Reduction by Direct Closed Loop Identification (Output Matching) A. KARIMI, I.D. LANDAU	291
Closed-Loop Parametrization Schemes: Identification of the K - B and d - Y Parameters L. KEVICZKY, Cs. BÁNYÁSZ	297

ROBUST CONTROL I

On the Set-Point Regulation of Uncertain Nonminimum-Phase Scalar Systems A. PIAZZI, A. VISIOLI	303
Some Preliminary Results on Robust Reflection Coefficients Placement Ü. NURGES	309

η - Robust Systems Synthesis via Phase Sets Variation V.N. PILISHKIN	317
Asymptotic Tracking with Infinite Gain Margin Through Linear Periodic Control S. GALEANI, O.M. GRASSELLI, L. MENINI	323
Unmatched Uncertainties in Robust LQ Control D. KROKAVEC, A. FILASOVÁ	329
The H_2 -Control for Markovian Jump Linear Systems with Cluster Observations of the Markov State J.B.R. do VAL, J.C. GEROMEL	335

ROBUST CONTROL II

On the Robustness of a Family of Discontinuous Controllers E. VALTOLINA, A. ASTOLFI	341
Lateral Acceleration Flight Control for a Quasi-Linear Parameter Varying Missile Model via Pseudolinearisation A. TSOURDOS, R. ŻBIKOWSKI, B.A. WHITE	347
Stability Analysis of a Class of Nonlinear Uncertain Systems via Pseudo Time Approach S. FARAHANI, M.J. YAZDANPANAH	355
Robust Interval-Based SISO Regulation of an Anaerobic Reactor V. ALCARAZ-GONZÁLEZ, J. HARMAND, J.P. STEYER, A. RAPAPORT, V. GONZÁLEZ-ALVAREZ, PELAYO-ORTIZ, C.	361
Polynomial-Time Random Generation of Uniform Real Matrices in the Spectral Norm Ball G. CALAFIORE, F. DABBENE	367

ROBUST CONTROL III

Simultaneous Stabilization of Two Plants with a Strictly Proper Compensator C. FONTE, M. ZASADZINSKI, C. BERNIER-KAZANTSEV	373
Robust Control of LTI Square MIMO Plants Using Two Crone Control Design Approaches P. LANUSSE, A. OUSTALOUP, B. MATHIEU	379
Gain Scheduled Control for Discrete-Time Systems Depending on Bounded Rate Parameters F. AMATO, M. MATTEI, A. PIRONTI	385
Robust Strategies for Trajectory Tracking of Nash Linear Quadratic Games F. AMATO, M. MATTEI, A. PIRONTI	391
Loop Gain Identification for Adaptive and Reconfigurable Control M. PACHTER, J. SILENCE	397
Deviations Estimates for Uncertain Time-Varying Discrete-Time Systems P. ORŁOWSKI	403

VOLUME 2

NONLINEAR SYSTEMS

Bi-Quadratic Stability for Nonlinear Systems A. TROFINO	409
Robust H_2 Performance of LPV Systems via Parameter Dependent Lyapunov Function J. de OLIVEIRA, A. TROFINO, C.E. de SOUZA	415

Robust Regulation for a Class of Dynamical Systems G. OBREGÓN-PULIDO, B. CASTILLO-TOLEDO, S. CELIKOVSKY	421
Performance Enhancement and Robustness for Linear Systems with Saturating Actuators R. REGINATTO, A.R. TEEL, E.R. De PIERI	427
Parametric Tuning of Control Systems via Phase Constraints Deformation Approach V.N. PILISHKIN	433

PLENARY PAPER

H_2 -Optimization – Theory and Applications to Robust Control Design H. KWAKERNAAK	437
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H_∞ AND H_2 CONTROL

On the Conditioning of the Mixed H_2/H_∞ -Optimisation Problem N.D. CHRISTOV, S. LESECQ, A. BARRAUD, M.M. KONSTANTINOV, P.Hr. PETKOV, D.W. GU	449
Robust Controller Design by Output Feedback against Uncertain Stochastic Disturbances C.W. SCHERER	453
Observer-Based Quadratic Stabilization with Dominant Pole Placement K. SUGIMOTO	459
On Upper Bounds on Worst-Case H_2 Performance for Unstructured Model Perturbations G.O. CORRÊA, A. CHAUBAH	465
Computation of Proper and Reduced Order Controllers via H^∞ Techniques P. DOBRA, D. MOGA	471
Approximate Optimization of H_∞ PID Controller on the Frequency Domain M. SAEKI, K. AIMOTO	477

APPLICATION I

Minimax LQG Optimal Control of a Flexible Beam I.R. PETERSEN, H.R. POTA	483
Design of μ -Optimal Robust Controller for a Thermoplastic Process A. KACHAŇAK, R. KRBAT'A, J. BELANSKÝ	489
Control of a Flexible Link Using H_∞ Design Methods D. FARRUGGIO, L. MENINI	495
A User Friendly Toolbox for the Analysis of Interval Systems N. TAN, D.P. ATHERTON	501
Robust Control of a Simple Mechanical System Subject to Impacts M. INDRI, A. TORNAMBÈ	507
Underactuated Manipulator Robot Control by State Feedback Linearization via H_∞ M.H. TERRA, B.C.O. MACIEL, P.H.R. NAKASHIMA, M. BERGERMAN	513

PARAMETER-SPACE TOOLS FOR ROBUST CONTROL

Mapping of Nyquist/Popov Theta-Stability Margins into Parameter Space T. BÜNTE	519
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Solving Nonlinear Parametric Mapping Equations D. KAESBAUER	525
Mapping of Frequency Response Performance Specifications into Parameter Space D. ODENTHAL, P. BLUE	531
Representing Multiple Objectives in Parameter Space Using Color Coding M. MUHLER, J. ACKERMANN	537
Parametric Systems Toolbox: A Tool for Analysis and Design of Uncertain Systems E. KONTOGIANNIS, N. MUNRO	543
Fast Calculation of Stabilizing PID Controllers for Uncertain Parameter Systems N. MUNRO, M.T. SOYLEMEZ	549
Robust Performance S.I. Engine Idle-Speed Control by a Mixed Sensitivity Parameter-Space Method A.T. SHENTON, V. BESSON	555
ROBAN: A Parameter Robustness Analysis Tool and Its Flight Control Applications F. AMATO, L. VERDE	561

APPLICATION II

Robust Control of Web Transport Systems H. KOÇ, D. KNITTEL, M. de MATHELIN, G. ABBA	567
Synthesis and Analysis of Robust Flux Observers for Induction Machines M. HILAIRET, C. DARENGOSSE, F. AUGER, P. CHEVREL	573
H_∞ Control for a Variable-Speed Stall-Regulated Wind Turbine Drive System R. ROCHA, P. RESENDE, J.L. SILVINO, M.V. BORTOLUS	579
Design and Implementation of μ Controllers for Real Industrial Plants M.F. MIRANDA, F.G. JOTA	585
When Quadratic Stability Meets PID Control M. MATTEI	591

FUZZY, QFT AND ADAPTIVE CONTROL

Plant Template Generation for Time-Delay Systems in Quantitative Feedback Control Theory C. HWANG, S.-F. YANG	597
Automatic Loop-Shaping of QFT Robust Controllers via Genetic Algorithms M. GARCÍA-SANZ, J.C. GUILLÉN	603
Design of PLL for Carrier Tracking in Uncertain OFDM Systems O. YANIV, D. RAPHAELI	609
Control of Unknown Order Uncertain Systems I. RUSNAK	615

APPLICATION III

Design of the Universal Digital Flight Controller for an Aircraft with not Exactly Known Parameters V.D. YURKEVICH, M.J. BŁACHUTA, K. WOJCIECHOWSKI	621
A Concept for Reduction of Structure Activation with Robust Control B. SATTLER	627
Periodic H_∞ Attitude Control for Satellites with Magnetic Actuators M. LOVERA	631

Robust Regulation via Sliding Modes of a Helicopter Model L.E. RAMOS, S. ČELIKOVSKÝ, V. KUČERA	637
Generalized Internal Model Control for a Single Flexible Beam D.U. CAMPOS-DELGADO, Y-P. HUANG, K. ZHOU	645

EUROPOLY I

A Descriptor Algorithm for the Spectral Factorization of Polynomial Matrices H. KWAKERNAAK	651
A Mixed $\text{GH}_2/\text{H}_\infty$ Approach for Stabilisation and Accurate Trajectory Tracking of Unicycle-Like Vehicles S. BITTANTI, F.A. CUZZOLA	657
Closed-Loop Identification of a Laboratory Chemical Reactor S. KOZKA, J. MIKLES, M. FIKAR, F. JELENCIAK, J. DZIVAK	663
Associated H_∞ -Problem for Sampled-Data Systems K.Y. POLYAKOV, E.N. ROSENWASSER, B.P. LAMPE	669
Robust Linear Controllers for Unstable Systems with Time Delay R. PROKOP, P. DOSTÁL, Z. PROKOPOVÁ	675
On the Standard H_2 Problem G. MEINSMA	681
Polynomial Approach to Stable Predictive Control M. FIKAR, H. UNBEHAUEN, J. MIKLEŠ	687
Inversion of Loudspeaker Dynamics by Polynomial LQ Feedforward Control M. STERNAD, M. JOHANSSON, J. RUTSTRÖM	693

EUROPOLY II

Robust Stability of Discrete Polynomials with Polynomic Structure of Coefficients R. DVOŘÁKOVÁ, P. HUŠEK, C. ELIZONDO	699
Robust Stability and Control Issues for NARX Models A. DZIELIŃSKI	705
H_2 Optimal Control via Pole Placement V. KUČERA, D. HENRION	711
A Separation Theorem for Polynomial Feedback Systems R. YLINEN	717
Fast Fourier Transform and Robustness Analysis with Respect to Parametric Uncertainties M. HROMČÍK, M. ŠEBEK	723
Polynomial Design of Simple Controllers for Time Delay Systems P. DOSTÁL, R. PROKOP, V. BOBÁL	727
LMIs for Linear Systems Control by Polynomial Methods D. HENRION	733

EUROPOLY III

Computation of the Set of Positive Solutions to Polynomial Matrix Equations T. KACZOREK, R. ŁOPATKA	739
Reachability and Controllability of 2D Positive Linear Systems with State Feedbacks T. KACZOREK	745

Robust Polynomial Assignment for Uncertain Periodic Discrete-Time Systems S. BITTANTI, P. COLANERI, V. KUČERA	751
Polynomial Toolbox 2.5 and Systems with Parametric Uncertainties M. ŠEBEK, M. HROMČIK, J. JEŽEK	757
Design of Dynamic Controllers for Electromechanical Hamiltonian Systems W. HAAS, K. SCHLACHER	763
Auto-Tuning of Predictive PI Controller I. GANCHEV, M. PETROV, K. HYNIOVA, A. STRIBRSKY	769
Output Robust Controller Design for Linear Parametric Uncertain Systems V. VESELÝ	775
Author Index	781

BI-QUADRATIC STABILITY FOR NONLINEAR SYSTEMS*

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Abstract: This paper deals with the robust stability of nonlinear systems with real time varying parameters having both magnitude and rate of variation which are confined to a given polytope. The system matrices may have entries which are rational functions of the states and uncertain parameters. LMI conditions are given, when they are feasible, they guarantee the asymptotic stability of the origin of the system through a Lyapunov function of the type $v(x, \delta) = x'P(x, \delta)x$ where the matrix function $P(x, \delta)$ depends quadratically on the states (x) and uncertain parameters (δ). The problem of maximizing an estimate of the region of attraction is also presented and a numerical example is used to show the potential of the proposed results. *Copyright ©2000 IFAC*

1. INTRODUCTION

Due to its nice properties, the LMI framework has been largely used to solve many robust control and filtering problems. In the context of linear uncertain systems many important results are now available (Boyd et al 1994). However, much work has to be done in order to extend these results to nonlinear systems.

In this last few years some interesting LMI based results for nonlinear systems have appeared in the literature. See for instance (Blanchini 1999), (El Ghaoui and Scorletti 1995).

For nonlinear systems, the following two aspects may contribute to the conservativeness of the existing LMI based results.

The first one is that LMIs, for nonlinear systems, are state dependent, and conditions of the type $x'P(x, \delta)x > 0$ cannot be directly replaced by the state & parameter dependent LMI $P(x, \delta) > 0$ without a great deal of conservatism.

Another potential source of conservatism is the use of quadratic Lyapunov functions for nonlinear systems. In general, quadratic Lyapunov functions

are suited to show the stability in a small neighbourhood of the origin. If the size of the region of attraction is an important issue, more complex Lyapunov functions could lead to larger estimates of the region of attraction. Of course, it comes at the expense of a more intensive computation.

In this paper we present results to reduce these two potential conservative aspects listed above. It is shown how to reduce the conservativeness of state dependent LMIs and, instead of quadratic Lyapunov functions, it is utilized a more general function of the type $x'P(x, \delta)x$ where $P(x, \delta)$ is a quadratic matrix function of (x, δ) .

The problem of concern consists of analysing the local stability with respect to a given polytopic neighbourhood of the origin under the assumption that the uncertain parameters and their respective rate of variation, are bounded by a given polytope that represents their admissible values. The problem of enlarging an estimate of the region of attraction inside the polytopic neighbourhood is also addressed and LMI solutions are proposed. The class of systems considered in this paper is described by a model in which the system matrices are allowed to have entries which are rational functions of the states and uncertain parameters. This is a large class of systems that includes

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the rational systems treated in (El Ghaoui and Scorletti 1995) and (Sasaki and Uchida 1997).

The paper is organized as follows. The next section is devoted to some preliminaries. Some definitions are presented and Lemma 2.1 shows how to reduce the conservativeness of state dependent LMIs. The problem of concern and the class of systems we deal with are presented in section 3. The stability results are stated in section 4 based on a notion called bi-quadratic stability. In section 5 we present the results for enlarging an estimate of the region of attraction and an example is presented to illustrate the results. Some concluding remarks end the paper. Due to space limitation, some results, examples and remarks and most part of the references were removed. We refer the reader to the full version of this paper for more details.

2. PRELIMINARIES

Let us start with some definitions.

Consider the system

$$\dot{x} = A(\delta, x)x \quad (1)$$

where $A(\delta, x)$ denotes a matrix function of x, δ which are the state and uncertain parameter vectors respectively. Let $\mathcal{B}_x, \mathcal{B}_\delta$ be given polytopes representing, respectively, a neighbourhood of the origin and the admissible values of the uncertain parameters. It is assumed that the right handside of (1) is bounded $\forall x \in \mathcal{B}_x, \delta \in \mathcal{B}_\delta$. Depending on the context, the polytope \mathcal{B}_δ will represent the admissible values of δ and its variation rate $\dot{\delta}$. In this case we use the notation $(\delta, \dot{\delta}) \in \mathcal{B}_\delta$. The notation $(x, \delta, \dot{\delta}) \in \mathcal{B}$ means that $x \in \mathcal{B}_x$ and $(\delta, \dot{\delta}) \in \mathcal{B}_\delta$.

The notation used in this paper is standard. I_r and 0_r denote the $r \times r$ identity and zero matrices. The matrices dimensions will not be specified when the dimensions can be deduced from the context. $A > 0$ (≥ 0) means that A is a symmetric positive definite (semi-definite) matrix. The time derivative of a function $R(t)$ will be denoted by $\dot{R}(t)$ and the argument (t) will be always omitted.

The problem of concern in this paper is to analyse the local stability of the equilibrium point $x = 0$ system (1). The stability notion that will be used is referred to as Bi-quadratic stability.

Definition 2.1. (Bi-Quadratic stability). Let $\mathcal{B}_x, \mathcal{B}_\delta$ be given polytopes.

The origin of the system (1) is said to be *locally bi-quadratically stable* if there exist class \mathcal{K} functions $\phi_1(\cdot), \phi_2(\cdot)$ and a function of the type $v(x, \delta) = x'P(x, \delta)x$, where $P(x, \delta)$ is a quadratic matrix function of (x, δ) , such that the following conditions are satisfied $\forall (\delta, \dot{\delta}) \in \mathcal{B}_\delta, \forall x \in \mathcal{B}_x$:

- $v(x, \delta) = x'P(x, \delta)x \geq \phi_1(\|x\|)$
- $\dot{v}(x, \delta) = x'\dot{P}(x, \delta)x + x'A(x, \delta)'P(x, \delta)x + x'P(x, \delta)A(x, \delta)'x \leq -\phi_2(\|x\|)$

In the affirmative case, $v(x, \delta) = x'P(x, \delta)x$ is said to be a local Lyapunov function for the origin.

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Note that Bi-Quadratic stability implies asymptotic stability of the origin and it may be viewed as an extension, to nonlinear systems, of a stability notion introduced in (Trofino and de Souza 1999) for the linear case. The interest of this definition is that the Lyapunov function employs a state¶meter dependent matrix instead of a fixed one used in the quadratic stability notion. Moreover, the usual quadratic stability definition can be recovered as a special case. As usual in nonlinear systems theory, once we have a Lyapunov function $v(x, \delta)$ we may get an estimate of the region of attraction of the equilibrium point. In section 5 we present techniques to solve the problem of maximizing the size of the region of attraction to be estimated.

Now we turn our attention to another problem that arises in the analysis of nonlinear system via LMI methods.

As previously mentioned, the resulting LMIs for nonlinear systems are state dependent and this may lead the standard techniques developed for linear systems to fail if they are directly applied to nonlinear systems. For instance, the condition $x'P(x, \delta)x > 0$, may be tested with the powerful algorithms developed for LMIs if $P(x, \delta)$ is affine in (x, δ) . In this case we just need to test whether the LMI condition $P(x, \delta) > 0$ is satisfied or not $\forall \delta \in \mathcal{B}_\delta, x \in \mathcal{B}_x$, i.e. at the vertices of the meta polytope $\mathcal{B} = \mathcal{B}_\delta \times \mathcal{B}_x$. However, due to the LMI state dependence, this last LMI condition is, in general, too conservative to be used in lieu of $x'P(x, \delta)x > 0$. To illustrate this fact, let us consider the nonlinear function $g(x) = x'H(x)x$ where $x = [x_1 \ x_2]'$ and

$$H(x) = \begin{bmatrix} 1 + x_2 & -\frac{x_1 + x_2}{2} \\ -\frac{x_1 + x_2}{2} & 1 + x_1 \end{bmatrix} \quad (2)$$

It is simple to verify that $g(x) = x_1^2 + x_2^2$ and thus $g(x) > 0 \forall x \neq 0$. However, the condition $H(x) > 0 \forall x \neq 0$ is not satisfied. The following lemma presents a less restrictive LMI test to check if the condition $x'H(x)x > 0$ holds $\forall x \in \mathcal{B}_x, x \neq 0$.

Lemma 2.1. Let $H(z)$ be a matrix whose entries are affine functions of $z = [z_1 \dots z_{n_z}]' \in \mathbb{R}^{n_z}$ and \mathcal{B}_z be a given convex polytope. Consider the nonlinear scalar function $z'H(z)z$ and the auxiliary matrix function $C_z : \mathbb{R}^{n_z} \mapsto \mathbb{R}^{(n_z-1) \times n_z}$ indicated below.

$$C_z = \begin{bmatrix} z_2 & -z_1 & 0 & 0 & \dots & 0 \\ 0 & z_3 & -z_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & z_{n_z} & -z_{n_z-1} \end{bmatrix} \quad (3)$$

Then the condition $z'H(z)z > 0$, $\forall z \in \mathcal{B}_z$, $z \neq 0$ is satisfied if there exists a matrix L , with the same dimension of C'_z , such that the following LMI condition is satisfied

$$H(z) + LC_z + C'_z L' > 0, \quad \forall z \in \mathcal{B}_z \quad (4)$$

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The proof is simple and may be viewed as an application of the Finsler lemma (Boyd et al 1994) if we notice that, by construction, we have $C_z z = 0, \forall z$. A formal proof may be found in (Trofino 2000).

To illustrate the interest of lemma 2.1, let us return to the nonlinear function $g(x) = x'H(x)x$ with $H(x)$ given by (2). It was shown that despite $g(x) > 0 \forall x \neq 0$, the condition $H(x) > 0 \forall x \neq 0$ is not satisfied. Using the Lemma 2.1 with $C_x = [x_2 \ -x_1]$ and the following choice for $L = [-0.5 \ 0.5]'$ we may show that indeed $g(x) > 0 \forall x \neq 0$ because $H(x) + LC_x + C'_x L' = I_2 > 0 \forall x$. Although the conditions of the lemma 2.1 are only sufficient, for this particular example they are not conservative.

3. SYSTEM REPRESENTATION

Let us consider that the system (1) may be rewritten in a equivalent form as indicated below.

$$\begin{cases} \dot{x} = A(\delta, x)x = \sum_{i=0}^q A_i \pi_i, & \pi_0 = x \\ \pi_{i+1} = \Theta_i(\delta, x) \pi_i, & i = 0, \dots, r-1 \\ \Omega(\delta, \pi) \pi = 0 \end{cases} \quad (5)$$

where $x \in \mathbb{R}^{m_0}$ denotes the state vector; $\delta \in \mathbb{R}^{n_\delta}$ the vector of uncertain parameters; $\pi_i \in \mathbb{R}^{m_i}$, $\pi = [\pi'_0 \dots \pi'_q] \in \mathbb{R}^m$ are auxiliary functions of (x, δ) representing the nonlinearities of the system; $\Theta_i(\delta, x) \in \mathbb{R}^{m_{i+1} \times m_i}$, for $i = 0, \dots, r-1$ ($r \leq q$) are affine function matrices of (δ, x) used to express the relations between the vectors π_i ; $A_i \in \mathbb{R}^{m_0 \times m_i}$, for $i = 0, \dots, q$ are fixed matrices of structure, and $\Omega(\delta, \pi) \in \mathbb{R}^{m_\Omega \times m}$ is an affine function matrix of (δ, π) used to express additional relations between the vectors π_i .

To simplify the notation we always use π_i, Θ_i, Ω without expliciting their respective dependence on x, δ , and time.

Hereafter we assume that the right handside of the diffential equation in (5) is bounded for all values of π of interest and that the system representation in terms of the auxiliary variable π is equivalent to

the representation $\dot{x} = A(\delta, x)x$. In other words we are assuming that the auxiliary variable π may be eliminated from the expressions and if this is done we recover the original system representation $\dot{x} = A(\delta, x)x$.

The system representation (5) may be used to represent a large class of uncertain nonlinear systems of the type $\dot{x} = A(\delta, x)x$.

For instance, the class of bi-linear system where $A(x) = \sum_{i=1}^{m_0} \tilde{A}_i x_i$, where $x_i \in \mathbb{R}$ are the entries of x and \tilde{A}_i are fixed matrices, may be written in the form (5) with the choice $q = 1$, $A_1 = [\tilde{A}_1 \dots \tilde{A}_{m_0}]$, $\Theta_0(x) = [x_1 I_{m_0} \dots x_{m_0} I_{m_0}]'$. Notice that, in general, the system representation in terms of the π variables is not unique. The dimension of the π vector plays an important role in the results of the next sections. In general, as the dimension of the π vector increases, the conservativeness decreases because more decision variables to be tuned are introduced in the underlying problem. However, this came at the expense of a more intensive computation.

The case where $A(x, \delta)$ is a polynomial function of (x, δ) may be also represented as in (5). This is shown in the example 5.1. In the following we consider the case where $A(\delta, x)$ is a rational function of (x, δ) .

Example 3.1. A simple fermentation process in a batch reactor may be described by the equations (Bastin and Douchain 1990)

$$\dot{x}_b = \mu x_b, \quad \dot{s} = -k_1 \mu x_b, \quad \mu = \frac{\mu_0 s}{k_m + s + \frac{s^2}{k_1}} \quad (6)$$

where the states are the biomass concentration x_b and the substrate concentration s ; μ is the population growth rate and k_1, k_m, μ_0 are constants of the fermentation process. Although we may consider these constants as uncertainties, for simplicity in the presentation we assume these constants are known.

In order to represent the system (6) as in (5) we may define $r = 0$ and

$$\Theta_0 = [0 \ s], \quad x = \pi_0 = \begin{bmatrix} x_b \\ s \end{bmatrix}, \quad \pi_1 = \Theta_0 \pi_0$$

$$\pi_2 = \mu \pi_0, \quad \pi' = [x' \ \pi_1' \ \pi_2' \ \mu]$$

From the expression of μ we get $\mu k_m + \mu s + \mu \frac{s^2}{k_1} - \mu_0 s = 0$ which in turn yields the representation (5) as follows:

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ -k_1 & 0 \end{bmatrix} \pi_2, \quad \pi_1 = \Theta_0 \pi_0$$

$$\Omega = \begin{bmatrix} \mu I_2 & 0 & -I_2 & 0 \\ [0 \ -\mu_0] & \mu/k_1 & [0 \ 1] & k_m \end{bmatrix}$$

4. BI-QUADRATIC STABILITY RESULTS

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In order to present the results, some considerations and some auxiliary notations are needed.

Notice from (5) that Θ_i are affine function matrices of (x, δ) . In particular, the matrix Θ_0 plays an important role in bi-quadratic stability methods since the Lyapunov function will be Θ_0 dependent, as can be seen from definition 2.1. Let us represent Θ_0 as follows:

$$\Theta_0 = \sum_{i=1}^{m_0} T_i x_i + \sum_{i=1}^{n_\delta} U_i \delta_i + V \quad (7)$$

where T_i, U_i, V are constant matrices of structure having the same dimensions of Θ_0 and x_i, δ_i are the entries of the vectors x and δ respectively.

Theorem 4.1. Consider the uncertain nonlinear system (5), the matrix C_x obtained from the definition (3), the representation of Θ_0 from (7), let E_i be the i -th row of I_{m_0} and the auxiliary notation

$$\begin{aligned} \tilde{\Theta}_0 &= \sum_{i=1}^{m_0} T_i x E_i, \quad \hat{\Theta}_0 = \sum_{i=1}^{n_\delta} U_i \delta_i \\ A_{a_1} &= \begin{bmatrix} A_0 & A_1 \\ \hat{\Theta}_0 + (\Theta_0 + \tilde{\Theta}_0)A_0 & (\Theta_0 + \tilde{\Theta}_0)A_1 \end{bmatrix} \\ A_{a_2} &= \begin{bmatrix} A_2 & \dots & A_q \\ (\Theta_0 + \tilde{\Theta}_0)A_2 & \dots & (\Theta_0 + \tilde{\Theta}_0)A_q \end{bmatrix} \\ C_{a_1} &= \begin{bmatrix} C_x & 0 \\ \Theta_0 & -I_{m_1} \end{bmatrix} \end{aligned} \quad (8)$$

$$C_{a_2} = \begin{bmatrix} \Omega & & & & \\ C_x & 0 & \dots & 0 \\ \Theta_0 & -I_{m_1} & 0 & \dots & 0 \\ 0 & \Theta_1 & -I_{m_2} & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \Theta_{r-2} & -I_{m_{r-1}} & 0 \\ 0 & \dots & 0 & \Theta_{r-1} & -I_{m_r} \end{bmatrix}$$

Let \mathcal{B} be a given convex polytope and suppose that $(\pi, \delta, \dot{\delta}) \in \mathcal{B}$. Then the system (5) is locally Bi-Quadratically stable if there exist matrices P, L_1, L_2 , with the same dimensions of $A_{a_1}, C'_{a_1}, C'_{a_2}$ respectively, such that the following LMIs are satisfied for $(\pi, \delta, \dot{\delta})$ at the vertices of the polytope \mathcal{B} .

$$\begin{bmatrix} A'_{a_1}P + PA_{a_1} & PA_{a_2} \\ A'_{a_2}P & 0 \end{bmatrix} + L_2 C_{a_2} + C'_{a_2} L_2' < 0 \quad (9)$$

$$P + L_1 C_{a_1} + C'_{a_1} L_1' > 0$$

In the affirmative case, the function $v(x, \delta) = x' \mathcal{P}(x, \delta) x$ with $\mathcal{P}(x, \delta)$ given below is a local Lyapunov function for the system (5).

$$\mathcal{P}(x, \delta) = \begin{bmatrix} I_{m_0} \\ \Theta_0(x, \delta) \end{bmatrix}' P \begin{bmatrix} I_{m_0} \\ \Theta_0(x, \delta) \end{bmatrix} \quad (10)$$

Proof: Consider the system (5) and define the following partition of the π vector:

$$\xi_1 = \begin{bmatrix} x \\ \pi_1 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} \pi_2 \\ \vdots \\ \pi_q \end{bmatrix}, \quad \pi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \quad (11)$$

Notice that, by construction, we have $C_x x = 0$. Moreover, the constraint $C_{a_1} \xi_1 = 0$ is equivalent to $\pi_1 = \Theta_0 \pi_0$ and $C_x x = 0$. Similarly, the constraint $C_{a_2} \pi = 0$ is equivalent to $\pi_{i+1} = \Theta_i \pi_i$, $C_x x = 0$ and $\Omega \pi = 0$. Except for the equality constraint $C_x x = 0$, which is always satisfied by construction, the remaining equalities are used to define the system representation in (5).

Suppose now the conditions of the Theorem 4.1 are satisfied. Then there exist a sufficient small positive scalar ϵ_1 such that $P + L_1 C_{a_1} + C'_{a_1} L_1' - \epsilon_1 I > 0$ still holds. In a similar way it is possible to add a term $\epsilon_2 I$ to the first LMI in (9). Now take the first LMI of (9) modified with the added term $\epsilon_2 I$. Pre and post multiply it by π' and its transpose respectively. Subsequently pre and post multiply $P + L_1 C_{a_1} + C'_{a_1} L_1' - \epsilon_1 I > 0$ by ξ_1' and its transpose respectively. These two operations lead the following to be satisfied.

$$\begin{aligned} \xi_1' P \xi_1 - \epsilon_1 \xi_1' \xi_1 &> 0, \quad \forall (\pi, \delta, \dot{\delta}) \in \mathcal{B} : C_{a_1} \xi_1 = 0 \\ \xi_1' (A'_{a_1} P + P A_{a_1}) \xi_1 + \xi_1' P A_{a_2} \xi_2 + \xi_2' A'_{a_2} P \xi_1 + \epsilon_2 \pi' \pi &< 0, \quad \forall (\pi, \delta, \dot{\delta}) \in \mathcal{B} : C_{a_2} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = 0 \end{aligned} \quad (12)$$

Since $\xi_1' = [x' \pi_1'] = [x' x' \Theta_0']$ we get $\|\xi_1\| \geq \|x\|$ and thus $v(x, \Theta_0) = \xi_1' P \xi_1 = x' \mathcal{P}(x, \delta) x > \epsilon_1 \|x\|^2$, $\forall (\pi, \delta, \dot{\delta}) \in \mathcal{B}$, $x \neq 0$, where $\mathcal{P}(x, \delta)$ is the state¶meter dependent matrix indicated in (10). Note in addition that $\|\pi\| \geq \|x\|$.

Now we use the equalities

$$\begin{aligned} \xi_1' P A_{a_1} \xi_1 &= x' \mathcal{P}(x, \delta) \begin{bmatrix} A_0 & A_1 \end{bmatrix} \xi_1 + \\ &+ x' \begin{bmatrix} I_{m_0} \\ \Theta_0 \end{bmatrix}' P \begin{bmatrix} 0 \\ \dot{\Theta}_0 \end{bmatrix} x \end{aligned}$$

$$\xi_1' P A_{a_2} \xi_2 = x' \mathcal{P}(x, \delta) \begin{bmatrix} A_2 & \dots & A_q \end{bmatrix} \xi_2$$

to show that the following holds.

$$\begin{aligned} \xi_1' (A'_{a_1} P + P A_{a_1}) \xi_1 + \xi_1' P A_{a_2} \xi_2 + \xi_2' A'_{a_2} P \xi_1 = \\ x' \dot{\mathcal{P}}(x, \delta) x + x' \mathcal{P}(x, \delta) \left(\sum_{i=0}^q A_i \pi_i \right) + \\ \left(\sum_{i=0}^q A_i \pi_i \right)' \mathcal{P}(x, \delta) x \end{aligned}$$

The above results together with (12) show that $\dot{v}(x, \delta) < -\epsilon_2 \|x\|^2$. Thus $v(x, \delta) = x' \mathcal{P}(x, \delta) x$ is a Lyapunov function for the system (5) and therefore its origin is locally bi-quadratically stable.

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