Mathematical Foundation for Computer Science

Alpha Science

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Dedicated to

In memory of *Mr. Durai Boopathy*, *Mr. Suganthan*

&

To my parents, parents-in-Law, beloved husband *D. R. Pockisam*

lovable babies

V. P. Jeya Harini, *A. G. Shruthi*, *B. Sophia*, *B. Oliviya*

and family members

PREFACE

Mathematical concepts have become the primary requirement in education and researcn .. science.

Books giving details of Introduction to Mathematical Concepts are available. However, many persons from science find the detailed course difficult, and consequently they conclude that mathematical concepts are the subjects of concern of engineers, the mathematicians and the physicists.

As a computer individual, I know the limitations of the teachers and the students of non-mathematical science. This book deals with elementary principles of the subject. All basic concepts and definitions introduced in the book are motivated from concrete examples with which most of the readers are familiar so that the abstract ideas and definitions become natural. Each concept is illustrated with a number of examples and solved problems. The book ends with the collection of revision and solved questions.

I have tried my best to optimize that the book should sufficiently cover the all the details and that too within the limitations of the teachers and students of science.

I am confident if they go step by step they will not find the mathematical concepts as difficult subjects.

My sincere and heartfelt thanks are to Mr. N.K. Mehra, Publisher and Managing Director, Narosa Publishing House Pvt. Ltd, for giving ready consent to publish this book.

Sky is the limit for perfection. Hence, I request the readers to come forward with their suggestions, which will go a long way in stepping up the quality of my work.

Wish you all good luck.

M. Vasanthi

CONTENTS

CHAPTER 1

LOGIC

1.1 INTRODUCTION

LOGIC (from the Greek λογική logikē) is the study of arguments. Logic is used in most intellectual activities, but is studied primarily in the disciplines of philosophy, mathematics, and computer science. Logic examines general forms which arguments may take which forms are valid, and which are fallacies. It is one kind of critical thinking. In philosophy, the study of logic figures in most major areas of focus: epistemology, ethics, and metaphysics. In mathematics, it is the study of valid inferences within some formal language.

It deals with the methods of reasoning. It provides rules and techniques for determining whether a given argument is valid or not. Logic is concerned with studying arguments and conclusions. A systematic study of arguments by making extensive use of symbols is known as symbolic logic. The algebra of symbolic logic represents one of the early applications of Boolean algebra.

In this unit, we introduce logical symbols for connectives such as "**and**", "**or**", "**if ... then...**" and with the aid of these symbols we state and apply rules of valid inference.

1.2 TF STATEMENTS

A tf- statement or a proposition is a declarative sentence to which it is meaningful to assign a truth value "True" or "False", but not both simultaneously. The two truth values are "true" and "false" and are denoted by the symbols 1 and 0.

If we cannot properly assign a definite truth value to a declarative statement then it is called as self-contradictory statement.

Examples

A hen has two legs	-	True
10 is a negative float	-	False
The moon is coolest	-	True
There are 32 days in a month	-	False
If you study hard then you will pass	-	True
Obey Your Parents	-	Command

When will you complete your degree?	-	Question (Not a declarative Statement)
A square has three sides	-	False
Triangle has three sides	-	True
Delhi is very hot	-	Depends on the Season
Everyone like icecream	-	Depends on the Person
Today is Monday	-	Self - Contradictory
In summer, day is shorter than night	-	False
This statement is false	-	Self – Contradictory
Do you like Coke?	-	Question (Not a declarative Statement)
Oh! How beautiful it is!	-	Exclamation
This Statement is unprovable	-	Self – Contradictory
Physics is a dull subject	-	Depends on the person

1.3 CONNECTIVES

For the systematic usage of certain keywords, "and", "or", "not", "if…then…", "if and only if", which are called sentential connectives. We should follow some rules to use them. The tf-statements (propositions) are represented by the letters p, q, r…., p1, p2, p3…..

1.3.1 Conjunction

The process of joining two statements p and q using "***and***" produces a new statement, denoted by $\mathbf{p \wedge q}$, which has the truth value T whenever both p and q have the truth value T, the truth value F otherwise. The statement $\mathbf{p \wedge q}$ is called the conjunction of the statements p and q.

Truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example 1: Consider the following statements:

P: I will buy a VCD

Q: I will buy a Bike

Then $P \wedge Q$ is the following statement

$P \wedge Q$: I will buy a VCD and I will buy a Bike

Example 2: Consider the following statements:

P: A dog has four legs

Q: He is a chain smoker

Then $P \wedge Q$ is the following statement

$P \wedge Q$: A dog has four legs and he is a chain smoker.

1.3.2 Disjunction

The process of joining two statements p and q using "*or*" produces a new statement, denoted by **$p \vee q$**, which has the truth value T whenever either p or q has the truth value T, the truth value F otherwise. The statement **$p \vee q$** is called the disjunction of the statements p and q.

Truth table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 1: Consider the following statements:

P: I will buy a VCD

Q: I will buy a Bike

Then $P \vee Q$ is the following statement

$P \vee Q$: I will buy a VCD or I will buy a Bike

Example 2: Consider the following statements:

P: A dog has four legs

Q: He is a chain smoker

Then $P \vee Q$ is the following statement

$P \vee Q$: A dog has four legs or he is a chain smoker.

Note: To form $P \wedge Q$ and $P \vee Q$, the statements P and Q need not be related to each other in one way or other.

1.3.3 Negation

The negation of a statement p is the statement obtained from p by prefixing the words "It is not true that". The negation of p is denoted by $\neg p$ and read as "Not p".

Truth table

P	$\neg P$
T	F
F	T

Example 1: Consider the following statement

P: Delhi is in Africa

Then **$\neg P$** is the following statement

$\neg P$: It is not true that Delhi is in Africa

(Or)

$\neg$ P: It is false that Delhi is in Africa

(Or)

$\neg$ P: Delhi is not in Africa

Note: The negation $\neg$ P of P is denoted by "**~P**" or "**not P**"

1.3.4 Conditional Statement

Let p and q be two given statements. The statement "**if p then q**", denoted by **p → q** is defined by the following Truth table.

Truth table

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Example 1: Consider the following statements:

P: Today is Monday

Q: It is raining.

Then P → Q is the following statement

P → Q: If today is Monday then it is raining.

Example 2: Consider the following statements:

P: $7 > 3$

Q: 10 is a prime number

Then P → Q is the following statement

P → Q: If $7 > 3$ then 10 is a prime number

Note 1:

The statement p → q is called a conditional statement or implication. The statement p is called the antecedent and q is called the consequent

Note 2: We shall also write p → q for

- p only if q
- q if p
- q provided that p
- q is implied by p
- p implies q
- p is a sufficient condition for q
- q is a necessary condition for p

1.3.5 Biconditional Statement

Let p and q be two given statements. The statement "**p if and only if q**" and abbreviated as "**p if q**" or "**q is necessary and sufficient for p**", denoted by **p ↔ q** is defined by the following Truth table.

Truth table

P	Q	P ↔ Q
T	T	T
T	F	F
F	T	F
F	F	T

Example 1: Consider the following statements:

P: The auditorium will be kept open
Q: There is cultural program.
Then P ↔ Q is the following statement
P ↔ Q: The auditorium will be kept open if and only if there is cultural program.

Example 2: Consider the following statements:

P: $7 > 3$
Q: 10 is a prime number
Then P ↔ Q is the following statement
P ↔ Q: $7 > 3$ iff 10 is a prime number

SOLVED PROBLEMS

1. Write the following statements in symbolic form, with P for 'Asha is smart' and q for 'Veena is smart'.
   a. Asha is smart and Veena is not smart.
   b. Asha and Veena are both smart.
   c. Neither Asha nor Veena is smart.
   d. It is not true that Asha and Veena are both smart.

Solution

Statements:

P: Asha is smart ┐P: Asha is not smart
Q: Veena is smart ┐Q: Veena is not smart.

Symbolic Form:

a. $P \wedge \neg Q$
b. $P \wedge Q$

c. $\neg P \wedge \neg Q$
d. $\neg(P \wedge Q)$

2. Write the following statements in symbolic form
   a. If either John takes Mathematics or Naveen takes Physics, then Lucy will take English.
   b. If it is raining, then we will not meet today.
   c. Jack and Jill went up the Hill
   d. There is something wrong with the bulb or with the wiring
   e. If I am not in mood, then I will go to a movie
   f. I will go to a movie only if I will not study today.
   g. If the sun is shining, I shall play tennis this afternoon.
   h. The sun is shining, and I shall play tennis this afternoon.

Solution

a. **If** either John takes Mathematics **or** Naveen takes Physics, **then** Lucy will take English.
Statements:

P: John takes Mathematics
Q: Naveen takes Physics
R: Lucy will take English

Symbolic Form:

$(P \vee Q) \rightarrow R$

b. **If** it is raining, **then** we will not meet today.
Statements:

P: It is raining
Q: We will meet today
$\neg$ Q: We will not meet today

Symbolic Form:

$P \rightarrow \neg Q$

c. Jack **and** Jill went up the Hill
Statements:

P: Jack went up the Hill
Q: Jill went up the Hill

Symbolic Form:

$P \wedge Q$

d. There is something wrong with the bulb **or** with the wiring
Statements:

P: There is something wrong with the bulb
Q: There is something wrong with the wiring

Symbolic Form:

$P \vee Q$

e. **If** I am not in mood, then I will go to a movie

Statements:

P: I am in Mood

┐P: I am not in Mood

Q: I will go to a Movie

Symbolic Form:

$\neg P \rightarrow Q$

f. I will go to a movie **only if** i will not study today.

Statements:

P: I will go to a movie

Q: I will study today

┐Q: I will not study today

Symbolic Form:

$P \rightarrow \neg Q$

g. **If** the sun is shining, I shall play tennis this afternoon.

Statements:

P: The sun is shining

Q: I shall play tennis this afternoon

Symbolic Form:

$P \rightarrow Q$

h. The sun is shining, **and** I shall play tennis this afternoon.

Statements:

P: The sun is shining

Q: I shall play tennis this afternoon

Symbolic Form:

$P \wedge Q$

3. Let p, q, r denote the following statements

p: Triangle ABC is isosceles

q: Triangle ABC is equilateral

r: Triangle ABC is equiangular

Translate each of the following into a statement in English statement.

1. $q \leftrightarrow p$
2. $\neg p \leftrightarrow \neg q$
3. $q \leftrightarrow r$

4. $p \wedge \neg q$
5. $r \rightarrow p$
6. $q \vee p$

Solution

1. $q \leftrightarrow p$

p: Triangle ABC is isosceles

q: Triangle ABC is equilateral

$q \leftrightarrow p$:

Triangle ABC is Equilateral ***if and only if*** it is isosceles.

2. $\neg p \leftrightarrow \neg q$

p: Triangle ABC is isosceles

q: Triangle ABC is equilateral

$\neg p \leftrightarrow \neg q$**:**

Triangle ABC is not isosceles ***if and only if*** it is not equilateral.

3. $q \leftrightarrow r$

q: Triangle ABC is equilateral

r: Triangle ABC is equiangular

$q \leftrightarrow r$**:**

Triangle ABC is equilateral ***if and only if*** it is equiangular.

4. $p \wedge \neg q$

p: Triangle ABC is isosceles

q: Triangle ABC is equilateral

$p \wedge \neg q$**:**

Triangle ABC is isosceles ***and*** it is not equilateral.

5. $r \rightarrow p$

p: Triangle ABC is isosceles

r: Triangle ABC is equiangular

$r \rightarrow p$**:**

If the triangle ABC is equiangular ***then*** it is isosceles.

6. $q \vee p$

p: Triangle ABC is isosceles

q: Triangle ABC is equilateral

$q \vee p$**:**

Triangle ABC is equilateral ***or*** isosceles

4. Let p stand for "Prices are high" and q for "Wages are rising". Translate the following into verbal form.
   1. $p \wedge q$
   2. $\neg p \wedge \neg q$
   3. $\neg(p \wedge q)$
   4. $p \vee \neg q$
   5. $\neg(\neg p \vee \neg q)$

Solution

Statements:

p: Prices are high
q: Wages are rising

Verbal Form:

1. $p \wedge q$
Prices are high ***and*** wages are rising.

2. $\neg p \wedge \neg q$
Prices are not high ***and*** wages are not rising.

3. $\neg(p \wedge q)$
It is not true that Prices are high ***and*** wages are rising.

4. $p \vee \neg q$
Prices are high ***or*** wages are not rising.

5. $\neg(\neg p \vee \neg q)$
It is not true that Prices are not high ***or*** wages are not rising.

5. Let p denote "It is cold",

   q denote "7 + 3 = 100" and

   r denote "It rains". Write the following statements in symbolic form.
   1. It is cold only if 7 + 3 = 100
   2. A necessary condition for it is to be cold is that 7 + 3 = 100.
   3. A sufficient condition for it to be cold is that 7 + 3 = 100.
   4. It rains and 7 + 3 is not 100.
   5. It never rains when 7 + 3 = 100.

Solution

Statements:

p: It is cold
q: 7 + 3 = 100
r: It rains

Symbolic Form:

1. It is cold only if 7 + 3 = 100

   $p \to q$
2. A necessary condition for it is to be cold is that 7 + 3 = 100.

   $p \to q$
3. A sufficient condition for it to be cold is that 7 + 3 = 100.

   $q \to p$
4. It rains and 7 + 3 is not 100.

   $r \wedge \neg q$
5. It never rains when 7 + 3 = 100.

   $q \to \neg r$

6. Give the negation of the following statements.
   a. $8 + 7 \leq 20$
   b. It is cold
   c. London is a city
   d. He is a good student
   e. 2 is an even integer and 8 is an odd integer
   f. If u drives then I will walk.
   g. 2 is even and -5 is negative.

Solution

Negation Statements:

a. $8 + 7 \leq 20$
   1. It is not true that $8 + 7 \leq 20$.
   2. $8 + 7 > 20$
   3. 8 + 7 not equal to 20.
   4. It is not false that $8 + 7 \leq 20$.

b. It is cold
   1. It is not cold.
   2. It is false that it is cold
   3. It is not true that it is cold

c. London is a city
   1. London is not a city
   2. It is false that London is a city
   3. It is not true that London is a city

d. He is a good student
   1. He is not a good student
   2. It is false that he is a good student
   3. It is not true that he is a good student