OPERATIONS RESEARCH: PROCESS AND STRATEGY

David S. Stoller

SCIENCE SURVEYS: 2

Operations Research: Process and Strategy

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OPERATIONS RESEARCH:
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SCIENCE SURVEYS

(*General Editor:* Edwin F. Beckenbach, Professor of Mathematics, University of California, Los Angeles)

1: Analysis and Synthesis of Linear Time-Variable Systems

2: Operations Research: Process and Strategy

ERRATA

OPERATIONS RESEARCH: PROCESS AND STRATEGY

Page 33, first equation: The third expression under the limit operator should read $\lambda/k = c$.

Page 33, last equation: The exponent $1/z$ should read $-1/z$.

Page 149, second equation: The right-hand side should read 17 tons of B instead of 25 tons of B.

PREFACE

Over the past few years I have had occasion to lecture on selected topics in operations research to mature and highly motivated audiences. The auditors represented diverse disciplines and backgrounds: production engineers, government officials, traffic engineers, military officers, industrial engineers, graduate students in business administration, and so forth. I share the conviction of these professional groups that the practitioner of operations research should be as interested in the business, governmental, or military operations that he is studying as he is in the scientific methods employed. I have therefore attempted to maintain a level of exposition that demands only a general technical background, rather than one requiring more specialized training. With this approach in mind, I have tried to arrange the material in this book so that only an introductory knowledge of calculus, statistics, probability, and theory of equations is required. Occasionally, more advanced material appears at the end of some of the chapters. This material may be skipped by the reader without loss of continuity.

A brief indication of the history of operations research and a discussion of the nature of operations research problems is given in Part I—Characteristics of Operations Research—chapters 1–3.

Out of the wealth of scientific theory and applications that have come to be associated with the field of operations research, material has been selected to illustrate two fundamental types of operations research problems. One type of problem may be characterized as pertaining to the operations of a relatively passive *system, in that the factors affecting the system are considered to be either those controllable by the management, or are considered to be statistical in nature and not subject to purposive control. This category would include the operations of such systems as production lines, repair shops, traffic flow, etc. Part II—Servicing—chapters 4–13, treats a variety of operational problems for which the concepts of the theory of queuing have been found to be appropriate as a theoretical framework.*

Another type of problem may be characterized as pertaining to the operations of a relatively nonpassive system, in that the factors affecting the system may include not only those which are statistical in nature but also those under the control of two or more decision-making entities, whose interests are partially or completely in conflict. This category would include the *operations of such systems as military engagements, bidding by competing firms, etc. Part III—Strategy—chapters 14–23, treats a variety of operational problems for which the theory of games of strategy has provided concepts appropriate as a theoretical framework. Because of the intimate connection between the theory of games and the theory of linear programming, a brief exposition of elements of linear programming is given in chapters 21–23.*

A number of colleagues have had positive and beneficial influence on the development of the material in this book, without of course being in any way responsible for any errors or shortcomings which may be found in the present version. My thanks go to Professor Philip M. Morse of the Massachusetts Institute of Technology; Professor Giuseppe Pompilj and Dr. Franca Baldessari of the University of Rome; Professor Morris Asimow and Professor James R. Jackson of the University of California, Los Angeles; Dr. Melvin Dresher and Dr. Albert Madansky of The RAND *Corporation; and Professor Herman Chernoff of Stanford University.*

D. S. S.

CONTENTS

PART I: CHARACTERISTICS OF OPERATIONS RESEARCH

Chapter 1

INTRODUCTION

Operations Research emerged as an identifiable activity during World War II, as a consequence of the endeavors of scientists of many different disciplines to solve military problems. These problems were often unrelated—at least superficially—to the disciplines of the investigators. After the war, the activity of Operations Research was extended to many business applications.

There has been a steadily growing number of professionals in military, government, and industrial organizations who are known by the title of "Operations Researchers," or an equivalent occupational title.‡ There has been a rapid growth in the number of Operations Research Societies throughout the world. There has been an increasing number of university-level courses of instruction in the subject. Finally, there are now University Departments and Institutes of Operations Research. (It should be borne in mind that established disciplines, such as Statistics, arrived at their present condition of vitality and maturity through just such a number of stages.)

Scientists and engineers have been involved with military activities for at least as long as recorded history. Perhaps the best known individual instance in ancient history occurred in the fourth century B.C., when Hieron, the King of Syracuse, employed Archimedes to devise means of breaking the naval siege of the city, which was then under attack by the Romans.

The scientists and engineers who brought about the development of the submarine, the machine gun, the wireless telegraph, and the airplane all had a profound impact on the nature of warfare. But in all of the instances above, the engineers and scientists were concerned with the *instrumentalities* of war, rather than with the *processes* of war.

The germination of the concept of Operations Research occurred on

‡ Some equivalents: Systems Engineer; Operations Analyst; Systems Analyst; Operational Analyst.

both sides of the Atlantic Ocean during World War I. In England, in the years 1914–1915, F. W. Lanchester attempted to treat military operations quantitatively. He obtained expressions relating the outcome of a battle to both the relative numerical strength of the combatants and their relative firepower. Just as Newton's equations can be said to describe certain fundamental relationships between force, mass, and motion, the "Lanchester Equations" can be said to describe certain fundamental relationships of warfare.

One form of the Lanchester equations states that the average rate of loss of a force in combat is proportional to the size of the opposing force times the firepower effectiveness per engagement of a unit of that force. For example, consider two opposed forces, Red and Blue, consisting of 100 "units" at the start of combat. (A "unit" might be an infantryman, a platoon of infantrymen, a tank, a squadron of aircraft, etc.) Each unit of the Red force will be considered to have a "firepower effectiveness" of 0.04; that is, each unit of Red force can, on the average, render ineffective 0.04 units of the Blue force per engagement. (An engagement might be a volley of fire, a time-interval, a sortie, etc.) Similarly, each unit of the Blue force will be considered, on the average, to have a "firepower effectiveness" of 0.03.

In the first engagement, the Red forces will inflict an average loss of $100 \times 0.04 = 4$ units on the Blue forces, and the Blue forces will inflict an average loss of $100 \times 0.03 = 3$ units on the Red forces. Assuming that the battle continues through a number of engagements until one of the forces has had 25 per cent of its units rendered ineffective, the table on the facing page illustrates the course of the battle.

Note that as the battle progresses, the ratio of Blue losses to Red losses per engagement, initially 33 per cent greater (four to three), increases to 44 per cent greater (3.35 to 2.33). This illustrates Lanchester's principle that for forces of equal size, the initial firepower effectiveness of the superior force is augmented in the course of the battle by losses of the inferior force, so that the rate of termination of the battle is appropriately accelerated.

Different forms of Lanchester's equations have been fitted to actual combat situations, and remarkably good concurrence with the theory has been obtained. A more formal treatment of a form of the Lanchester equations is given in the Appendix at the end of this chapter.

During the period when Lanchester was pioneering military Operations Research in Great Britain, Thomas Alva Edison in America was studying the *process* (rather than the instrumentalities) of antisubmarine warfare. He initiated the compilation of statistics to be used in analyzing maneuvers whereby surface ships could evade and destroy submarines. He devised a war game to be used for simulating problems of naval maneuver. He analyzed the merits of "zig-zagging" as a submarine countermeasure for merchant ships.

It must be noted, however, that the work of Lanchester and Edison did not have any noticeable impact on military operations in World War I, and therefore we must look to a later period—World War II—for the beginnings of effective military Operations Research. (The criterion for effective Operations Research work is that it not only increase

THE COURSE OF A BATTLE

	RED		*BLUE*	
Number of engagements	*Average losses*	*Average surviving force size*	*Average losses*	*Average surviving force size*
0	0	100	0	100
1*	3	97	4	96
2†	2.88	94.12	3.88	92.12
3	2.76	91.36	3.76	88.36
4	2.65	88.71	3.65	84.71
5	2.54	86.17	3.55	81.16
6	2.43	83.74	3.45	77.71
7‡	2.33	81.41	3.35	74.36

* The Red loss is calculated as $100 \times 0.03 = 3$; the Blue loss is calculated as $100 \times 0.04 = 4$.

† The Red loss is calculated as $96 \times 0.03 = 2.88$; the Blue loss is calculated as $97 \times 0.04 = 3.88$, and so on.

‡ The Blue force has been reduced below 75%; the battle terminates.

the body of knowledge about operations but also be effectively utilized by the executives in charge of the operations.)

Since Britain was at war for two years before the United States became involved in the hostilities, it was almost inevitable that the first effective military Operations Research studies occurred there. At the beginning of the war, the Royal Air Force Fighter Command was very much involved with the newly developing radar system of warning against enemy air attacks. The critical problem was to devise means of utilizing this new instrumentality of warfare and to integrate this new system into the older types of warning systems (which then were based principally on sightings from the ground by skilled observers). The scientists assigned to this problem examined the question not only from the point of view of the technical performance of the radars but also from the point of view of the total efficiency of an information-transmitting system. In other words, they considered it not only from the point of view of the operators of the equipment but also from the point of view of the military decision-makers involved in the entire network.

This study was so successful in increasing the operational capability of the aircraft-detection network that within two years after the beginning of the war all three of Britain's military services had acquired formal Operations Research Groups. One of the most publicized of these groups was under the direction of P. M. S. Blackett, F.R.S., Nobel Laureate.

In 1942, following the lead of the Royal Air Force, officers of the U.S. Air Force established Operations Analysis teams in many of the Air Force Commands, the first one being in the Eighth Bomber Command stationed in England. Almost simultaneously, the U.S. Navy formed Operations Research teams (in two organizations—the Naval Ordnance Laboratory and the Tenth Fleet). The main problems attacked by these groups were naval mining operations and antisubmarine warfare. The Army Ground Forces did not make as much use of Operations Research as did the Air Forces and the Navy, although by the end of the war there were a few evaluation groups analyzing ground warfare in the Pacific Theater.

This type of activity was considered to be so valuable by American military leaders that military Operations Research functions were not discontinued at the end of the war. The U.S. Army continued its Operations Research functions through the agency of the Operations Research Office (now called the Research Analysis Corporation). The U.S. Navy established the Operations Evaluations Group under the direction of Professor P. M. Morse of the Massachussetts Institute of Technology. The U.S. Air Force continued to employ Operations Analysis groups as part of the various commands and further established Project RAND, administered by The RAND Corporation, for long-range studies of aerial warfare.

Because of the rapid growth and spread of Operations Research activities in military applications, many people have gained the impression that Operations Research is a type of analysis applicable only to military problems. This is a regrettable misconception. It must be emphasized that, in common with many other instrumentalities of war, Operations Research as a field of application is effective regardless of the context of the application. Just as the radar systems that control military aerial operations are virtually the same as those controlling civil aviation, a scientific analysis of a particular military operation is often virtually identical to an analysis of an industrial operation. The techniques of Operations Research can be used to study problems of "peace" just as well as those of "war," and it is the responsibility of mankind, not of the scientists alone, to determine how much research effort goes into each of these categories.

Some of the techniques of Operations Research were already employed in business applications before World War II. One of the first proponents of business Operations Research in the U.S.A. was Dr. Horace C. Levinson, who began his Operations Research work in the decade of 1920–1930. He applied the methods of science to the problems of commerce—and studied such problems as the relationship between advertising and sales and the relationship between customers' incomes and home locations and types of articles sold. Even earlier than this period, we observe that such pioneers of scientific management as Taylor and Gantt started their work at the end of the nineteenth century. However, just as World War II marked the first large-scale application of military Operations Research methods, the post-World War II era marked the first large-scale nonmilitary employment of Operations Research.

What lies ahead for Operations Research? Many economists feel that Operations Research methods have become increasingly important in the management of large-scale enterprises. It is even visualized that significant contributions can be made by the field of Operations Research to the stabilization and growth of the national economy—far beyond the relatively modest scope of application that most scientists consider to be the province of Operations Research at the present.

Appendix: Supplemental Section on Lanchester's Equations

Each "unit" of the Red force is considered to have a "firepower effectiveness" of value m. That is, the average rate at which one "unit" of the Red force renders ineffective "units" of the opposing Blue force is equal to m. Similarly, the quantity n is defined as the "firepower effectiveness" of a "unit" of the Blue force. That is, the average rate at which one "unit" of the Blue force renders ineffective "units" of the opposing Red force is equal to n.

Therefore at time t, the rate at which the number of effective units in the Red force is changing is given by

$$\frac{dM}{dt} = \begin{cases} -nN & \text{if} \quad MN > 0; \\ 0 & \text{if} \quad MN = 0, \end{cases}$$

where M = number of effective units of the Red force at time t, and
N = number of effective units of the Blue force at time t.

Similarly, for the Blue force,

$$\frac{dN}{dt} = \begin{cases} -mM & \text{if} \quad NM > 0. \\ 0 & \text{if} \quad NM = 0. \end{cases}$$

By differentiating both of the above equations with respect to time, one obtains

$$\frac{d^2M}{dt^2} = -n\frac{dN}{dt},$$

$$\frac{d^2N}{dt^2} = -m\frac{dM}{dt}.$$

By substitution, this system of two simultaneous first-order differential equations can be changed to a simpler system of simultaneous second-order differential equations:

$$\frac{d^2M}{dt^2} = mnM, \quad \text{if} \quad MN > 0; \quad = 0 \quad \text{if} \quad MN = 0;$$

$$\frac{d^2N}{dt^2} = mnN, \quad \text{if} \quad MN > 0; \quad = 0 \quad \text{if} \quad NM = 0.$$

The general solutions for equations of this form are

$$M = a_1e^{-ct} + a_2e^{ct},$$

$$N = b_1e^{-ct} + b_2e^{ct},$$

where

$$c = \sqrt{mn}.$$

First solving for M, by substituting in $dM/dt = -nN$, we obtain

$$-a_1ce^{-ct} + a_2ce^{ct} = -nb_1e^{-ct} - nb_2e^{ct}.$$

This equation must hold for all values of t, whereby

$$b_1 = \sqrt{m/n}\, a_1, \qquad b_2 = -\sqrt{m/n}\, a_2.$$

But also, at $t = 0$,

$$M_0 = a_1 + a_2, \quad \text{where} \quad M_0 = M(0),$$

$$N_0 = b_1 + b_2, \quad \text{where} \quad N_0 = N(0).$$

Solving for a_1 and a_2, and substituting in the general solution, we get

$$M = \frac{M_0\sqrt{m/n} + N_0}{2\sqrt{m/n}} e^{-\sqrt{mn}\,t} + \frac{M_0\sqrt{m/n} - N_0}{2\sqrt{m/n}} e^{\sqrt{mn}\,t},$$

when $MN > 0$.

By symmetry, the solution for N is

$$N = \frac{N_0\sqrt{n/m} + M_0}{2\sqrt{n/m}} e^{-\sqrt{mn}\,t} + \frac{N_0\sqrt{n/m} - M_0}{2\sqrt{n/m}} e^{\sqrt{mn}\,t},$$

when $MN > 0$.

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