



# **symposia on theoretical physics and mathematics**

Lectures presented at the  
1967 Fifth Anniversary Symposium  
of the Institute  
of Mathematical Sciences  
Madras, India

Edited by  
**ALLADI RAMAKRISHNAN**  
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**symposia on  
theoretical  
physics  
and mathematics**

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**8**

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## Introduction

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This volume comprises the lectures given at the Fifth Anniversary Symposium held at the Institute of Mathematical Sciences, Madras, India, during January 1967.

Professor Dallaporta of Padua delivered the inaugural address on the fundamental problem of quasars "whose study appears to hold implications for cosmology." He presented a critical review of several attempts to understand their exceptionally large red shifts and also discussed the physical theories concerning the cause of the explosions which give rise to the quasars and to the tremendous energy output they require — questions which still remain unanswered. He stated, in concluding, that we may have to invoke certain aspects of the present theories of elementary particles in order to unravel these mysteries.

Professor Mercier, well known for his studies on the philosophical foundations of modern physics, critically examined the various attempts, such as that of Einstein, to formulate a unified field theory. Ramakrishnan and Raghavacharayulu described the group theoretical significance of the hierarchy of  $L$ -matrices and the  $\sigma$ -operation introduced earlier by Ramakrishnan, and Mathews spoke about quantizing the relativistic "Schrödinger type" equations for higher spin which were described in Volume VI of these proceedings. Horváth lectured on his contributions to a relativistic generalization of the kinetic theory of gases, in which the relativistic phase-space represents a geometrization of the dynamics of gases. Jochain's paper was devoted to a comprehensive review of both the variational and minimum principles in scattering theory and their utility in obtaining bounds on scattering length and phase shifts. Charpak presented the results of recent experiments at CERN on the absorption of pions by a pair of nucleons and discussed its significance for the study of nuclear structure. Rho analyzed muon capture as a means of probe nuclear structure.

In his lectures on Raikov systems, Williamson described how they “provide yet another instance, at a quite elementary level, of just how complicated a system the real numbers are.” With their origin in a study by D. A. Raikov of the structure of the measure algebra  $\mathfrak{M}(R)$  of real line  $R$ , the main interest of the subject is perhaps due to its bearing on the structure of  $\mathfrak{M}(R)$  or, more generally, of  $\mathfrak{M}(G)$ , where  $G$  is a locally compact Abelian topological group. Williamson also stressed the need for studying Raikov systems for their own sake, independent of any possible applications. The ideas and results described here are still in a state of development.

Shapiro proposed a formal definition of generalized analytic continuation and gave various examples of function classes which admit generalized analytic commutation in this sense. Later he discussed such topics as continuations by matching boundary values and the Laplace transform of almost periodic functions.

The other contributions on mathematical topics were by Eliezer on convex functions and related inequalities, Deo on functional differential equations, Nirmala Prakash on harmonic differential forms in a general manifold, Padmanabhan on developments in the theory of univalent functions, and Subrahmanyam on congruent embedding in Boolean vector spaces. The volume also includes a contribution on the construction of total programmed-motion systems by Galiullin.\*

*Alladi Ramakrishnan*

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\*The original manuscript in Russian was translated by the Publishers, Plenum Press, N. Y.

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# A General Outlook on the Quasars

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Since my main field of research is astrophysics, I suppose that when asking me to present this paper, Professor Ramakrishnan probably intended to stress the point that the Institute of Mathematical Sciences, which is so rapidly and vigorously developing from year to year, after having extended the domain of its activities from theoretical physics to pure and applied mathematics, is now just starting to include astrophysics. And therefore, the real meaning of my paper should be to show that this new extension is worthwhile and that astrophysics may actually provide the theoretical physicist with problems which are no less important, exciting, and fundamental as those related to elementary particles. For this reason, I have chosen as the subject of my paper the problem of the so-called quasi-stellar sources, or quasars, whose study appears to hold implications for cosmology and the type of model which we use to explain our world. Therefore, I shall try, in a summary of this field, to present its most outstanding features, and the important steps in our knowledge of the universe which have been accomplished by the radioastronomical techniques developed in the last decades, among which the discovery of the quasars appears actually to be one of the most significant.

As is well known, powerful radiotelescopes have permitted us to demonstrate the presence of sources in the sky emitting on the range of the cm wavelength, and measurements on several wavelengths have yielded information on their spectra. In most cases

these spectra are not thermal, but follow a power law of the type  $S_\nu \sim \nu^{-\alpha}$ , typical of the so-called synchrotron phenomenon, that is, deceleration of relativistic electrons moving in magnetic fields.

The question then was where are these fields and electrons generated?

An answer to it has been suggested by the galactic radio sources, especially the Crab nebula. This strong source is located in the exact position where a supernova exploded in the year 1054. Thus, we know that a radio source is what is left from a star's explosion about a thousand years after the outburst. This result, obtained on a stellar scale, has been extended and extrapolated to interpret also the extragalactic radio sources, most of which, as observation reveals, are connected to the central parts, or nuclei, of galaxies. We know today that many galaxies are radio sources, including ours; however, most of them are only weak sources (output between  $10^{38}$ – $10^{41}$  ergs/sec) and only few, mostly giant elliptical galaxies, are strong ones (output between  $10^{41}$ – $10^{44}$  ergs/sec). According to the previous extrapolation, radio sources in galaxies are now thought to be the remnants of explosions which have occurred in their nuclei, and a whole scale of different galaxy types are now interpreted as showing the different phases of such explosions:

*Initial phase:* A bright and concentrated nucleus, with little radio emission (Seyfert galaxies).

*Central phase:* Evidence of output of gases and material from the nucleus with increased radio emission (M 82).

*Final phase:* Characterized frequently by two radio sources situated generally on the normal to the plane of the galaxy; as the distance of the sources increases, the output of the radio power declines.

Moreover, there are several reasons to suppose that these explosions are probably recurrent, with a period of the order of  $10^6$ – $10^7$  years. It may also be remarked that they appear to occur only in the largest and brightest of the galaxies.

It is in such a general frame that the discovery of the quasi-stellar sources (QSS) has to be located.

As soon as very precise measurements concerning the exact position of radio sources became available, it was discovered by Sandage that some sources (3C 48, 196, 286), which had escaped identification with any known galaxy, were practically coincident

with point source images completely similar to stars. Hence, the name of QSS or quasars given to these sources. More cases were later gradually found, so that on the whole about 80 of such sources have now been recognized. Although they may differ in several details from each other, one may try to summarize their general characteristics as follows:

*Radio data:* For about half of the sources (57%) the radio spectrum follows the power law of the synchrotron type  $\nu^{-\alpha}$ . For about 25% the spectrum has an upward convex curvature, which is typical of the self-absorption phenomenon of the synchrotron radiation by the electrons themselves. The frequency corresponding to the maximum of the spectrum  $\nu_m$  depends on the magnetic field  $H$  and the angular diameter of the source  $\theta$  as  $\nu_m \sim (H^{1/2} \theta^{-2})^{2/5}$ . For a remaining 10%, the curvature is opposite (convex downward), and this corresponds to a more complex case to be mentioned later. The remaining 8% are still unclassified.

A further remark of interest is that the radio source is not always centered on the optical point source. There are several cases of double radio sources, more or less symmetrically disposed with respect to the optical image, as for galaxies. Moreover, one sometimes sees jets or wisps emanating from the point source and connected to the radio spectrum.

*Optical data:* The spectrum consists of a continuous background with broad emission lines. Only recently, Oke has succeeded in separating the contributions of these two spectra for some of the most luminous sources. It has thus turned out that the continuous spectrum may also be fitted by a kind of power law, either straight or with some curvature, and this again stresses that, as for radio spectrum, these spectra are not thermal but are compatible with being either synchrotron radiation or emission of quanta in free-free collisions. In some but not in all cases, it has been possible to fit both the optical and the radio spectra with a single power law. For what concerns the color, the main characteristic is an ultraviolet excess in respect to normal main sequence stars, to be found only in some peculiar stellar states as white dwarfs or old novae.

The emission line spectra have for a long time been a puzzle, as no one was able to identify these lines with those of any of the known elements. Finally, Schmidt succeeded in recognizing the Balmer series and several forbidden lines of different origin on the spectrum of 3C 273, with the assumption of a high red shift



$z = \Delta\lambda/\lambda$  of 0.158; and this was the key to the general understanding of the phenomenon. However, it turned out that the red shifts, required for the interpretation, were on the whole extremely high (up to  $z \simeq 2$ ), even with respect to the highest red shifts related to the expansion of the universe observed for the most remote galaxies ( $z \sim 0.5$ ).

Thus, it became evident that the outstanding feature of the quasars were their exceptionally large red shifts, and all the interest concentrated in trying to understand their origin.

A first assumption was that the red shift could be gravitational, a well-known consequence of general relativity being that the spectral lines of a source of mass  $M$  and radius  $R$  are shifted toward the red by an amount of

$$z = \frac{\Delta\lambda}{\lambda} = \frac{GM^2}{c^2 R}$$

(where  $G$  is the gravitation constant and  $c$  is the velocity of light), which may turn out to be very large if a high mass is concentrated in a small volume. In an important paper related to the two best observed cases, 3C 48 and 3C 273, Greenstein and Schmidt were able to show, by an argument which admits of no brief summary, that it was extremely unlikely, when assuming the quasars to be either dense stars or compact galaxies, that the gravitational effect could explain the red shift, and their conclusion was generally accepted.

The red shift had then to be thought as due to the Doppler effect, and the simplest assumption (as all observed shifts were red and none blue) was to suppose it was cosmological as for galaxies. It turned out on this line that the distances of 3C 48 and 3C 273 were of the order of 1000 Mpc and their absolute luminosity of  $\sim 10^{46}$  ergs/sec, that is, 100 times greater than for the most luminous galaxies and, therefore, by far the most luminous objects in the world.

The further consequences of the cosmological assumption were discussed by Greenstein and Schmidt,<sup>1</sup> and may be summarized as follows: The radius of the radio emitting region can be deduced to be of the order of 1000 pc (small in comparison with the radius of 15,000 pc of our galaxy). The seat however of the optical lines is much narrower and of the order of only 1–10 pc. Further, it is likely that the continuum is emitted by a still narrower region. The