L. RESHETOV

SELF-ALIGNING MECHANISMS

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Л. Н. Решетов

Самоустанавливающиеся механизмы

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L. RESHETOV

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The Russian Alphabet and Transliteration

А а	a	К к	k	Х х	kh
Б б	b	Л л	l	Ц ц	ts
В в	v	М м	m	Ч ч	ch
Г г	g	Н н	n	Ш ш	sh
Д д	d	О о	o	Щ щ	shch
Е е	e	П п	p	ъ	''
Ё ё	e	Р р	r	ы	y
Ж ж	zh	С с	s	ь	'
З з	z	Т т	t	Э э	e
И и	i	У у	u	Ю ю	yu
Й й	y	Ф ф	f	Я я	ya

The Greek Alphabet

Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Δ β	Beta	Κ ϰ	Kappa	Σ σ	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	ϒ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ ϑ	Theta	Π π	Pi	Ω ω	Omega

На английском языке

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FOREWORD

Statically indeterminate trusses have been long studied. It has been proved that it takes a redundant link for static indeterminacy to evolve. This is believed to be true of mechanisms as well. However, one encounters mechanisms with a redundant link but very rarely, and therefore their study has been considered unnecessary. It has been found, though, that static indeterminacy may take place in mechanisms having no redundant links, should their kinematic pairs impose redundant constraints, i.e. those which fail to reduce the mobility of the mechanism (at one time such constraints were also called passive). It appears that the majority of mechanisms used in machinery are subject to such constraints.

Redundant constraints do considerable harm. They call for higher accuracy in manufacture and this makes the machine considerably more costly, and more often than not such accuracy is altogether unattainable on acount of deformations. The load-bearing capacity and efficiency are reduced so the weight and size have to be increased. Therefore, in engineering design, the scientific selection of kinematic pairs (the connections of the links) must strive to eliminate redundant constraints. Mechanisms free of redundant constraints are self-aligning, which explains the title of this book. Its content is dedicated to the methods of analysis and design of such mechanisms and to their applications in various fields.

Section 1.11 was written by E. Y. Kachalova (Budyka); Sections 1.12 through 1.16 were written by N. E. Shamaidenko, Cand. Tech. Sc.; Section 4.13 was written by the author in collaboration with J. J. Gaipel; Section 5.13 was written by the author in collaboration with G. A. Chernova; Section 5.4 was written by the author in collaboration with L. L. Rusak, Cand. Tech. Sc.; the rest of the book was written by the title author with the assistance of N. L. Reshetov.

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L. N. Reshetov, D. Sc.,
Honorary Inventor of
the Russian Federation

Chapter 1

REDUNDANT CONSTRAINTS AND MOBILITIES IN MECHANISMS

1.1. General

In the quantity production of machines, in automotive industry in particular, the process of assembly is fairly simple, since it involves merely bringing together the components and fixing the threaded connections, with no manual adjustment required. In individual production of large-size machinery, however, notwithstanding multi-pass machining, the accuracy of the component parts more often than not is insufficient. Therefore, the assembling entails manual fitting which is extremely labor-consuming and far from readily susceptible to mechanization.

It is particularly difficult to attain the precise dimensions of links that are made up of several parts which, when connected, might have their tolerances added together beyond the permissible value. It is, therefore, very important to select the design of a mechanism so that the accuracy requirements, put before its links, should be relatively low. The said is true of statically determinate mechanisms, i.e. those devoid of redundant[1] (passive) constraints and having their links self-aligning.

Redundant (passive) constraints are those whose elimination would not step up the mobility of a mechanism.

The dimensions of links may vary throughout the service life on account of sagging of the foundation, of wearout and adjustments of clearance in joints, of elastic deformation (e.g. deflection of shafts), of thermal expansion, and also due to maintenance and assembling errors (for instance, if bearing inserts have been confused). The variation of the link dimensions would not affect a statically determinate mechanism. Hence, statically determinate mechanisms save labor and enhance the reliability.

Redundant constraints in a mechanism are harmful, for they increase the labor consumption in the manufacture and operation of mechanisms and affect their reliability.

---

[1]We will use here the expression "redundant constraints" (suggested by N. I. Kolchin) as conveying their true essence.

In a kinematic pair redundant constraints are harmless since in most cases the pair can be fabricated with adequate accuracy. Examples of such pairs are a splined joint made by broaching the hole and milling the shaft with a hob, gears with the teeth engagement factor in excess of 1.0 (modern techniques of making teeth profiles enable simultaneous operation of two pairs of teeth, a great engagement factor enhancing the gear performance), and an antifriction bearing.

The performance of kinematic pairs can be enhanced, however, by other means. Thus, rattling in four-stroke diesel engines can be kept under control by observing adequate clearances in connecting-rod ends. In two-stroke diesels, where forces in the ends do not alternate, the clearances can be not as strict for they do not influence the rattling. Unfortunately, this advantage of two-stroke diesels is not shared by two-stroke carburettor engines with their throttle control.

The required precision of gears, the dynamic loads and noise are dependent on the peripheral velocity. In some planetary gearing patterns the velocity is very high. It may be significantly reduced, however, by appropriate selection of the design, which is of essential importance for high-speed mechanisms.

When toothed and worm gearings are arranged in series, the high-speed stage should be the worm one, and the low-speed stage should be the toothed gearing. This provides a relatively low peripheral velocity of the toothed gearing, and the latter may be manufactured to a lesser precision.

Statically determinate and indeterminate systems associated with trusses have been thoroughly studied. It has been found that statically determinate systems do not yield significant advantages. The dimensions of the bars or rods are always fitted to suit job by drilling-out (in riveted structures), or by welding the members (in welded ones). Therefore, a statically determinate system associated with a truss would not bring down the accuracy requirements. One practically never encounters a case where bars are expected to perform at temperatures differing by hundreds of degrees. With the propagation of welded structures the advantages of a statically indeterminate system become more pronounced. With a great number of rivets, a loose one would affect the load-bearing capacity of a truss but slightly, whereas a loose weld would impair this capacity significantly greater. This is particularly true of a statically determinate truss where the load-bearing capacity can be reduced to the critical value.

The foregoing refers to the inner static indeterminacy of a truss. There also exists the outer static indeterminacy as, for example, in a multisupport truss. Such a truss is responsive to various degrees of sagging of the foundations of its supports. To avoid this, statically determinate Herber's and Wichert's designs are used in bridge construction.

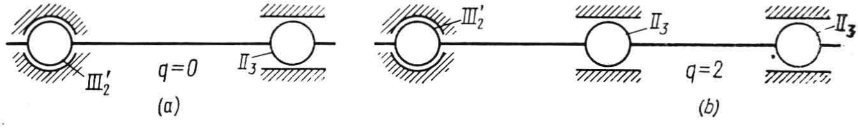

Fig. 1.1. Diagram of shaft bearings
(*a*) correct; (*b*) wrong

Now and again, mechanisms free of redundant constraints were developed by design engineers ignorant of the theory of the structure of mechanisms; more often than not they failed to eliminate all the redundant constraints.

Let us consider the advantages of mechanisms devoid of redundant constraints in two simplified exemplary cases. In a common slider-crank mechanism the lengths of the members may vary from the nominal values by several millimeters. This is used in practice for adjusting the spacing of the piston and the cover by setting an insert or spacer between the body and the end of the connecting-rod. However, in a twin mechanism, where two connecting-rods are associated with the common slider, the tolerances on the lengths of the members are measured not in millimeters, but in microns. It also takes great accuracy to ensure parallelism of the cranks.

Let us take for our second example a shaft supported by two bearings (Fig. 1.1*a*). If the bearings are made as shown in this drawing, they may be set with fairly broad tolerances (up to several millimeters). If the shaft is supported by three bearings (Fig. 1.1*b*), the required setting accuracy will be in the micron range, irrespective of the construction of the bearings.

Tolerances on the links depend on whether the mecanism is statically determinate. When the mechanism is statically determinate, the dimensions of the links all but fail to influence the forces being transmitted, and the tolerances on their dimensions may be loose, which is always desirable. With the statically indeterminate mechanism, the forces being transmitted depend on the deformation of the links. The deformation being very small (with the stress of the material being 200 kg(f)/cm², which is often the case in practice, considering longitudinal bending, deformation of steel with $E = 2 \times 10^6$ is 0.1 mm per metre), the dimensions of the members are to be maintained with a great accuracy, lest the members intended for joint performance may strongly interfere each other in operation (in some cases the very assembling of the mechanism might prove impossible). The friction would grow considerably, and the efficiency factor would diminish. Thus, it is ill-advisable to use statically indeterminate mechanisms.

Consequently, it can be stated that for a mechanism provided with broad tolerances on the dimensions of its members to perform reliably, it should have no redundant constraints. Exceptions may be solely the cases where the links incorporate springs, or where the links are intended to work in bending with considerable deformation.

The harmful role played by redundant constraints in steam boilers is commonly known. Of particularly ill fame in this respect was the steam engine boiler. Engine-drivers were well aware of the fact that mere blow-holes in the furnace (with the grate exposed) resulted in smoke-tube leaks. This was caused by streams of the cold air reaching some of the smoke-tubes and bringing about their cooling-down, and, hence, their contraction leading to leaks. Straight smoke and flue tubes set into two rigid tube sheets present a system that is multiply statically indeterminate. As their thermal duty, and, consequently, their heat expansion cannot be the same, great strains are developed in the tubes, causing their leaking in the areas where they are secured by expansion or flaring-out. A similar phenomenon was observed in Garbe's stationary vertical water-tube boilers.

For this reason straight tubes have been replaced in present-day boilers with bent ones whose flexibility does away with the strains causing leaks. It goes without saying that even some of the tubes could not be left to remain straight.

For the same reason, V. G. Shukhov's water-tube boilers were rejected due to their straight tubes, although they had come as a breakthrough in boiler engineering and had been widely used for many years.

The first constructions of machinery had a considerable number of redundant constraints. Those were the times when the consumption of labor and time by their manufacture was of small importance. Then came the period of gradual introduction of mechanisms free of redundant constraints (Sellers's bearings, self-aligning antifriction bearings, cylindrical crossheads, etc.). Redundant constraints had been in abundance in steam locomotives, but the trend toward reducing their number had made itself seen likewise long ago (Tate's hinged connections in boilers, axle boxes of the Tsar's system, the Hugyins ball inserts incorporated by prominent designer A. Y. Malokhovski in the "C" Series of steam locomotives). However, redundant constraints linger today in many a machine and mechanism for no reason whatsoever, complicating as they do the manufacture and operation and rising the costs involved.

It is particularly important to avoid redundant constraints in the design of mechanisms, in which some of the members are expected to operate under elevated temperatures (e.g. the regulating and control mechanisms of steam and gas turbines), for this would enhance their performance reliability and reduce friction. A major advantage of self-aligning mechanisms is the fact that the value and location of forces are independent of the tolerances, clearances and preloads.

It enables one to create mechanisms with uniform loading of the kinematic pairs—of bearings, gear teeth, brake shoes, etc. The load-bearing capacity and durability rise accordingly. Design engineers should be also aware of the fact that the design analysis of self-aligning mechanisms involves no deformation equations which are hard to solve, so the designing process is facilitated.

For numerous mechanisms, the number of redundant constraints equals that of dimensions requiring stringent tolerances; sometimes, the last-mentioned number is considerably greater than the number of redundant constraints. Therefore, it is the very presence of redundant constraints in a mechanism that is of concern, not their amount.

The examples quoted above to show the harm done by redundant constraints enable us to devise two major rules of designing self-aligning mechanisms:

—each shaft should be supported by no more than two bearings;

—twin or double mechanisms should be avoided at all costs, i.e. with the input and output links being the same, there should be no two transmitting mechanisms between them (an exception can be made solely in cases where an equalizing device is provided).

1.2 Checking Mechanisms for Redundant Constraints

It has been explained above that linear and angular deviations in the sizes of the members would not affect the performance of a mechanism free of redundant constraints. This gives rise to a general concept of designing rational mechanisms: a rational mechanism should be assembled without preloads and interference fits even if the dimensions of the members deviate from nominal ones (both linear and angular). In other words, variation in the angular and linear dimensions of the members in a rational mechanism should cause no preload (meaning variation in the linear and angular dimensions of the members alone and not of the kinematic pairs).

The rule can be also applied as follows. Let us cut one of the links. The kinematics of a rational mechanism should provide the possibility of reconnecting it without a preload by bringing together the halves of the cut link by moving in the three directions and by their relative rotation around the three orthogonal axes without deformation of the links.

When the method suggested by S. A. Popov is used, the process of assembling the mechanisms should be considered. The kinematic pair in the loop, which is the last to be assembled, should be considered, and the linear bringing together of the links should be followed-up in the directions of the three orthogonal axes and by rotation about the three axes. The amount of these motions does not include the relative motion of the links yielded by the mobility of the pair itself, but they should be considered while evaluating the mobility remaining in the mechanism.

If approaching displacement of links can be performed by moving one link to another or vice versa, then one of the motions is used in assembling of the mechanism (to close the loop), while the other remains in the mechanism as a mobility. If, on the other hand, some of the motions involved in the bringing together would not be attained by the mobilities of the kinematic pairs, but solely at the cost of deformation of the members, this indicates the existence of redundant constraints.

Let us consider it in case of crank-and-rocker mechanism $OABC$ of Fig. 1.2 (the pairs are denoted by the numerator) having two rotatable pairs O and C and two spherical-joint pairs A and B. The pair to be assembled last in this system is the one denoted B.

Let us consider the approaching motions of the links in assembling:

along x-axis—by rotating the crank AO and the rocker BC; there remains the mobility—the main one in the mechanism;

along y-axis—by rotating the connecting-rod AB about the spherical-joint pair A;

along z-axis—by rotating the connecting-rod AB about the spherical-joint pair A;

about x-axis—by rotating the connecting-rod AB about the spherical-joint pair A. When one considers the angular mobility of the pair B itself, there remains the local mobility, *viz.* the rotation of the connecting-rod about the axis AB;

about y-axis—attainable owing to the mobility of the pair B itself;

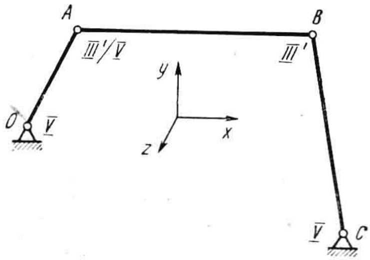

Fig. 1.2. Diagram of a four-joint mechanism for detecting preloads in assembling

about z-axis—attainable owing to the mobility of the pair B itself.

For comparison sake, let us discuss a similar mechanism where one of the spherical joints—the pair A—has been replaced by a revolute or turning joint

The bringing together of the links in assembling is achieved:

along x-axis—by rotating the crank OA and the rocker BC; the mobility remains—the main one in the mechanism;

along y-axis—by rotating the connecting-rod AB about the pair A;

along z-axis—at the cost of prestraining and distorting the links.

No mobilities are required about x, y and z axes, since they are attained by the mobilities of the pair itself.

It appears expedient from the said that the last pair should be the one with the least amount of the constraint conditions, so as to reduce the number of motions involved in the assembling. An alternative way of detecting redundant constraints during assembling is the method of counting angular mobilities which is advisable in case of toothed gearings.

In the meshing of cylindrical teeth, to provide for a linear contact by self-aligning with a loading uniformly spread along the length of the tooth, one angular mobility per each engagement is required.

The number of angular mobilities in the mechanism being short of the number of engagements means that redundant constraints are present, interfering with the uniform distribution of the load longitudinally of the tooth. This will be dealt with in more detail in

Section 4. Another solution of the same problem has been suggested by A. F. Popov [29].

Besides, a mechanism may be checked for the presence of redundant constraints by counting them with the use of structural formulas. It is known that the authors of structural formulas have meant them for determination of the mobility of a mechanism, assuming the number of redundant constraints to be taken into consideration as known. The mobility of the mechanism is taken to be the number of independent parameters which should be specified to define the motion. In practice, however, it is far more easier to evaluate the mobility of a mechanism by its external inspection which is more simple and reliable than using a structural formula. It is, thus, more expedient to evaluate the mobility and introduce the value thus obtained into a structural formula, to find from the latter the number of redundant constraints. An error in evaluating the mobility is easily detectable by an unseemly number of redundant constraints (e.g. a negative one).

There are general and local (passive) mobilities of a mechanism. Let us call a mobility which does not influence the mobility of a mechanism, as a whole, a local mobility. Local mobilities are found in rollers (on account of their slipping), blocks, pulleys, floating bushes and pins, connecting-rods and cross-heads with spherical heads, and also in the races of antifriction bearings, if they are assembled with a slide fit (when the bearings are considered). Balls in a chute have three local mobilities in addition to the slip, i.e. rotation about the three orthogonal axes. Links with a local mobility (floating pins and bushes, flat followers in cam mechanisms) are sometimes used to provide for uniform wear of kinematic pairs.

Local mobility can be also found in a group of members. In most cases this should be avoided. We will discuss it in more detail below.

To derive a structural formula, let us make use of the classes of the kinematic pairs. We define the class of a kinematic pair as the number of constraints introduced by a given pair. The constraints in a kinematic pair are thus the restrictions to relative motion along a given axis or angular displacements about the same axis.

A limited linear displacement implies the necessity of transmitting an effort in the kinematic pair between its links, while a limited angular displacement implies the necessity of transmitting a torque between the links of the pair. Therefore, the notion of "kinematic constraints" in a structure has as counterparts the notions of "transmitted forces" or "transmitted torques" in dynamics.